Operations on Intuitionistic Trapezoidal Fuzzy Numbers using Interval Arithmetic

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Abstract. In this paper operations based on \((\alpha, \beta)\) — cut for intuitionistic trapezoidal fuzzy numbers are defined as addition, multiplication, scalar multiplication, subtraction etc., including exponentiation, extracting nth root, and taking logarithm using the degree of acceptance and rejection.

Keywords: Intuitionistic trapezoidal fuzzy number, \((\alpha, \beta)\) — cuts.

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1 Introduction
Transportation problem is a special kind of linear programming problem in which goods are transported from a set of sources to destinations with respect to the supply and demand respectively, such that the total cost of transportation is minimized. Intuitionistic fuzzy set (IFS) is one of the components of fuzzy set theory [10] and also introduced by Atanassov [1,2]. IFS is exaggerated by a degree of acceptance and a degree of rejection function so that the sum of both values is less than one [1]. Mahapatra and Roy [6] defined the triangular intuitionistic fuzzy number (TrIFN) and trapezoidal intuitionistic fuzzy number (TrIFN) and their arithmetic operations based on intuitionistic fuzzy extension principle using \((\alpha, \beta)\) — cuts method. Li [4] developed the ratio ranking method for ranking trapezoidal intuitionistic fuzzy numbers.

This paper is organized as follows, in section 2, the representation of trapezoidal intuitionistic fuzzy numbers and \((\alpha, \beta)\) — cut sets. In section 3, operations including exponentiation, logarithms and nth root of trapezoidal intuitionistic fuzzy number.

2 Preliminaries
Definition 2.1. Let X be the universal set. An intuitionistic fuzzy set (IFS) \(\tilde{A}^I\) in X is given by
\[\tilde{A}^I = \{(x, \mu_{\tilde{A}^I}(x), \nu_{\tilde{A}^I}(x) / x \in X\}\] where the function \(\mu_{\tilde{A}^I}(x), \nu_{\tilde{A}^I}(x)\) define the degree of acceptance and degree of rejection of the element \(x \in X\) to the set \(\tilde{A}^I\), for every \(x \in X\),
\[0 \leq \mu_{\tilde{A}^I}(x) + \nu_{\tilde{A}^I}(x) \leq 1.\]
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**Definition 2.2.** Let $\tilde{A}$ be the Trapezoidal intuitionistic fuzzy number with parameters $b_1 \leq a_1 \leq b_2 \leq a_2 \leq a_3 \leq b_3 \leq a_4 \leq b_4$ which is denoted by $\tilde{A} = (b_1, a_1, b_2, a_2, a_3, b_3, a_4, b_4).$ The degree of acceptance and degree of rejection functions are defined as

$$
\mu_A(x) = \begin{cases} 
\frac{x-a_1}{a_2-a_1}, & a_1 \leq x \leq a_2 \\
1, & a_2 \leq x \leq a_3 \text{ and } a_3 \leq x \leq a_4 \\
\frac{x-a_4}{a_4-a_3}, & a_3 \leq x \leq a_4 \\
0, & x > a_4 
\end{cases}
$$

$$
\gamma_A(x) = \begin{cases} 
\frac{x-b_1}{b_2-b_1}, & b_1 \leq x \leq b_2 \\
0, & b_2 \leq x \leq b_3 \\
\frac{x-b_4}{b_4-b_3}, & b_3 \leq x \leq b_4 \\
1, & x > a_4 
\end{cases}
$$

**Definition 2.3.** The $(\alpha, \beta)$-cut trapezoidal intuitionistic fuzzy numbers is defined as usually, by $\tilde{A}^{i,\alpha}_\beta = \left\{ [\tilde{A}_1^i(\alpha), \tilde{A}_2^i(\alpha)], [\tilde{A}_1^{i,\alpha}_\beta(\beta), \tilde{A}_2^{i,\alpha}_\beta(\beta)] \right\}, \alpha + \beta \leq 1, \alpha, \beta \in [0,1]$ where $\tilde{A}_1^i(\alpha) = a_1 + \alpha(a_2 - a_1), \tilde{A}_2^i(\alpha) = a_3 - \alpha(a_4 - a_3)$ and $\tilde{A}_1^{i,\alpha}_\beta(\beta) = b_2 - \beta(b_3 - b_1), \tilde{A}_2^{i,\alpha}_\beta(\beta) = b_3 + \beta(b_4 - b_3)$.

**Definition 2.4.** A $\alpha$-cut sets of $\tilde{A} = (b_1, a_1, b_2, a_2, a_3, b_3, a_4, b_4)$ is a crisp subset of $R$, which is defined as $\tilde{A}^i_\alpha = \{x/\mu_{\tilde{A}}(x) \geq \alpha \}$. Using $\mu_{\tilde{A}}(x)$ and definition of $\alpha$-cut it follows that $\tilde{A}^i_\alpha$ is a closed interval, denoted by $\tilde{A}^i_\alpha = [L_\alpha(\tilde{A}^i), R_\alpha(\tilde{A}^i)] = [a_1 + \alpha(a_2 - a_1), a_4 - \alpha(a_4 - a_3)]$.

**Definition 2.5.** A $\beta$-cut sets of $\tilde{A} = (b_1, a_1, b_2, a_2, a_3, b_3, a_4, b_4)$ a crisp subset of $R$, which is defined as $\tilde{A}^i_\beta = \{x/\gamma_{\tilde{A}}(x) \geq \beta \}$. Using $\gamma_{\tilde{A}}(x)$ and definition of $\beta$-cut, it follows that $\tilde{A}^i_\beta$ is a closed interval, denoted by $\tilde{A}^i_\beta = [L_\beta(A), R_\beta(A)] = [b_2 + \beta(b_3 - b_2), a_4 - \beta(b_4 - b_3)]$.

3 Operations on TrIFNs

**Theorem 3.1.** If $\tilde{A} = (b_1, a_1, b_2, a_2, a_3, b_3, a_4, b_4)$ and $\tilde{B} = (b_1', a_1', b_2', a_2', a_3', b_3', a_4', b_4')$ are two TrIFNs, then $\tilde{A} + \tilde{B} = (b_1 + b_1', a_1 + a_1', b_2 + b_2', a_2 + a_2', a_3 + a_3', b_3 + b_3', a_4 + a_4', b_4 + b_4')$ is also a TrIFN.

**Proof:** For every $\alpha, \beta \in [0,1],$

$$
\tilde{A}^{i,\alpha}_\beta + \tilde{B}^{i,\alpha}_\beta = \left[ a_1 + a_1' + \alpha((a_2 - a_1) + (a_2' - a_1')), a_3 + a_3' - \alpha(a_4' - a_3') + (a_4 - a_3) \right] \quad (3.1)
$$

$$
\tilde{A}^{i,\beta}_\beta + \tilde{B}^{i,\beta}_\beta = \left[ b_2 + b_2' - \beta((b_2 - b_1) + (b_2' - b_1')), b_3 + b_3' + \beta((b_3 - b_2) + (b_3' - b_2')) \right] \quad (3.2)
$$

x = a_1 + a_1' + \alpha((a_2 - a_1) + (a_2' - a_1')) and

$$
x = a_4 + a_4' - \alpha((a_4 - a_3') + (a_4 - a_3))
$$

which gives, $\mu_{\tilde{A} + \tilde{B}}(x) = 

$$
\begin{cases} 
\frac{x-(a_1+a_1')}{(a_2+a_2')-(a_1+a_1')}, & a_1 + a_1' \leq x \leq a_2 + a_2' \\
1, & a_2 + a_2' \leq x \leq a_3 + a_3' \\
\frac{x-(a_1+a_1')}{(a_4+a_4')-(a_4+a_4')}, & a_3 + a_3' \leq x \leq a_4 + a_4' \\
0, & otherwise
\end{cases}
$$

(3.3)
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\[ x = b_2 + b_2' - \beta ((b_2 - b_2') + (b_2' - b_2')) \]
and
\[ x = b_3 + b_3' + \beta ((b_4 - b_3) + (b_4' - b_3')) \]

which gives,

\[ y_{A+B}(x) = \begin{cases} 
\frac{x - (b_2 + b_2')}{(b_1 + b_1') - (b_2 + b_2')} , & b_1 + b_1' \leq x \leq b_2 + b_2' \\
0 , & b_2 + b_2' \leq x \leq b_3 + b_3' \\
\frac{(b_4 + b_4') - (b_3 + b_3')}{(b_4 + b_4') - (b_3 + b_3')} , & b_3 + b_3' \leq x \leq b_4 + b_4' \\
1 , & \text{otherwise}
\end{cases} \tag{3.4} \]

So \( A + B \) represented by (3.3) and (3.4) is a trapezoidal shaped IFN. It can be approximated to TrIFN. Thus we have \( \hat{A} + \hat{B} = (b_1 + b_1', a_1 + a_1', b_2 + b_2', a_2 + a_2', a_3 + a_3', b_3 + b_3', a_4 + a_4', b_4 + b_4') \) is also a TrIFN.

**Theorem 3.2.** If \( \hat{A} = (b_1, a_1, b_2, a_2, a_3, b_3, a_4, b_4) \) and \( \hat{B} = (b_1', a_1', b_2', a_2', a_3', b_3', a_4', b_4') \) are two TrIFNs, then \( \hat{A} - \hat{B} = [b_1 - b_4', a_1 - a_4', b_2 - b_3', a_2 - a_3', a_3 - a_2', b_3 - b_2', a_4 - a_1', b_4 - b_1'] \) is also a TrIFN.

**Proof.** For every \( \alpha, \beta \in [0, 1] \). Let the additive image of \( \hat{B}_\alpha \) and \( \hat{B}_\beta \) be \( -\hat{B}_\alpha = [-a_4' + \alpha (a_4' - a_3'), -a_1' - \alpha (a_2' - a_1')] \) and \( \hat{B}_{\beta} = [-b_3' - \beta (b_3' - b_2'), -b_2' \beta (b_2' - b_1')] \).

\[ \hat{A}_\alpha - \hat{B}_\alpha = (a_1 - a_4') + \alpha [(a_2 - a_1) + (a_4' - a_3')], (a_4 - a_1') - \alpha [(a_4 - a_3) + (a_2' - a_1')] \tag{3.5} \]

\[ \hat{A}_\beta - \hat{B}_{\beta} = (b_2 - b_2') - \beta [(b_2 - b_2') - (b_1 - b_1')], (b_3 - b_2') + \beta [(b_4 - b_1') - (b_3 - b_2')] \tag{3.6} \]

\[ x = (a_1 - a_4') + \alpha [(a_2 - a_1) + (a_4' - a_3')] \quad \text{and} \quad \mu_{A-B}(x) = \begin{cases} 
\frac{x - (a_1 - a_4')}{(a_2 - a_2') - (a_1 - a_4')} , & (a_1 - a_4') \leq x \leq (a_2 - a_2') \\
1 , & (a_2 - a_2') \leq x \leq (a_3 - a_2') \\
\frac{(a_4 - a_1') - x}{(a_4 - a_1') + (a_3 - a_2')} , & (a_3 - a_2') \leq x \leq (a_4 - a_1') \\
0 , & \text{otherwise}
\end{cases} \tag{3.7} \]

\[ x = (b_2 - b_2') - \beta [(b_2 - b_2') - (b_1 - b_1')] \quad \text{and} \quad \mu_{A-B}(x) \]

Now expressing \( \beta \) in terms of \( x \) and setting \( \beta = 0 \) and \( \beta = 1 \) in (3.6) which gives,
Proof. For every $a, b \in [0, 1]$. To calculate the multiplication of TrIFNs $\tilde{A}$ and $\tilde{B}$, we first multiply the $(a, b)$ -cuts of $\tilde{A}$ and $\tilde{B}$ using interval arithmetic.

\[
\tilde{A}_{\alpha} \ast \tilde{B}_{\beta} = \left[ a_1 + \alpha(a_2 - a_1) \ast (a'_1 + \alpha(a'_2 - a'_1)), (a_4 - \alpha(a_4 - a_3)) \ast (a'_4 - \alpha(a'_4 - a'_3)) \right]
\]

\[
\tilde{A}_{\beta} \ast \tilde{B}_{\alpha} = \left[ (b_2 - \beta(b_2 - b_1)) \ast (b'_2 - \beta(b'_2 - b'_1)), (b_3 + \beta(b_4 - b_3)) \ast (b'_3 + \beta(b'_4 - b'_3)) \right]
\]

\[
x = \begin{cases} 
    [(a_2 - a_1)(a'_2 - a'_1)]a^2 + (a_1(a_2 - a_1) + (a'_1(a'_2 - a'_1))a + a_1a_4' & (3.9) \\
    [(a_4 - a_3)(a'_4 - a'_3)]a^2 - [(a'_4 - a'_3) + (a'_1(a'_2 - a'_1))a + a_4'a_4'] & (3.10)
\end{cases}
\]

\[
\mu_{A \ast B}(x)
\]

\[
\begin{align*}
\frac{x - (b_2 - b_1')}{(b_1 - b_1') - (b_2 - b_1')} , & \quad (b_1 - b_1') \leq x \leq (b_2 - b_1') \\
0 , & \quad (b_2 - b_2') \leq x \leq (b_3 - b_2') \\
\frac{(b_4 - b_1')(b_3 - b_2')}{(b_3 - b_2') - (b_4 - b_1')} , & \quad (b_3 - b_2') \leq x \leq (b_4 - b_1') \\
1 , & \quad \text{otherwise}
\end{align*}
\]

So $A - B$ represented by (3.7) and (3.8) is a trapezoidal shaped IFN. It can be approximated to TrIFN. Thus we have $\tilde{A} - \tilde{B} = [b_1 - b_4', a_1 - a_4', b_2 - b_4', a_2 - a_4', a_3 - a_2', b_3 - b_2', a_4 - a_3']$ is also a TrIFN.

Theorem 3.3. If $\tilde{A} = (b_1, a_1, b_2, a_2, a_3, b_3, a_4, b_4)$ and $\tilde{B} = (b_1', a_1', b_2', a_2', a_3', b_3', a_4', b_4')$ are two TrIFNs, then $\tilde{A} \ast \tilde{B} = (b_1, a_1', b_2, a_2, a_3, b_3, a_4, b_4, b_4')$ is also a TrIFN.

Proof. Let $\tilde{A} = (b_1, a_1, b_2, a_2, a_3, b_3, a_4, b_4, b_4')$ be TrIFN and $reR$

So $A * B$ represented by (3.11) and (3.12) is a trapezoidal shaped IFN. It can be approximated to TrIFN. Thus we have $\tilde{A} * \tilde{B} = (b_1, a_1', b_2, a_2, a_3, b_3, a_4, b_4, b_4')$ is also a TrIFN.

Theorem 3.4. Let $reR$. If $\tilde{A} = (b_1, a_1, b_2, a_2, a_3, b_3, a_4, b_4)$ is a TrIFN then $S = r \tilde{A}$ is also a TrIFN and is given by $D = r \tilde{A} = (rb_1, ra_1, rb_2, ra_2, ra_3, rb_3, ra_4, rb_4, r > 0$

Proof. Let $\tilde{A} = (b_1, a_1, b_2, a_2, a_3, b_3, a_4, b_4)$ be TrIFN and $reR$
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Case (i) $r > 0$
To calculate the scalar multiplication of TrIFN $\tilde{A}^i$ we first multiply $r$ and $(\alpha, \beta)$-cuts of $\tilde{A}^i$ gives

\[ r \tilde{A}^i = \begin{cases} \left[r a_1 + a(r a_2 - r a_1), r a_4 - a(r a_4 - r a_3)\right] & \text{if } r > 0 \end{cases} \]

Case (ii) $r < 0$
Similarly, it can be shown that $S = r\tilde{A}^i$ if $r < 0$ where

\[ \mu_S(x) = \begin{cases} \frac{x - ra_1}{ra_2 - ra_1}, & ra_1 \leq x \leq ra_2 \\ 1, & ra_2 \leq x \leq ra_3 \\ \frac{x - ra_4}{ra_3 - ra_4}, & ra_3 \leq x \leq ra_4 \\ 0, & \text{otherwise} \end{cases} \]

Thus, scalar multiplication can be given as $S = \left[r b_1, r b_2, r a_2, r a_3, r b_3, r a_4, r b_4\right]$ which is also a TrIFN when $r > 0$.

Proof. To calculate division of TrIFNs $\tilde{A}^i$ and $\tilde{B}^i$ we first divide the $(\alpha, \beta)$-cuts of $\tilde{A}^i$ and $\tilde{B}^i$ using interval arithmetic.

\[
\begin{align*}
\tilde{A}^i_\alpha &= \frac{a_1 + a(a_2 - a_1), a_4 - a(a_4 - a_3)}{a_1 + a(a_2 - a_1), a_4 - a(a_4 - a_3)} \\
\tilde{A}^i_\beta &= \frac{b_2 - \beta(b_2 - b_1), b_3 + \beta(b_3 - b_2)}{b_2 - \beta(b_2 - b_1), b_3 + \beta(b_3 - b_2)} \\
\end{align*}
\]

This is an excerpt from a document discussing operations on intuitionistic trapezoidal fuzzy numbers using interval arithmetic. The text details how to perform scalar multiplication and division of TrIFNs. It includes mathematical expressions and cases for scalar multiplication and division, with conditions for when $r > 0$ and $r < 0$. The proof involves calculating the division of two TrIFNs using interval arithmetic.
Proof.

\[
\mu_{A_i/B_i}(x) = \begin{cases} 
\frac{x a_4' - a_1}{(a_2 - a_1) + x(a_4' - a_1')}, & \frac{a_1}{a_4'} \leq x \leq \frac{a_2}{a_3} \\
1, & \frac{a_2}{a_3} \leq x \leq \frac{a_3}{a_2} \\
\frac{a_4 - xa_4'}{x(a_4' - a_1') + (a_4 - a_3)}, & \frac{a_3}{a_2} \leq x \leq \frac{a_4}{a_1'} \\
0, & \text{otherwise}
\end{cases} 
\] (3.17)

Let \(x = \frac{b_2 - \beta(b_2 - b_1)}{b_2' - \beta(b_2' - b_1')}\) and \(x = \frac{b_3 + \beta(b_3 - b_2)}{b_3' - \beta(b_3' - b_1')}\)

\[
\gamma_{A_i/B_i}(x) = \begin{cases} 
\frac{b_2 - xb_2'}{x(b_2' - b_1) + (b_2 - b_1)}, & \frac{b_1}{b_4'} \leq x \leq \frac{b_2}{b_3} \\
0, & \frac{b_2}{b_3} \leq x \leq \frac{b_3}{b_2'} \\
\frac{xb_3' - b_3}{x(b_3' - b_1') + (b_3 - b_2)}, & \frac{b_3}{b_2'} \leq x \leq \frac{b_4}{b_1'} \\
1, & \text{otherwise}
\end{cases} 
\] (3.18)

So \(\bar{A}/\bar{B}\) represented by (3.17) and (3.18) is a trapezoidal shaped IFN. It can be approximated to TrIFN, \(\bar{A}/\bar{B} = \left(\frac{b_1}{b_4'}, \frac{a_1}{a_4'}, \frac{b_2}{b_3'}, \frac{a_2}{a_3'}, \frac{b_3}{b_2'}, \frac{a_3}{a_2'}, \frac{b_4}{b_1'}, \frac{a_4}{a_1'}\right)\).

**Theorem 3.6.**

If \(\bar{A} = (b_1, a_1, b_2, a_2, a_3, b_3, a_4, b_4)\) is a positive TrIFN, then

\[
\bar{A}^{-1} = \left(\frac{1}{b_4'}, \frac{1}{a_4'}, \frac{1}{b_3'}, \frac{1}{a_3'}, \frac{1}{b_2'}, \frac{1}{a_2'}, \frac{1}{b_1'}, \frac{1}{a_1'}\right) 
\]

**Proof.** Let \(\bar{A} = (b_1, a_1, b_2, a_2, a_3, b_3, a_4, b_4)\) be a positive TrIFN. To calculate inverse of TrIFN \(\bar{A}\), we first take the inverse of the \((\alpha, \beta)\)-cuts of \(\bar{A}\) using interval arithmetic.

\[
\begin{align*}
\frac{1}{\bar{A}_\alpha} &= \left[\frac{1}{a_4 - a(a_4 - a_3)}, \frac{1}{a_4 + a(a_4 - a_3)}\right] = \left[\frac{1}{a_4 - (a_4 - a_3)}, \frac{1}{a_4 + (a_4 - a_3)}\right] \\
\frac{1}{\bar{A}_\beta} &= \left[\frac{1}{b_2 - \beta(b_2 - b_1)}, \frac{1}{b_2 + \beta(b_2 - b_1)}\right] = \left[\frac{1}{b_2 - \beta(b_2 - b_1)}, \frac{1}{b_2 + \beta(b_2 - b_1)}\right]
\end{align*} 
\] (3.19) (3.20)

\[
x = \frac{1}{a_4 - a(a_4 - a_3)} \quad \text{and} \quad x = \frac{1}{a_4 + a(a_4 - a_3)} \quad \left(\frac{a_4 - a(a_4 - a_3) - 1}{x(a_4 - a_3)}, \frac{1}{a_4} \leq x \leq \frac{1}{a_3}\right) \\
\mu_{\bar{A}^{-1}}(x) = \begin{cases} 
\frac{1}{a_4 - a(a_4 - a_3)} \quad \left(\frac{1}{a_4} \leq x \leq \frac{1}{a_3}\right) \\
1, \quad \left(\frac{1}{a_3} \leq x \leq \frac{1}{a_2}\right) \\
\frac{1}{a_2} \quad \left(\frac{1}{a_2} \leq x \leq \frac{1}{a_1}\right) \\
0, \quad \text{otherwise}
\end{cases} 
\] (3.21)

\[
x = \frac{1}{b_2 + \beta(b_2 - b_1)} \quad \text{and} \quad x = \frac{1}{b_2 - \beta(b_2 - b_1)}
\]
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\[ y'_{\bar{A}}(x) = \begin{cases} \frac{1-xb_2}{b_4-b_3} & \frac{1}{b_4} \leq x \leq \frac{1}{b_3} \\ 0 & \frac{1}{b_3} \leq x \leq \frac{1}{b_2} \\ \frac{xb_2-1}{b_2-b_1} & \frac{1}{b_2} \leq x \leq \frac{1}{b_1} \\ 1 & \text{otherwise} \end{cases} \] (3.22)

So \( \bar{A}^{\pm} \) represented by (3.21) and (3.22) is a trapezoidal shaped IFN. It can be approximated to TRIFN \( \bar{A}^{\pm} = \left( \frac{1}{b_4}, \frac{1}{b_3}, \frac{1}{b_2}, \frac{1}{b_1}, \frac{1}{b_4}, \frac{1}{b_3} \right) \).

**Theorem 3.7.** If \( \bar{A} = (b_1, a_1, b_2, a_2, a_3, b_3, a_4, b_4) \) is a trapezoidal intuitionistic fuzzy number then \( e^{\bar{A}} = (e^{b_1}, e^{a_1}, e^{b_2}, e^{a_2}, e^{a_3}, e^{b_3}, e^{a_4}, e^{b_4}) \) is also a TrIFN.

**Proof.** To calculate exponential of the TrIFN \( \bar{A} \) we first take the exponential of the \((\alpha, \beta)\) -cut of \( \bar{A} \) using interval arithmetic.

\[ e^{\bar{A}}^\alpha = e^{a_1+\alpha(a_2-a_1)}, e^{a_4-\alpha(a_4-a_3)} \] \( e^{\bar{A}}^\beta = e^{b_2-\beta(b_2-b_1)}, b_3+\beta(b_4-b_3)) \) (3.23) & (3.24)

\[ x = e^{a_1+\alpha(a_2-a_1)} \text{ and } \mu_{e^{\bar{A}}}(x) = \begin{cases} \frac{lnx-a_1}{a_2-a_1} & e^{a_1} \leq x \leq e^{a_2} \\ 1 & e^{a_2} \leq x \leq e^{a_3} \\ \frac{a_4-lnx}{a_4-a_3} & e^{a_3} \leq x \leq e^{a_4} \\ 0 & \text{otherwise} \end{cases} \] (3.25)

\[ x = e^{b_2-\beta(b_2-b_1)} \text{ and } \gamma_{e^{\bar{A}}}(x) = \begin{cases} \frac{lnx-b_2}{b_2-b_1} & -\frac{e^{b_1}}{b_2-b_1} \leq x \leq e^{b_2} \\ 0 & e^{b_2} \leq x \leq e^{b_3} \\ \frac{lnx-b_3}{b_4-b_3} & e^{b_3} \leq x \leq e^{b_4} \\ 1 & \text{otherwise} \end{cases} \] (3.26)

So \( e^{\bar{A}} \) represented by (3.25) and (3.26) is a trapezoidal shaped IFN. It can be approximated to TRIFN \( e^{\bar{A}} = (e^{b_1}, e^{a_1}, e^{b_2}, e^{a_2}, e^{a_3}, e^{b_3}, e^{a_4}, e^{b_4}) \).

**Theorem 3.8.** If \( \bar{A} = (b_1, a_1, b_2, a_2, a_3, b_3, a_4, b_4) \) is a positive TrIFN. Then \( \ln(\bar{A}) = \left[ \ln(b_1), \ln(a_1), \ln(b_2), \ln(a_2), \ln(b_3), \ln(a_4), \ln(b_4) \right] \) is also a TrIFN.

**Proof.** To calculate logarithm of the TrIFN \( \bar{A} \) we first take the logarithm of the \((\alpha, \beta)\) -cut of \( \bar{A} \) using interval arithmetic.

\[ \ln(\bar{A})^\alpha = \ln(a_1+\alpha(a_2-a_1)), a_4-\alpha(a_4-a_3) \] \( \ln(\bar{A})^\beta = \ln(b_2-\beta(b_2-b_1)), b_3+\beta(b_4-b_3)) \) (3.27) & (3.28)
So also a TrIFN as 
\[
\frac{e^{x-a_1}}{a_2-a_1}, \quad \ln(a_1) \leq x \leq \ln(a_2)
\]
\[
1, \quad \ln(a_2) \leq x \leq \ln(a_3)
\]
\[
\frac{a_4-e^x}{a_4-a_3}, \quad \ln(a_3) \leq x \leq \ln(a_4)
\]
\[
0, \quad \text{otherwise}
\]

(3.29)

\[x = \ln(b_2 - \beta (b_2 - b_1)) \quad \text{and} \quad x = \ln(b_2 + \beta (b_4 - b_3)).\]

\[Y_{ln(\hat{A})}(x) = \begin{cases} 
\frac{b_2-e^x}{b_2-b_1}, & \ln(b_1) \leq x \leq \ln(b_2) \\
0, & \ln(b_2) \leq x \leq \ln(b_3) \\
\frac{e^x-b_2}{b_4-b_3}, & \ln(b_3) \leq x \leq \ln(b_4) \\
1, & \text{otherwise} 
\end{cases}
\] (3.30)

So \( \ln(\hat{A}) \) represented by (3.29) and (3.30) is a trapezoidal shaped IFN. It can be approximated to \( \ln(\hat{A}) = [\ln(b_1), \ln(a_1), \ln(b_2), \ln(a_2), \ln(a_3), \ln(b_3), \ln(a_4), \ln(b_4)] \)

**Theorem 3.9.** If \( \hat{A}^i = (b_2, a_1, b_2, a_2, a_3, b_3, a_4, b_4) \) is a positive TrIFN then \( \hat{A}^{i/\sqrt{n}} \) is also a TrIFN as

\[
(\hat{A})^{1/\sqrt{n}} = \left[ (b_2)^{1/\sqrt{n}}, (a_1)^{1/\sqrt{n}}, (b_2)^{1/\sqrt{n}}, (a_2)^{1/\sqrt{n}}, (a_3)^{1/\sqrt{n}}, (b_3)^{1/\sqrt{n}}, (a_4)^{1/\sqrt{n}}, (b_4)^{1/\sqrt{n}} \right].
\]

**Proof.** To calculate the n\(^{th}\) root of the TrIFN \( \hat{A}^i \) we first take the n\(^{th}\) root of the \((a, \beta) - \text{cuts of} \hat{A}^i \) using interval arithmetic.

\[
(\hat{A}_a)^{1/\sqrt{n}} = [a_1 + \alpha(a_2 - a_1)]^{1/\sqrt{n}}, \quad [a_2 - \alpha(a_4 - a_3)]^{1/\sqrt{n}}
\] (3.31)

\[
(\hat{A}_\beta)^{1/\sqrt{n}} = [b_2 - \beta(b_2 - b_1)]^{1/\sqrt{n}}, \quad [b_3 + \beta(b_4 - b_3)]^{1/\sqrt{n}}
\] (3.32)

\[x = [a_1 + \alpha(a_2 - a_1)]^{1/\sqrt{n}} \quad \text{and} \quad x = [a_4 - \alpha(a_4 - a_3)]^{1/\sqrt{n}}.
\]

\[\mu_{(\hat{A})^{1/\sqrt{n}}}(x) = \begin{cases} 
\frac{x^n-a_1}{a_2-a_1}, & (a_1)^{1/\sqrt{n}} \leq x \leq (a_2)^{1/\sqrt{n}} \\
0, & (a_2)^{1/\sqrt{n}} \leq x \leq (a_3)^{1/\sqrt{n}} \\
\frac{a_4-x^n}{a_4-a_3}, & (a_3)^{1/\sqrt{n}} \leq x \leq (a_4)^{1/\sqrt{n}} \\
1, & \text{otherwise}
\end{cases}
\] (3.33)

\[x = [b_2 - \beta(b_2 - b_1)]^{1/\sqrt{n}} \quad \text{and} \quad x = [b_3 + \beta(b_4 - b_3)]^{1/\sqrt{n}}.
\]

\[\gamma_{(\hat{A})^{1/\sqrt{n}}}(x) = \begin{cases} 
\frac{b_2-x^n}{b_2-b_1}, & (b_1)^{1/\sqrt{n}} \leq x \leq (b_2)^{1/\sqrt{n}} \\
0, & (b_2)^{1/\sqrt{n}} \leq x \leq (b_3)^{1/\sqrt{n}} \\
\frac{x^n-b_2}{b_4-b_3}, & (b_3)^{1/\sqrt{n}} \leq x \leq (b_4)^{1/\sqrt{n}} \\
1, & \text{otherwise}
\end{cases}
\] (3.34)

So \( (\hat{A})^{1/\sqrt{n}} \) represented by (3.41) and (3.42) is a trapezoidal shaped IFN by

\[
(\hat{A})^{1/\sqrt{n}} = \left[ (b_2)^{1/\sqrt{n}}, (a_1)^{1/\sqrt{n}}, (b_2)^{1/\sqrt{n}}, (a_2)^{1/\sqrt{n}}, (a_3)^{1/\sqrt{n}}, (b_3)^{1/\sqrt{n}}, (a_4)^{1/\sqrt{n}}, (b_4)^{1/\sqrt{n}} \right].
\]
4. Conclusion
In this paper we have deliberated many operations of TrIFN based on \((\alpha, \beta)\) —cut method including \(n^{th}\) root and exponentiation.

REFERENCES