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Knowledge Sharing Strategies between Advanced Manufacturers and Disadvantaged Suppliers in Supply Chain Digital Transformation

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Abstract. In this paper, a differential game model was established to describe the supply chain digitalization context formed with a leading manufacturer and a disadvantaged supplier, by focusing on the knowledge sharing problem. The Hamilton-Jacobi-Bellman equations are used to solve the optimal decisions, subjective benefits, and the overall knowledge sharing of the supply chain between the manufacturer and the supplier in three different scenarios: the non-cooperative scenario, the Stackelberg game scenario, and the cooperative scenario. By comparing and analyzing the optimal decision-making results under three different game scenarios, it can be concluded that, when transitioning from the non-cooperative game to the Stackelberg game in which the cost of knowledge sharing is subsidized by the manufacturer to the supplier, there is a significant increase in the amount of knowledge sharing by the supplier, and there is no significant change in the degree of effort to perform knowledge conversion, so it can be seen that the cost-subsidizing mechanism has a very good incentive role for knowledge-sharing behaviors, and the increase is equal to the cost subsidy coefficient of the manufacturer to the supplier. At the same time, the cost-sharing coefficient β is related to the proportion of revenue sharing, and there is a threshold to achieve the Pareto optimality of the two subjects' decisions. All decision results in the fully cooperative scenario reachin optimality. Finally, the analysis of arithmetic examples is used to verify the conclusions of the resulting propositional derivations.

Keywords: digital transformation; collaborative innovation; knowledge sharing; cost sharing

1. Introduction

In recent time, the digital economy is becoming a key force in reorganizing global factor resources, reshaping the global economic structure and changing the pattern of global competition. Under the current new development pattern, the digital economy plays an important role in expanding effective investment, increasing effective supply, stabilizing economic growth, etc., and shows a trend of development towards the new, the real, and the outward. In the year of 2020, the Chinese government proposed to build a new development pattern, adjust the layout of China's industrial chain and supply chain, and then promote the optimization and upgrading of the industrial chain and supply chain. According to Gartner's survey data, 70% of enterprises accelerated the pace of digital change during the epidemic, and more enterprises regarded "digitization" as an important strategy for future supply chain development.

In the security and stable development of the supply chain, supply chain co-innovation is one of the accelerators, and co-innovation is the requirement that each participant in the supply chain actively take part in innovative behaviors and implement upstream and downstream technological connectivity in the supply chain. The existing situation is that only a few enterprises have mastered the digital transformation capabilities, most SMEs admit that digitalization is a general trend, but lack guidance and assistance on how to transform. At the same time, there is a lack of professionals who can't operate specific digital upgrades. To field research Changan Automobile Company's digital transformation found that as a manufacturing enterprise needs upstream suppliers to provide timely production of the corresponding production to ensure the daily production operations, for the weak ability of the main suppliers, Changan will provide them with the appropriate digital facilities, and teach them how to use to enhance the production information in the supply chain to improve the rapid flow of the supply chain, to better achieve the overall digitalization of the supply chain. It can be seen that knowledge sharing can well eliminate the knowledge barriers between the subjects, and has a certain role in promoting the collaborative innovation between the supply chain subjects as well as the overall digitalization level of the supply chain. Similarly, for the disadvantaged suppliers, giving certain cost subsidies can reduce their cost pressure in the process of knowledge sharing, in order to better realize the transformation of the source. Therefore, this paper applies the differential game model to explore what kind of knowledge sharing strategy between the leading manufacturers and the disadvantaged suppliers can better promote the flow of knowledge to enhance the overall supply chain digitization level and revenue.

2. Literature review

Haken first proposed the theory of synergy, which argues that different systems in the environment can not only interact with each other, but also have a relationship of mutual cooperation, and that the subsystems form a certain working system in the process of interacting with each other, so as to realize the effect of the overall effect brought about by the interaction and integration of the role of the overall effect is higher than the sum of the effects of each element alone [1]. Roberts compared hierarchical, competitive and collaborative strategies and found that multi-subject collaboration is superior to hierarchy and competition in developing and implementing innovative solutions [2]. Feranita et al. concluded that collaborative innovation can improve a firm's own innovation performance by helping it to solve its own resource constraints and to leverage knowledge, financial capital, technology, and information from other organizations [3]. Esposito et al. used contract theory to analyze the relationship between co-innovation strategies and digital platforms and found that the combination of digital platforms and co-innovation can facilitate the creative process and be a driver of operational synergies, and on the other hand, it can reduce transaction costs [4]. Saeed et al. also considered the conditioning effect of absorptive capacity on the relationship between co-innovation and innovation capabilities and found that collaboration with different partners can effectively improve a firm's product and process innovation capabilities only if the firm has the ability to acquire external knowledge itself [5]. Wang et al. revealed the co-innovation process by examining the complex relationship between key factors affecting performance in the supply chain network, and concluded that there is a positive correlation between co-innovation activities, knowledge sharing, co-innovation capabilities and firm performance, with knowledge sharing playing a mediating role and co-innovation capabilities playing a moderating role [6]. Xie et al. categorized the influence relationship between supply chain collaborative innovation and enterprise performance into three latitudes content dimension, subject dimension and enterprise performance dimension. Collaborative knowledge innovation, technological innovation and management innovation all have a positive impact on enterprise performance, and when suppliers, customers or cross-sectoral participation in collaborative innovation can significantly improve enterprise performance [7].

Collaborative innovation has three main characteristics, scalability, sharing ability and facilitation, in which sharing ability is that collaborative innovation can share resources between different regions and enterprises to maximize the utility of resources, and also share the results of innovation to maximize the value of innovation [8]. Knowledge as a kind of tacit enterprise resources, some scholars believe that knowledge is usually embedded in the process of innovation, with "sticky" and difficult to spread, without a high degree of knowledge sharing, it is difficult to achieve the expected level of innovation performance [9-10]. Abbas also believes that knowledge sharing is a basic element of innovation strategy development [11]. Castaneda et al. argued that knowledge sharing related behaviors positively influence the innovation capability of knowledge sharers in terms of their propensity and ability to promote and implement new ideas [12]. Taminiau et al. found that the most productive route to innovation is informal knowledge sharing [13]. Kamaşak et al. investigated the effects of two types of knowledge sharing, knowledge donation and knowledge collection, on exploratory and exploitative innovation in organizations and concluded that knowledge collection has a significant effect on both types of innovation [14]. Yeşil et al. concluded that knowledge sharing has a significant effect on both innovation capability and innovation performance of firms and also innovation capability affects innovation performance of firms[15]. Radaelli et al. studied how knowledge sharing affects innovative work behavior; knowledge sharing triggers the direct effect of knowledge reorganization and transformation, which promotes innovation, and knowledge sharing creates an indirect effect for the recycling of new knowledge [16]. Li et al. believe that the value of the enterprise's products depends largely on the enterprise's intellectual capital, and in order to develop rapidly, the enterprise must strengthen its investment in intellectual capital, and it must share its knowledge with the other members of the network [17].

This paper introduces a differential game model to study the knowledge sharing behavior between leading manufacturers and weak suppliers in the supply chain from a dynamic perspective, and uses the Hamilton-Jacobi-Bellman equation to compare and analyze the optimal amount of knowledge sharing, optimal knowledge transformation execution, individual and overall benefits, and the overall amount of knowledge sharing between suppliers and manufacturers in three different scenarios, namely, the Nash non-cooperative game, the Stackelberg game, and the cooperative game, to explore the main reasons affecting the collaborative knowledge sharing problem between suppliers and manufacturers, to study the effect of the cost sharing mechanism provided by manufacturers on knowledge sharing behavior, and to find the Pareto optimal game scenarios of collaborative sharing of digital transformation knowledge between subjects in a dynamic framework.

3. Problem description and modeling

This study focuses on an advanced manufacturer and a disadvantaged supplier in a supply chain. As a weak subject in digital transformation, suppliers have weak transformation knowledge and need the knowledge sharing from leading manufacturer, so the differential game model of collaborative innovation between suppliers and manufacturers is constructed. Knowledge mutually shared between supplier and manufacturer, and the manufacturer provides subsidies for supplier's transformation efforts. We consider the following hypotheses when build our model.

Hypothesis 1: The amount of knowledge shared by the manufacturer is $X_m(t)$, and the amount of knowledge shared by the supplier is $X_s(t)$, $X_m(t)$、$X_s(t)$ can measure the degree of effort of the manufacturer and the supplier in the process of knowledge sharing. Both of them will create new knowledge in the process of knowledge sharing, and the cumulative variable $K(t)$ is used to represent the amount of knowledge innovation at time t. Transformational knowledge innovation over time satisfies the following differential equation:

$$\frac{dK(t)}{dt} = r_s(t)X_s(t) + r_m(t)X_m(t) - \delta K(t) \quad (1)$$

$$K(0) = K_0$$

$r_s(t)$、$r_m(t)$ denote the coefficients of innovation capability of suppliers and manufacturers respectively, and δ denotes the coefficient of knowledge decay in the process of knowledge sharing.

Hypothesis 2: Knowledge sharing costs are incurred when knowledge sharing between subjects, which is expressed as:

$$\mathrm{C_s[X_s(t), t]} = \frac{1}{2}\mathrm{c_s(t)X_s(t)^2}$$

$$\mathrm{C_m[X_m(t), t]} = \frac{1}{2}\mathrm{c_m(t)X_m(t)^2} \quad (2)$$

$c_s(t)$、$c_m(t)$ denote the cost coefficients of knowledge sharing between suppliers and manufacturers. The manufacturer subsidizes the cost of suppliers in order to encourage them to share actively in the transformation process, and the subsidy ratio is $\beta(t)$.

Hypothesis 3: In the process of collaborative knowledge sharing between suppliers and manufacturers in the supply chain, the execution ability of each member has a strong impact on the overall degree of transformation and innovation of the supply chain. Assuming that the execution efforts of both suppliers and manufacturers are one-dimensional variables, the total benefit of the supply chain is a linear function of the execution efforts of both parties:

$$\Pi\,[\mathrm{Y_s(t), Y_m(t), t}] = \lambda_s(t)\mathrm{Y_s(t)} + \lambda_m(t)\mathrm{Y_m(t)} + \mu\mathrm{K(t)} \quad (3)$$

$Y_s(t)$、$Y_m Liu\ Xiao - yu(t)$ denotes the execution effort of knowledge utilization of suppliers and manufacturers, $\lambda_s(t)$、$\lambda_m(t)$ denotes the marginal output coefficients of the execution effort of suppliers and manufacturers, and μ is the impact coefficient of the knowledge sharing of the supply chain on the overall revenue of the supply chain. In order to better facilitate the distribution of the gain obtained between the two parties, the distribution coefficients are based on the degree of contribution and execution effort of the participating parties in the process of collaborative knowledge sharing, and the distribution ratio of the gain between the two parties is $\alpha(t)$、$1-\alpha(t)$，$\alpha(t)\in(0,1)$.

Hypothesis 4: Suppliers and manufacturers also incur costs in the implementation process, as expressed below:

$$D_s[Y_s(t),t]=\frac{1}{2}d_s(t)Y_s(t)^2$$

$$D_m[Y_m(t),t]=\frac{1}{2}d_m(t)Y_m(t)^2 \tag{4}$$

$d_s(t)$、$d_m(t)$ are the cost coefficients of innovation implementation for suppliers and manufacturers.

Hypothesis 5: Suppliers and manufacturers in the supply chain have the same and positive discount rate ρ.

In summary, the supplier's objective function is:

$$Js=\int_0^{\infty}e^{-pt}\left[\alpha(t)\left(\lambda_s(t)Y_s(t)+\lambda_m(t)Y_m(t)+\mu K(t)\right)-\frac{1}{2}\left(1-\beta(t)\right)c_s(t)X_s(t)^2-\frac{1}{2}d_s(t)Y_s(t)^2\right] \tag{5}$$

The manufacturer's objective function is:

$$Jm=\int_0^{\infty}e^{-pt}[(1-\alpha(t))\left(\lambda_s(t)Y_s(t)+\lambda_m(t)Y_m(t)+\mu K(t)\right)-\frac{1}{2}\beta(t)c_s(t)X_s(t)^2-\frac{1}{2}c_m(t)X_m(t)^2-\frac{1}{2}d_m(t)Y_m(t)^2]\,dt \tag{6}$$

4. Knowledge sharing differential game model analysis

(1) Nash non-cooperative knowledge sharing game situation

In the Nash non-cooperative game situation, suppliers and manufacturers are equal and independent of each other, each maximizing their own revenue as the criterion to make the optimal decision independently, at this time, the manufacturer does not subsidize the cost of the supplier, that is $\beta=0$. At this time, the objective function of the supplier and the manufacturer are:

$$Js=\int_0^{\infty}e^{-pt}\left[\alpha(t)\left(\lambda_s(t)Y_s(t)+\lambda_m(t)Y_m(t)+\mu K(t)\right)-\frac{1}{2}c_s(t)X_s(t)^2-\frac{1}{2}d_s(t)Y_s(t)^2\right]dt \tag{7}$$

$$Jm=\int_0^{\infty}e^{-pt}[(1-\alpha(t))\left(\lambda_s(t)Y_s(t)+\lambda_m(t)Y_m(t)+\mu K(t)\right)-\frac{1}{2}c_m(t)X_m(t)^2-\frac{1}{2}d_m(t)Y_m(t)^2]\,dt \tag{8}$$

Proposition 1: The Nash equilibrium strategies of the supplier and the manufacturer in the Nash non-cooperative game scenario are respectively:

$$X_s^N = \frac{\alpha\mu r_s}{c_s(\rho+\delta)}$$

$$Y_s^N = \frac{\alpha\lambda_s}{d_s}$$

$$X_m^N = \frac{(1-\alpha)\mu r_m}{c_m(\rho+\delta)}$$

$$Y_m^N = \frac{(1-\alpha)\lambda_m}{d_m}$$

Proof: To obtain a Markov-refined Nash equilibrium for the Nash non-cooperative game, assume that the objective function of the supplier and the manufacturer is a continuous bounded differential revenue function $V_i(k)(i \in \{s,m\})$, and that the HJB equation is satisfied for any $k \geq 0$:

$$\rho V_s(k) = \max\left\{\alpha(\lambda_s Y_s + \lambda_m Y_m + \mu k) - \frac{1}{2}c_s X_s^2 - \frac{1}{2}d_s Y_s^2 + V_s'(k)(r_s X_s + r_m X_m - \delta k)\right\} \quad (9)$$

$$\rho V_m(k) = \max\left\{(1-\alpha)(\lambda_s Y_s + \lambda_m Y_m + \mu k) - \frac{1}{2}c_m X_m^2 - \frac{1}{2}d_m Y_m^2 + V_m'(k)(r_s X_s + r_m X_m - \delta k)\right\} \quad (10)$$

Equation (9) (10) are all about X_s、Y_s、X_m、Y_m concave function, using the utility maximization of the first-order conditions, on the right part of equation (9) to find its first-order partial derivatives with respect to X_m、Y_m, on the right part of equation (10) to find its first-order partial derivatives with respect to X_m, Ym, and make them all equal to 0, you can obtain the optimal strategy of the supplier and the manufacturer are respectively:

$$X_s = \frac{V_s'(k) r_s}{c_s}$$

$$Y_s = \frac{\alpha\lambda_s}{d_s}$$

$$X_m = \frac{V_m'(k) r_m}{c_m}$$

$$Y_m = \frac{(1-\alpha)\lambda_m}{d_m} \quad (11)$$

Substituting X_s、Y_s、X_m、Y_m into function (9) and (10), then the optimal value function coefficients are obtained as followed:

$$\omega_1 = \frac{\alpha\mu}{\rho+\delta}$$

$$\theta_1 = \frac{(1-\alpha)\mu}{\rho+\delta}$$

$$\omega_2 = \frac{\alpha^2\lambda_s^2}{2\rho d_s} + \frac{\alpha(1-\alpha)\lambda_m^2}{\rho d_m} + \frac{\mu^2\alpha^2 r_s^2}{2\rho c_s(\rho+\delta)^2} + \frac{\alpha(1-\alpha)\mu^2 r_m^2}{\rho c_m(\rho+\delta)^2}$$

$$\theta_2 = \frac{\alpha(1-\alpha)\lambda_s^2}{\rho d_s} + \frac{(1-\alpha)^2\lambda_m^2}{2\rho d_m} + \frac{(1-\alpha)^2\mu^2 r_m^2}{2\rho c_m(\rho+\delta)^2} + \frac{\alpha(1-\alpha)\mu^2 r_s^2}{\rho c_s(\rho+\delta)^2} \quad (12)$$

This is obtained by substituting the value obtained in equation (12) into (11), we can obtain:

$$X_s^N = \frac{\alpha\mu r_s}{c_s(\rho+\delta)}$$

$$Y_s^N = \frac{\alpha\lambda_s}{d_s}$$

$$X_m^N = \frac{(1-\alpha)\mu r_m}{c_m(\rho+\delta)}$$

$$Y_m^N = \frac{(1-\alpha)\lambda_m}{d_m}$$

which

$$\frac{dK(t)}{dt} = r_s(t)X_s(t) + r_m(t)X_m(t) - \delta K(t)$$

$$K(0) = K_0$$

Let $A = r_s(t)X_s(t) + r_m(t)X_m(t)$, then $K' = A - \delta K$, and solve the functional expression according to the first order differential equation, $K(t) = \frac{A}{\delta} + (K_0 - \frac{A}{\delta})e^{-\delta t}$is the amount of knowledge sharing in the supply chain as a whole.

$$K^N = \frac{A^N}{\delta} + (K_0 - \frac{A^N}{\delta})e^{-\delta t}$$

$$A^N = \frac{\alpha\mu {r_s}^2}{c_s(\rho+\delta)} + \frac{(1-\alpha)\mu {r_m}^2}{c_m(\rho+\delta)}$$

The optimal value functions for suppliers and manufacturers can be obtained by substituting the values obtained in equation (16) into $V_s(\mathrm{k}) = \omega_1 k + \omega_2$, $V_m(k) = \theta_1 k + \theta_2$ as follows:

$$V_s^N(k) = \frac{\alpha\mu}{\rho+\delta}K^N + \frac{\alpha^2\lambda_s^2}{2\rho d_s} + \frac{\alpha(1-\alpha)\lambda_m^2}{\rho d_m} + \frac{\mu^2\alpha^2 r_s^2}{2\rho c_s(\rho+\delta)^2} + \frac{\alpha(1-\alpha)\mu^2 r_m^2}{\rho c_m(\rho+\delta)^2}$$

$$V_m^N(k) = \frac{(1-\alpha)\mu}{\rho+\delta}K^N + \frac{\alpha(1-\alpha)\lambda_s^2}{\rho d_s} + \frac{(1-\alpha)^2\lambda_m^2}{2\rho d_m} + \frac{(1-\alpha)^2\mu^2 r_m^2}{2\rho c_m(\rho+\delta)^2} + \frac{\alpha(1-\alpha)\mu^2 r_s^2}{\rho c_s(\rho+\delta)^2}$$

$$V^N(k) = \frac{\mu}{\rho+\delta}K^N + \frac{(2\alpha-\alpha^2)\lambda_s^2}{2\rho d_s} + \frac{(1-\alpha^2)\lambda_m^2}{2\rho d_m} + \frac{(2\alpha-\alpha^2)\mu^2 r_s^2}{2\rho c_s(\rho+\delta)^2} + \frac{(1-\alpha^2)\mu^2 r_m^2}{2\rho c_m(\rho+\delta)^2}$$

(2) Stackelberg game scenario

In the Stackelberg game, the manufacturer is the leader of knowledge sharing in the digital transformation process, and the supplier is the follower in the transformation process. In order to promote suppliers' active participation in the transformation knowledge sharing, the manufacturer subsidizes the suppliers' knowledge sharing cost \beta in order to motivate suppliers to actively participate in the knowledge flow of the supply chain as a whole. In this case, the manufacturer, as the leader, determines its own knowledge sharing cost, implementation effort and subsidy ratio, and the suppliers, in order to maximize their own benefits, make their own countermeasures by observing the information of the manufacturer's decisions. In this case, the objective function of the supplier and the manufacturer is:

$$\mathrm{Js} = \int_0^\infty e^{-pt}\left[\alpha(\mathrm{t})\left(\lambda_s(\mathrm{t})\mathrm{Y_s}(\mathrm{t}) + \lambda_m(\mathrm{t})\mathrm{Y_m}(\mathrm{t}) + \mu\,\mathrm{K}(\mathrm{t})\right) - \frac{1}{2}\left(1 - \beta(\mathrm{t})\right)\mathrm{c}_s(\mathrm{t})\mathrm{X}_s(\mathrm{t})^2 - \frac{1}{2}\mathrm{d_s}(\mathrm{t})\mathrm{Y_s}(\mathrm{t})^2\right]dt \quad (5)$$

$$\mathrm{Jm} = \int_0^\infty e^{-pt}[(1-\alpha(\mathrm{t}))\left(\lambda_s(\mathrm{t})\mathrm{Y_s}(\mathrm{t}) + \lambda_m(\mathrm{t})\mathrm{Y_m}(\mathrm{t}) + \mu\,\mathrm{K}(\mathrm{t})\right) - \frac{1}{2}\beta(\mathrm{t})\mathrm{c_s}(\mathrm{t})\mathrm{X_s}(\mathrm{t})^2 - \frac{1}{2}\mathrm{c_m}(\mathrm{t})\mathrm{X_m}(\mathrm{t})^2 - \frac{1}{2}\mathrm{d_m}(\mathrm{t})\mathrm{Y_m}(\mathrm{t})^2]\,dt \quad (6)$$

Proposition 2: The Nash equilibrium strategies of suppliers and manufacturers in the Stackelberg game model are:

$$X_s^Z = \frac{\alpha\mu r_s}{(1-\beta)(\rho+\delta)c_s}$$

$$Y_s^Z = \frac{\alpha\lambda_s}{d_s}$$

$$X_m^Z = \frac{(1-\alpha)\mu r_m}{(\rho+\delta)c_m}$$

$$Y_m^Z = \frac{(1-\alpha)\lambda_m}{d_m}$$

$$\beta = \frac{2-3\alpha}{2-\alpha}$$

Proof: Assuming that the objective functions of both the supplier and the manufacturer are continuously bounded differential revenue functions $V_i(k)(i \in \{s,m\})$ and satisfy the HJB equation for any $k \geq 0$, the optimal decision-making outcome of the supplier can be obtained by first solving the supplier's optimal decision using the inverse induction method:

$$\rho V_s(k) = \max\left\{\alpha(\lambda_s Y_s + \lambda_m Y_m + \mu k) - \tfrac{1}{2}(1-\beta)c_s X_s^2 - \tfrac{1}{2}d_s Y_s^2 + V_s'(k)(r_s X_s + r_m X_m - \delta k)\right\} \tag{13}$$

where both Eq. (13) are concave functions about X_s、Y_s. Using the first-order condition of utility maximization, the first-order partial derivatives about X_s and Ys are solved for the right-hand side of Eq. (13) and made equal to 0, which can be used to find out the degree of knowledge-sharing and the degree of execution effort of the supplier:

$$X_s = \frac{V_s'(k) r_s}{(1-\beta)c_s}$$

$$Y_s = \frac{\alpha\lambda_s}{d_s} \tag{13}$$

The manufacturer as a leader can effectively predict the supplier's optimal strategy choice, and thus the manufacturer decides its own optimal strategy and subsidy ratio based on the supplier's reaction function (13), and the manufacturer's HJB equation is:

$$\rho V_m(k) = \max\left\{(1-\alpha)(\lambda_s Y_s + \lambda_m Y_m + \mu k) - \tfrac{1}{2}\beta c_s X_s^2 - \tfrac{1}{2}c_m X_m^2 - \tfrac{1}{2}d_m Y_m^2 + V_m'(k)(r_s X_s + r_m X_m - \delta k)\right\} \tag{14}$$

Substituting the optimal decision of the supplier sought in (18) into (19), and according to the first-order condition of utility maximization, taking the first-order partial derivatives of the right-most end of Eq. (14) for X_m、$Y_m$and β, respectively, and making them equal to 0, the optimal decision of the manufacturer can be obtained.

$$\rho V_m(k) = \max\left\{(1-\alpha)\left(\frac{\alpha\lambda_s^2}{d_s} + \lambda_m Y_m + \mu k\right) - \frac{\beta V_s'^2(k) r_s^2}{2c_s(1-\beta)^2} - \frac{1}{2}c_m X_m^2 - \frac{1}{2}d_m Y_m^2 + V_m'(k)\left(\frac{V_s'(k) r_s^2}{(1-\beta)c_s} + r_m X_m - \delta k\right)\right\}$$

Maximization is available:

$$X_m = \frac{r_m V_m'(k)}{c_m}$$

$$Y_m = \frac{(1-\alpha)\lambda_m}{d_m}$$

$$\beta = \frac{2V_m'(k) - V_s'(k)}{2V_m'(k) + V_s'(k)} \tag{15}$$

Substituting X_s、Y_s、X_m、Y_m、βinto (13)(14), and then the optimal value function coefficients can be obtained as followed:

$$\omega_1 = \frac{\alpha\mu}{\rho+\delta}$$

$$\theta_1 = \frac{(1-\alpha)\mu}{\rho+\delta}$$

$$\omega_2 = \frac{\alpha^2\lambda_s^2}{2\rho d_s} + \frac{\alpha(1-\alpha)\lambda_m^2}{\rho d_m} + \frac{\alpha^2\mu^2 r_s^2}{2\rho(1-\beta)(\rho+\delta)^2 c_s} + \frac{\alpha(1-\alpha)\mu^2 r_m^2}{\rho(\rho+\delta)^2 c_m}$$

$$\theta_2 = \frac{\alpha(1-\alpha)\lambda_s^2}{\rho d_s} + \frac{(1-\alpha)^2\lambda_m^2}{2\rho d_m} + \frac{(1-\alpha)^2\mu^2 r_m^2}{2\rho(\rho+\delta)^2 c_m} + \frac{\alpha(1-\alpha)\mu^2 r_s^2}{(1-\beta)(\rho+\delta)^2\rho c_s} - \frac{\beta\alpha^2\mu^2 r_s^2}{2\rho c_s(\rho+\delta)^2(1-\beta)^2} \tag{25}$$

Substituting the obtained ω_1、ω_2、θ_1、θ_2 from the solution into (14)(15) :

$$X_s^Z = \frac{\alpha\mu r_s}{(1-\beta)(\rho+\delta)c_s}$$

$$Y_s^Z = \frac{\alpha\lambda_s}{d_s}$$

$$X_m^Z = \frac{(1-\alpha)\mu r_m}{(\rho+\delta)c_m}$$

$$Y_m^Z = \frac{(1-\alpha)\lambda_m}{d_m}$$

$$\beta = \frac{2-3\alpha}{2-\alpha}$$

The amount of knowledge shared by the supply chain as a whole is $K^Z = \frac{A^Z}{\delta} + (K_0 - \frac{A^Z}{\delta})e^{-\delta t}$, where $A^Z = \frac{\alpha\mu {r_s}^2}{(1-\beta)(\rho+\delta)c_s} + \frac{(1-\alpha)\mu {r_m}^2}{(\rho+\delta)c_m}$.

Substituting Eq. (24) into $V_s(\mathrm{k}) = \omega_1 k + \omega_2$, $V_m(k) = \theta_1 k + \theta_2$, the optimal value functions of the supplier and the manufacturer can be obtained:

$$V_s^Z(\mathrm{k}) = \frac{\alpha\mu}{\rho+\delta}K^Z + \frac{\alpha^2\lambda_s^2}{2\rho d_s} + \frac{\alpha(1-\alpha)\lambda_m^2}{\rho d_m} + \frac{\alpha^2\mu^2 r_s^2}{2\rho(1-\beta)(\rho+\delta)^2 c_s} + \frac{\alpha(1-\alpha)\mu^2 r_m^2}{\rho(\rho+\delta)^2 c_m}$$

$$V_m^Z(k) = \frac{(1-\alpha)\mu}{\rho+\delta}K^Z + \frac{\alpha(1-\alpha)\lambda_s^2}{\rho d_s} + \frac{(1-\alpha)^2\lambda_m^2}{2\rho d_m} + \frac{(1-\alpha)^2\mu^2 r_m^2}{2\rho(\rho+\delta)^2 c_m} + \frac{\alpha(1-\alpha)\mu^2 r_s^2}{(1-\beta)(\rho+\delta)^2\rho c_s} - \frac{\beta\alpha^2\mu^2 r_s^2}{2\rho c_s(\rho+\delta)^2(1-\beta)^2}$$

$$V^Z = \frac{\mu}{\rho+\delta}K^Z + \frac{(2\alpha-\alpha^2)\lambda_s^2}{2\rho d_s} + \frac{(1-\alpha^2)\lambda_m^2}{2\rho d_m} + \frac{(2\alpha-\alpha^2)\mu^2 r_s^2}{2\rho(1-\beta)(\rho+\delta)^2 c_s} + \frac{(1-\alpha^2)\mu^2 r_m^2}{2\rho(\rho+\delta)^2 c_m} - \frac{\beta\alpha^2\mu^2 r_s^2}{2\rho c_s(\rho+\delta)^2(1-\beta)^2}$$

(3) Cooperative game scenario

In order to further promote the knowledge sharing behavior between suppliers and manufacturers, and to provide the digital level of the overall supply chain, the relationship between suppliers and manufacturers from the manufacturer to subsidize the knowledge sharing costs of suppliers to the mode of cooperation between the two, in which the suppliers and manufacturers to maximize the overall interests of the two sides as the goal, and jointly determine the optimal strategy of the participating subjects. The objective function at this time is as follows:

$$J = \int_0^\infty e^{-pt}[\lambda_s Y_s + \lambda_m Y_m + \mu k - \frac{1}{2}c_s X_s^2 - \frac{1}{2}d_s Y_s^2 - \frac{1}{2}c_m X_m^2 - \frac{1}{2}d_m Y_m^2]dt$$

Proposition 3: The feedback Nash equilibrium strategies of the supplier and the manufacturer in the case of collaborative cooperation game are respectively:

$$X_s^C = \frac{\mu r_s}{(\rho+\delta)c_s}$$

$$Y_s^C = \frac{\lambda_s}{d_s}$$

$$X_m^C = \frac{\mu r_m}{(\rho+\delta)c_m}$$

$$Y_m^C = \frac{\lambda_m}{d_m}$$

Proof: Assuming that the objective function of the supplier-manufacturer contractual cooperation ecosystem is a continuous bounded differential revenue function $V(k)$, and that the HJB equation is satisfied for any $k \geq 0$, it can be obtained:

$$\rho V(k) = \max\left[\lambda_s Y_s + \lambda_m Y_m + \mu k - \tfrac{1}{2}c_s X_s^2 - \tfrac{1}{2}d_s Y_s^2 - \tfrac{1}{2}c_m X_m^2 - \tfrac{1}{2}d_m Y_m^2 + V'(k)(r_s X_s + r_m X_m - \delta k)\right] \quad (16)$$

Eq. (16) is a concave function with respect to X_s、Y_s、X_m、Y_m, and according to the first-order condition of utility maximization, the optimal strategies of the supplier and the manufacturer can be found as follows:

$$X_s = \frac{V'(k)r_s}{c_s}$$

$$Y_s = \frac{\lambda_s}{d_s}$$

$$X_m = \frac{V'(k)r_m}{c_m}$$

$$Y_m = \frac{\lambda_m}{d_m} \tag{17}$$

Substituting the result obtained in (17) into (16), the following relation is obtained by simplification:

$$\rho V(k) = [\mu - \delta V'(k)]k + \frac{\lambda_s^2}{2d_s} + \frac{\lambda_m^2}{2d_m} + \frac{V'^2(k)r_s^2}{2c_s} + \frac{V'^2(k)r_m^2}{2c_m} \tag{18}$$

From (18), the linear optimal functional equation with respect to k is a solution of the HJB equation, and hence let:

$$\mathrm{V(k) = \omega k + \theta}$$

where ω、θ are constants, substituting $V(k)$ and its first-order partial derivatives into Eq. (18) and collapsing gives:

$$\rho(\mathrm{\omega k + \theta}) = [\mu - \delta\omega]k + \frac{\lambda_s^2}{2d_s} + \frac{\lambda_m^2}{2d_m} + \frac{\omega^2 r_s^2}{2c_s} + \frac{\omega^2 r_m^2}{2c_m} \tag{19}$$

The optimal value function coefficients can be obtained from (19):

$$\omega = \frac{\mu}{\rho + \delta}$$

$$\theta = \frac{\lambda_s^2}{2\rho d_s} + \frac{\lambda_m^2}{2\rho d_m} + \frac{\mu^2 r_s^2}{2\rho(\rho+\delta)^2 c_s} + \frac{\mu^2 r_m^2}{2\rho(\rho+\delta)^2 c_m} \tag{20}$$

Substituting (20) into (17):

$$X_s^C = \frac{\mu r_s}{(\rho + \delta)c_s}$$

$$Y_s^C = \frac{\lambda_s}{d_s}$$

$$X_m^C = \frac{\mu r_m}{(\rho + \delta)c_m}$$

$$Y_m^C = \frac{\lambda_m}{d_m}$$

At this point, the amount of knowledge shared by the supply chain as a whole is $K^C =$

$\frac{A^C}{\delta}+(K_0-\frac{A^C}{\delta})e^{-\delta t}$, where $A^C=\frac{\mu {r_s}^2}{c_s(\rho+\delta)}+\frac{\mu {r_m}^2}{c_m(\rho+\delta)}$.

Substituting ω、θ into V(k) = ωk + θ yields the optimal total return for the supplier-manufacturer synergy in this scenario:

$$V^C(k)=\frac{\mu}{\rho+\delta}k^C+\frac{\lambda_s^2}{2\rho d_s}+\frac{\lambda_m^2}{2\rho d_m}+\frac{\mu^2 r_s^2}{2\rho(\rho+\delta)^2 c_s}+\frac{\mu^2 r_m^2}{2\rho(\rho+\delta)^2 c_m}$$

5. Results and discussion

By comparing the optimal decision, the optimal benefit and the overall optimal benefit of knowledge sharing between suppliers and manufacturers under the three game models, the following propositions can be obtained.

Proposition 4: The cost-sharing coefficient is related to the benefit-sharing coefficient, when the benefit-sharing coefficient is less than $\frac{2}{3}$, the manufacturer will engage in knowledge-sharing cost sharing; when the benefit-sharing coefficient is greater than $\frac{2}{3}$, at this time, the benefit distribution is not equal, and the manufacturer will not take the behavior of subsidizing the cost of knowledge-sharing.

Proposition 5: The results of the analysis of the optimal decisions of the supplier and the manufacturer under the three game scenarios are as follows: (1) Comparison of the supplier's optimal knowledge sharing if the profit sharing ratio $0\le\alpha\le\frac{2}{3}$ in the supply chain: $X_s^C\ge X_s^Z>X_s^N$; (2) Comparison of the supplier's optimal knowledge transformation execution effort: $Y_s^C\ge Y_s^Z=Y_s^N$; (3) Comparison of manufacturer's optimal knowledge sharing: $X_m^C\ge X_m^Z=X_m^N$; (4) Comparison of manufacturer's optimal knowledge transformation execution effort: $Y_m^C\ge Y_m^Z=Y_m^N$.

Proof: (1) The optimal decision of the supplier can be obtained based on the required three game scenarios:

$$X_s^C-X_s^Z=\frac{\alpha\mu c_s}{2(\rho+\delta)c_s}\ge 0$$

$$X_s^Z-X_s^N=\frac{\mu c_s(2-3\alpha)}{2(\rho+\delta)c_s}=\beta * X_s^Z>0$$

At this point the optimal decision $X_s^C\ge X_s^Z>X_s^N$ for different scenarios of suppliers.

(2) The optimal decision of the supplier can be obtained based on the required three game scenarios:

At this point the optimal decision $Y_s^C\ge Y_s^Z=Y_s^N$ for different scenarios of suppliers.

(3) The optimal manufacturer's decision based on the three game scenarios sought can be obtained:

$$X_m^C - X_m^Z = \frac{\alpha\mu r_m}{(\rho+\delta)c_m} \geq 0$$

At this point the manufacturer's optimal decisions for different scenarios $X_m^C \geq X_m^Z = X_m^N$.

(4) The optimal manufacturer's decision can be obtained based on the required manufacturer's optimal decision in the three game scenarios:

$$Y_m^C - Y_m^Z = \frac{\alpha\lambda_m}{d_m} \geq 0$$

At this point the manufacturer's optimal decision for different scenarios $Y_m^C \geq Y_m^Z = Y_m^N$.

Corollary 1: When the knowledge sharing situation between suppliers and manufacturers is transitioned from a Nash non-cooperative game to a Stackelberg master-slave game in which the cost of knowledge sharing of suppliers is subsidized by the manufacture, the manufacturer stays unchanged in terms of the amount of knowledge sharing and the level of effort in knowledge implementation, and when the manufacturer subsidizes the cost of knowledge sharing of the supplier, the amount of knowledge sharing of the supplier increases significantly and the increase is equal to the coefficient of the manufacturer's subsidy of its cost, which suggests that the subsidizing behavior of the manufacturer can be good incentives for suppliers to engage in knowledge sharing.

Corollary 2: In the collaborative game, the amount of knowledge sharing and the effort of knowledge implementation between the manufacturer and the supplier reach the highest state, and the results obtained are better than the optimal strategies in the other two scenarios, so it can be seen that the collaborative mode of sharing transformational knowledge between the supplier and the manufacturer in the supply chain is an effective mechanism to improve the digitization degree of the overall supply chain.

Proposition 6: A comparison of the amount of knowledge shared in the supply chain as a whole under the three models of non-cooperation, manufacturer subsidy, and collaborative cooperation results in $K^C \geq K^Z > K^N$.

$r_s X_s^C + r_m X_m^C \geq r_s X_s^Z + r_m X_m^Z > r_s X_s^N + r_m X_m^N$, and then $A^C > A^Z > A^N$.

According to $\frac{dK}{dA}=\frac{1}{\delta}\left(1-e^{-\delta t}\right)>0(\delta>0)$ it can be obtained that K is an increasing function with respect to A, and thus the conclusion of Proposition 7 can be verified that $K^C \geq K^Z > K^N$.

Corollary 3: First, the cost subsidy of knowledge sharing from manufacturers to suppliers can increase the amount of knowledge sharing in the overall supply chain, which facilitates a better transfer of knowledge related to digital transformation in the supply chain; second, when manufacturers and suppliers collaborate in knowledge sharing, the amount of digital knowledge sharing is maximized, which effectively avoids redundancy in the process of knowledge transfer, and maximizes the overall effect.

Proposition 7: The comparison of the optimal returns of the manufacturer and the supplier as well as the total returns of the supply chain as a whole under the three game models is as follows: (1) Comparison of the optimal returns of the supplier: $V_s^Z > V_s^N$; (2) Comparison of the optimal returns of the manufacturer: $V_m^Z > V_m^N$; (3) Comparison of the total returns of the two cooperating entities: $V^C > V^Z > V^N$.

Proof: (1)(2) Depending on the optimal returns of suppliers and manufacturers obtained in different scenarios, it can be obtained:

$$V_s^Z - V_s^N = \frac{\alpha\mu}{\rho+\delta}(k^Z-k^N)+\frac{\beta\alpha^2\mu^2 r_s^2}{2\rho(1-\beta)(\rho+\delta)^2 c_s}>0$$

$$V_m^Z - V_m^N = \frac{(1-\alpha)\mu}{\rho+\delta}(k^Z-k^N)+\frac{\alpha\beta(1-\alpha)\mu^2 r_s^2}{(1-\beta)(\rho+\delta)^2\rho c_s}+\frac{\beta\alpha^2\mu^2 r_s^2}{2\rho c_s(\rho+\delta)^2(1-\beta)^2}>0$$

Therefore the inequality relations $V_s^Z > V_s^N, V_m^Z > V_m^N$ both hold.

(2) Depending on the total revenue of the supplier and the manufacturer obtained in different scenarios, it can be obtained:

$$V^Z - V^N = \frac{\mu}{\rho+\delta}(k^Z-k^N)+\frac{\beta(2\alpha-\alpha^2)\mu^2 r_s^2}{2\rho(1-\beta)(\rho+\delta)^2 c_s}++\frac{\beta\alpha^2\mu^2 r_s^2}{2\rho c_s(\rho+\delta)^2(1-\beta)^2}>0$$

$$V^C - V^Z = \frac{\mu}{\rho+\delta}(k^C-k^Z)+\frac{(\alpha-1)^2\lambda_s^2}{2\rho d_s}+\frac{\alpha^2\lambda_m^2}{2\rho d_m}+\frac{\alpha^2\mu^2 r_m^2}{2\rho(\rho+\delta)^2 c_m}+\frac{\beta\alpha^2\mu^2 r_s^2}{2\rho c_s(\rho+\delta)^2(1-\beta)^2}>0$$

Thus the inequality relation $V^C > V^Z > V^N$ holds.

6. Conclusion

This paper analyzes the knowledge sharing strategy between suppliers and manufacturers in the supply chain from a dynamic perspective, and establishes a differential game model of knowledge sharing between manufacturers and suppliers to investigate what kind of strategy can be adopted between the leading manufacturers and the weak suppliers to achieve the optimal effect of each subject and the supply chain as a whole. By applying the HJB equations, we solved the optimal knowledge sharing volume and knowledge conversion effort of each player, the optimal revenue of each subject and the optimal revenue of the supply chain as a whole, and the optimal subsidy coefficient of the manufacturer in subsidizing the knowledge sharing cost of the suppliers under the three different scenarios, including non-cooperative, the Stackelberg game, and the cooperative game context. The following conclusions were obtained.

First, the optimal strategy in the Stackelberg game is significantly better than the optimal strategy in the Nash non-cooperative scenario, which can show that the manufacturer's subsidy on suppliers' knowledge sharing cost can significantly increase the suppliers' knowledge sharing amount and the optimal returns of the inter-subjects and the supply chain as a whole and the increase of the suppliers' knowledge sharing amount is equal to the manufacturer's subsidy on the suppliers' knowledge sharing cost. The increase in the amount of knowledge sharing is equal to the manufacturer's subsidy coefficient, but this subsidy behavior has no significant incentive effect on other decisions. Manufacturers as a leading enterprise requires its source suppliers to actively participate in the transformation of knowledge sharing process, suppliers as a vulnerable subject, the transformation of thinking is weak lack of appropriate technical support, making it difficult to really step into the ranks of digital transformation, this time between the subject of the transformation of knowledge sharing is particularly important, and at the same time, the cost sharing strategy is a more effective incentives to give Technical and cost support can better drive the upstream disadvantaged subjects to actively participate in the transformation of innovation.

Second, the optimal subsidy coefficient of the manufacturer is closely related to the coefficient of benefit distribution of knowledge sharing, when the coefficient of benefit distribution to the supplier is greater than $\frac{2}{3}$, the manufacturer will not take the sharing strategy. It can be seen that the premise of cost sharing should also ensure that the main body to obtain a certain amount of revenue, set a more reasonable revenue sharing coefficient can better promote the subsidy behavior.

Third, when taking the fully cooperative game, the optimal decision-making is better than the decision-making behavior under the Nash non-cooperative and Stackelberg game. Cooperation and information sharing between the two subjects with the goal of maximizing the overall benefit is the optimal decision for knowledge sharing behavior.

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