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The Optimal Contract Complexity for Coordination Mechanisms of Supply Chain

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Abstract. This paper considers a supply chain system which consists of a supplier and a retailer. The purpose is to investigate the impact of contract complexity on supply chain coordination under complete information. The supplier drafts contracts, which include wholesale price contracts and simple quantity discount contracts. These contracts are of different complexity. The retailer chooses one of the supplier-designed contracts to optimize its profit. This study shows that a complex contract with an infinite number of price breaks can achieve the coordination of a general supply chain. It can also arbitrarily distribute supply chain profit under mild conditions. Theoretically, this is the optimal contract. However, it is difficult to implement in practice. Complex contracts with limited price breaks can improve the performance of the decentralized supply chain system compared to simple contracts (i.e. wholesale price contracts), but neither can coordinate the general supply chain. In addition, as the complexity of the contract increases, the performance of the decentralized supply chains continues to decline. This means that the increased in contract complexity does not necessarily increase the efficiency of supply chain contracts. Our study suggest that a three-price contract (all-unit quantity discount contract with two price breaks), although theoretically suboptimal, is sufficient for a general supply chain and should be preferred in practice.

Keywords: supply chain coordination; contract complexity; all-unit quantity discount contract; wholesale price contract

1. Introduction

Today's global supply chains are mostly characterized by decentralized systems. Decentralization has many advantages, such as lower production costs and shortening time to market [1]. Thus, decentralized supply chain management has become one of the key factors to successfully address the growing complexity of the current business environment [2]. However, supply chain, as a complex network, is difficult to manage [3]. In addition, the current decentralized supply chain faces the following challenge: supply chain members are primarily focused on optimizing their own goals [4]. However, their self-interested attitude often leads to poor performance.

To achieve the optimal performance of the general supply chain, a series of precise actions need to be implemented under the incentive mechanism. Furthermore, there is a need to be able to effectively manage the complexity of the supply chain [5]. This can be achieved if the firms coordinate through a set of transfer payment contracts such that the objective of each firm is achieved and aligned with that of the supply chain [6]. Compared to the wholesale price contract [7], which is a simple contract because it requires only a single parameter (the wholesale price), many coordination contracts have proved to be effective for the general supply chain under newsvendor setting. These include buy-back contracts (e.g., Hou et al. [8]), quantity flexibility contracts (e.g., Tsay and Lovejoy [9]), sales-rebate contracts (e.g., Gong-bing et al. [10]), compensation contracts (e.g., Chen et al.[11]), quantity discount contracts (e.g., Zhang [12]) and revenue-sharing contracts (e.g., Cachon and Lariviere [13], Zhao et al. [14]). Although these contracts can coordinate supply chain under mild conditions and are theoretically optimal under certain conditions, they can be rather complex [15, 16]. Thus, their applicability may be a problem, because in practice, it is difficult for decision makers to response effectively to such complex issues. According to the statement made by Zadeh (1973, p. 28) [17], "as the complexity of a system increases, our ability to make precise and yet significant statements about its behavior diminishes." Therefore, in the context of supply chain coordination, decision makers need to consider the complexity of the contract when designing the contract. The complexity is measured by the number of price breaks (or price blocks) [18].

Lim and Ho [19] observed that the efficiency of the supply chain in a complete information setting can be continuously improved when the number of price breaks in a quantity discount contract increases to more than two. Kalkanci et al. [20] examined contract complexity as a design issue through human subject experiments in the supplier-buyer supply chain and showed that quantity discount contracts with a small number of price blocks are sufficient for the supply chain. Based on the above discussions, this paper aims to provide decision makers with some useful suggestions in the contract design by considering the relationship between the complexity and efficiency of the coordination contract.

This study uses the method proposed by Kalkanci et al. [20] to design contracts, in which contract complexity is considered as a design dimension of coordinating contract in a decentralized supply chain, consisting of a single supplier and a single buyer. They characterized the impact of contract complexity on performance using simple contracts (wholesale price contract) and complex contracts (all-quantity discount contracts with two or three price blocks). Their results show that the optimization effect of complex contracts on supplier profit is flawed and requires in-depth theoretical research. This study expands their research by designing another complex contracts (all-quantity discount contracts with arbitrary price breaks) to capture the impact of contract complexity on supply chain coordination under complete information.

This paper examines the coordination of a supply chain, which consists of one supplier and one retailer. The supplier designs contracts, including the simple contract without price break and complex contracts with infinite or a finite number of price breaks. The retailer selects one of the contracts to maximize its profit under the scenario that the market demand is random. Three key questions will be addressed as follows.

The first question is "what are the complex contracts which can coordinate a general supply chain and achieve a win-win outcome under complete information?" A contract is said to be able to coordinate a supply chain if the optimal action set of the supply chain reaches its Nash equilibrium, meaning that no firm can attain a profitable unilateral by deviating from the optimal action set of the supply chain. Therefore, the coordination of retailer's optimal order quantity is under consideration in this paper.

The second question is "can the coordination contract's efficiency be improved by increasing the complexity of the contract?" The coordination contract's efficiency (i.e. the efficiency of the supply chain under the coordination contract) is the ratio of the decentralized supply chain's profit under the coordination contract to the centralized supply chain's optimal profit. This paper aims to explore the relationship between coordination contract's efficiency and complexity of the contract under complete information.

The third question is "which complex contracts are preferred in practice?" Managing a complex contract is often costly. Thus, contract designers may actually be more willing to offer simpler contracts, even if these contracts do not optimize the supply chain's performance. A simple contract is particularly desirable if the contract's efficiency is high and the contract designer can gain a sufficiently large share of the supply chain profit. The goal of this paper is to find a complex contract usable in practice despite being theoretically suboptimal.

The rest of this study is organized as follows. In Section 2, related results in existing literature are reviewed. Details of the proposed model are presented in Section 3. In Section 4, we conduct the analysis of optimal contracts, including the simple contract in Section 4.1, the complex contract with an infinite number of price breaks in Section 4.2 and the complex contracts with a small number of price breaks in Section 4.3. Section 5 discusses the value of contract complexity. Section 6 concludes this study with discussion and managerial insights.

2. Literature review

We shall review some of the relevant results in the literature on contract complexity and supply chain coordination contracts. In the literature, the theory of all-unit quantity discounts is mainly due to Munson and Rosenblatt [21] and Altintas et al. [22]. See also relevant references cited therein.

2.1. Literature on contract complexity

A supply chain is a complex network of business entities that involves upstream and downstream flows of products and/or services, as well as related financial transactions and information [23]. Due to the complexity of the supply chain, it is to coordinate these flows with a simple contract mechanism [3]. Therefore, for the accomplishment of supply chain coordination, it is required to take into consideration of the complexity of the coordination contract. This is almost a common sense.

As described by Melumad et al. [24], contract complexity is defined by the number of contingent events in the contract, followed by the number of decisions specified in each contingency. Buvik and Halskau [25] argued that contractual complexity is referred to the complexity of established rules and guidelines for handling performance assessments and quality control issues (e.g., Hannås et al. [26]), monitoring procedures (e.g., Rokkan and Buvik [27]), price incentives clauses (e.g., Klein [28]). Kalkanci et al. [18, 20] studied complexity as a factor in contract design, measured by the number of price blocks and the number of contract parameter decisions. This is similar to the definition of contract complexity in this study.

Although there are difficulties in dealing with contract complexity, the issue of contract complexity has received much attention. Barthelemy and Quélin [29] described the connection between the three specificities and the complexity of the outsourcing contract, because specificity is often considered as the most important transaction cost attribute. More specifically, a method was developed to study the relationships between contract complexity and exchange hazards (i.e. specificity and environmental uncertainty), contractual aspects of outsourcing (control, incentives, penalties, price and flexibility clauses) and ex post transaction cost level , respectively. Kalkanci et al. [20] studied the impact of contract complexity on the performance of members of the supply chain, where theoretical predictions were compared with the actual behaviors in human subject experiments. The conclusion was that the notion that complex contracts can optimize the supplier's profit is flawed. Kalkanci et al. [18] adopted this approach to study the contract design issues and expanded their research by taking into account the interaction between a human supplier and a human buyer to regain the impacts of social preferences. Buvik and Halskau [25] explored the concept of contractual complexity for small -sized service contractors in an offshore market. More specifically, how does the complexity of the contract and its impact on the cooperation of small service supplier alliance affect the perception of contractual difficulties in which these small service suppliers serve contracts?

2.2. Literature on supply chain coordination with contracts

Supply chain coordination requires each firm to take optimal action and share information accurately based on a coordinated contractual mechanism [6].

Cachon [6] reviewed and expanded the supply chain literature on managing incentive conflicts arising from contracts under uncertain demand. Under random demand environment (i.e., the newsvendor environment), Cachon and Lariviere [13] demonstrated that revenue-sharing contracts can coordinate supply chain and distribute profits arbitrarily. It is also found that the revenue-sharing contracts can coordinate a supply chain under the influence of retailers competition, e.g., Cournot competitors or competing newsvendors with fixed prices. Huang et al. [30] proved that the quantity discount contracts can coordinate the supply chain and prevent potential incentives for retailers to encourage returns. Partha et al. [31] developed a combined contract model for coordinating a two-stage supply chain where the retailers' demand is price-sensitive and stock-dependent. It has been shown that the proposed coordination mechanism can achieve coordination and win-win outcome for both the members of the supply chain. Chiu et al. [32] proposed a sophisticated menu of target sales rebate taking into account minimum order quantity and quantity discount contracts. This menu can not only coordinate supply chain, but also provide each retailer with a higher degree of freedom in the selection of order quantity. Giri and Bardhan [33] studied the three-layer supply chain coordination and the sub-supply chain coordination. They also observed that the coordination at any stage can increase the total profit of the chain, and if coordination occurs at the upstream level, this enhancement will be even stronger. Lin et al. [34] examined three modifications to confirm Warehouse Financing (CWF): cash-advance discount compensation CWF, deposit withholding CWF, and two-way compensation CWF. They showed that, although all the three modes can give rise to the set of Pareto solutions, only the two-way compensation CWF can properly coordinate supply chain.

In the existing literature, the role of complexity in contracts (e.g., Lim and Ho [19], Kalkanci et al. [20]) has been extensively studied. However, this paper is the first to study the influence of contract complexity on supply chain coordination in operation management.

3. Model details

The setting in this study is similar to that of Kalkanci et al. [18, 20], where the model composes of two risk-neutral firms, a single supplier (he) and a single retailer (she). The retailer procures seasonal products, such as seasonal vaccine, fresh fish and soft drink, from the supplier and sells it to end customers at p dollars per unit. For the supplier, he has ample capacity and produces products at a cost of c dollars per unit to meet order requirements, where $p > c$. The retailer is a newsvendor facing a market demand, D, which is random being uniformly distributed between $u - v$ and $u + v$, i.e., $D \sim U(u - v, u + v)$. Here, u ($u > 0$) is the mean demand, and v ($v > 0$) defines the range of demand. Without loss of generality, it is assumed that the lowest possible demand is zero, i.e., $u = v$. A goodwill penalty for lost sales (i.e., cost of lost sales) is not included in this model. There is no salvage value for leftover products. The supplier determines the pricing scheme of seasonal products and the retailer has only one opportunity to place the order before the start of a single selling season to maximize her expected profit, with no replenishment opportunity. The amount the retailer chooses to order depends on the terms of the contract between supplier and the retailer. Also, it is assumed that all the information mentioned above is available to both the parties.

Different coordination contracts between the supplier and the retailer are studied. The supplier offers a menu of contracts: $\{w_{(n),i}, Q_{(n),i-1}\}_{i=1,\cdots,n,\ n=1,2,\cdots}$, where n, which is a positive integer, is the number of wholesale prices in the contract, $w_{(n),i}$ represents the wholesale price and $Q_{(n),i-1}$ represents the price breaks. In the one-price contract ($n = 1$, i.e., wholesale price contract), the supplier sets a single wholesale price $w_{(1),1}$, and the retailer procures seasonal products for the quantity she chooses. This constitutes a simple contract because it only requires the specification of a single parameter, i.e., the wholesale price. In the two-price contract ($n = 2$, quantity discount contract with two prices and one price break), the supplier quotes two wholesale prices $w_{(2),1}$ and $w_{(2),2}$ ($w_{(2),1} \geq w_{(2),2}$), and a single price break $Q_{(2),1}$. If the retailer orders less than $Q_{(2),1}$, she pays $w_{(2),1}$ per unit. Otherwise, she pays a cheaper unit price of $w_{(2),2}$. In the three-price contract ($n = 3$, quantity discount contract with three prices and two price breaks), the supplier quotes three wholesale prices $w_{(3),1}$, $w_{(3),2}$, and $w_{(3),3}$ ($w_{(3),1} \geq w_{(3),2} \geq w_{(3),3}$), and two price breaks $Q_{(3),1}$ and $Q_{(3),2}$ ($Q_{(3),1} \leq Q_{(3),2}$). If the retailer orders less than $Q_{(3),1}$, she pays $w_{(3),1}$ per unit. If she orders more than $Q_{(3),1}$ but less than $Q_{(3),2}$, she pays a cheaper unit price of $w_{(3),2}$. If she orders more than $Q_{(3),2}$, she pays an even cheaper unit price of $w_{(3),3}$. Similar interpretations are applied to the four-price contract ($n = 4$, quantity discount contract with four wholesale prices and three price breaks), the five-price contract ($n = 5$, quantity discount contract with five wholesale prices and four price breaks) and so on. The detailed description of n-price contract is given in Section 4.2, where n is an arbitrary positive integer.

This study also examines a more complex contract with an infinite number of price breaks ($n \to +\infty$, infinite-price contract) and compares it with the quantity discount contracts with small numbers of price breaks, where $w_{(+\infty),i}(q)$ is a decreasing function with respect to q. In practice, $w_{(+\infty),i}(q)$ is typically a step function, but, for simplicity, it is assumed to be continuous and differentiable [6]. In addition, the wholesale prices are assumed to be equal to the discount ratio under the infinite-price contract, i.e., $w_{(\mathrm{n}),i}=\alpha_{(\mathrm{n})}w_{(\mathrm{n}),i-1}=\cdots=\alpha_{(\mathrm{n})}^{n-1}w_{(\mathrm{n}),1}$, $i=1,\cdots,n$, where $\alpha_{(\mathrm{n})}$ is the discount ratio between prices.

The proposed model is focused on exploring the relationship between contract complexity and supply chain coordination. Firstly, the supplier's and the retailer's profit functions are given, under the n-price contract, by

$$\pi_{(n),S}\left(w_{(n),i}\right)=\left(w_{(n),i}-c\right)q_{(n)} \tag{1}$$

$$\pi_{(n),R}\left(q_{(n)}\right)=p\min\left(q_{(n)},D\right)-w_{(n)}q_{(n)} \tag{2}$$

respectively, where $w_{(n),i}$ is the wholesale price corresponding to the order quantity $q_{(n)}$ under the n-price contract . The whole supply chain can be viewed as a newsvendor. Thus, the centralized supply chain's profit function is

$$\pi_{CSC}=p\min\left(q_{SC},D\right)-cq_{SC} \tag{3}$$

where q_{SC} is the order quantity under the centralized decision-making setting.

4. Analysis of optimal contracts

In the setting of this model, the supplier offers his contracts with wholesale prices and price breaks, while the retailer chooses her optimal procurement quantities. We analyze the equilibrium obtained under the settings of a simple contract, a contract without price breaks, and complex contracts with price breaks. The results obtained are reported.

4.1. Simple contract

Under the simple contract (i.e., one-price contract without price break) setting, supplier simply charges a single wholesale price $w_{(1),1}$ to the retailer for each unit of seasonal products. The profits of the supplier and retailer are $\pi_{(1),S}=\left(w_{(1),1}-c\right)q_{(1)}$ and $\pi_{(1),R}=p\min\left(q_{(1)},D\right)-w_{(1),1}q_{(1)}$ respectively. The solution approach to this problem follows the approach proposed by Cachon [6]. In a decentralized supply chain, this is a dynamic game under complete information, where the supplier moves first and the retailer follows. Hence, this dynamic game can be solved by using backward induction.

Given the wholesale price $w_{(1),1}$, it is necessary to find the retailer's best response $q_{(1)}\left(w_{(1),1}\right)$. She will choose $q_{(1)}\left(w_{(1),1}\right)$ to maximize $\pi_{(1),R}$. Clearly, $\pi_{(1),R}$ is a concave function with respect to $q_{(1)}\left(w_{(1),1}\right)$. Hence, under the first-order condition, the optimal $q_{(1)}\left(w_{(1),1}\right)=2u\left(\left(p-w_{(1),1}\right)/p\right)$ can be obtained. Next, given the retailer's best response $q_{(1)}\left(w_{(1),1}\right)$, the supplier will maximize $\pi_{(1),S}$. This is a concave function with respect to $w_{(1),1}$, and $w_{(1),1}=\left(p+c\right)/2$ can be obtained using the first-order condition. Furthermore, the optimal order quantity $q_{(1)}^{*}=u\left(\left(p-c\right)/p\right)$ can also be obtained. From the equilibrium solution for this decentralized supply chain, it gives $\pi_{(1),S}=\left(u\left(p-c\right)^{2}\right)/\left(2p\right)$ and

$\pi_{(1),R} = \left(u\left(p-c\right)^2\right)\big/\left(4p\right)$, which are listed in Table 2. We then move on to consider a centralized (integrated) supply chain, where both the retailer and the supplier are parts of the same organization and are managed by the same entity. To be more precise, it is assumed that there is a single decision maker who is concerned with maximizing the entire chain's profit $\pi_{CSC} = p\min\left(q_{SC}, D\right) - cq_{SC}$. Evidently, π_{CSC} is a concave function of q_{SC}. Hence, under the first-order condition, the optimal order quantity $q_{SC}^{*} = 2u\left(\left(p-c\right)/p\right)$ is obtained. Therefore, the entire supply chain's optimal profit is: $\pi_{CSC} = \left(u\left(p-c\right)^2\right)\big/\left(p\right)$.

It can be seen that in the centralized scheme, the sales volume is greater than that of the decentralized scheme, i.e., $q_{SC}^{*} > q_{(1)}^{*}$. In addition, the supplier gets half of the supply chain profits, while the retailer only gets one fourth. This is partly due to the advantage that the supplier enjoys for being the leader. A closer look reveals that the wholesale price $w_{(1),1}$ has no impact on quantity or supply chain profit. However, the choice of $w_{(1),1}$ in the decentralized supply chain is still important, because it determines the distribution of the profit between the supplier and the retailer. The wholesale price $w_{(1),1}$ can be interpreted as a form of transfer payment made by the retailer to the supplier.

Compared with the centralized supply chain, the decentralized one allows the supplier and the retailer to maximize their own profits; i.e., they each try to secure their respective margins $p - w_{(1),1}$ and $w_{(1),1} - c$. This is known as "double marginalization". In Definition 1 given below, the double marginalization loss for n-price contract is defined; and furthermore, the relationship between simple contract (i.e., one-price contract or wholesale price contract) and supply chain coordination is characterized.

Definition 1. *Under an n-price contract, where $n = 1, 2, \cdots$, the double marginalization loss (also referred to as the supply chain system profit loss) is defined by*

$$\phi(n) = \left(1 - \frac{\pi_{(n),S} + \pi_{(n),R}}{\pi_{CSC}}\right) \times 100\% \tag{4}$$

When $n = 1$, the double marginalization loss $\phi(1)$ is 25% (see Table 2), implying that the supply chain cannot be coordinated under one-price contract (i.e., simple contract).

Note that $1 - \phi(1)$ denotes the supply chain efficiency. Clearly, the firm's interest is to reduce or eliminate double marginalization, such as by allocating the extra profits in such a way that both players will benefit. Moreover, when we change the terms (wholesale prices and price breaks) of the contract, do the independently managed companies act as if they are vertical integration. Next, we explore ways to eliminate or reduce double marginalization loss under complex contract settings.

4.2. Complex contracts

Under the setting of complex contracts (i.e., contracts with price breaks), the supplier provides a detailed payment scheme for each quantity that the retailer may choose when procuring seasonal products [18]. Specifically, the focus is restricted to a menu of contracts $\{w_{(n),i}, Q_{(n),i-1}\}_{i=1,\cdots,n,\ n=1,2,\cdots}$. Section 3 provides detailed descriptions of n-price contract, particularly one-price contract, two-price contract, and three-price contract. Suppose that individually, both the supplier and the retailer are individually rational players. Then, eliminate trivial cases, we assume that $c \le w_{(n),n} \le w_{(n),n-1} \le \cdots \le w_{(n),1} \le p$ and $0 \le Q_{(n),1} \le Q_{(n),2} \le \cdots \le Q_{(n),n-1}$ under n-price contract. Figure 1 characterizes the n-price contract, which has n wholesale prices and $n-1$ price breaks.

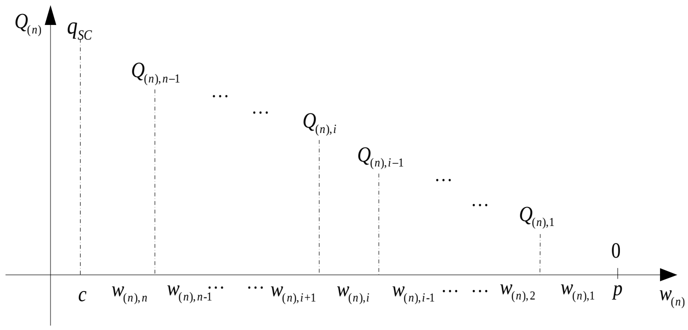

Figure 1: The menu $\{w_{(n),i}, Q_{(n),i-1}\}$ of n-price contract

If the retailer orders less than $Q_{(n),i-1}$, she needs to pay $w_{(n),i\text{-}1}$ per unit seasonal product. If she orders more than $Q_{(n),i-1}$ and less than $Q_{(n),i}$, the price drops to $w_{(n),i}$. If she orders more than $Q_{(n),i}$ and less than $Q_{(n),i+1}$, the price further drops to $w_{(n),i+1}$ per unit seasonal product. In the operation management literature, for a given n, the menu $\{w_{(n),i}, Q_{(n),i-1}\}$ is called "all-unit quantity discount contract". Note that the complexity of the contract is measured by the total number of the wholesale prices n and price breaks $n-1$, meaning that the contract complexity of the n-price contract is $2n-1$.

This section explores the relationship between contract complexity and supply chain coordination. The next section studies complex contracts with an infinite number of price breaks.

4.2.1. Complex contracts with an infinite number of price breaks

Under an infinite-price contract (i.e., complex contract with an infinite number of price breaks, $n \to +\infty$), we follow the approach proposed by Cachon in [6]. We assume that the supplier charges the wholesale price $w_{(+\infty)}\left(q_{(+\infty)}\right)$, where $w_{(+\infty)}$ is a decreasing function of the retailer's order quantity $q_{(+\infty)}$. In practice, $w_{(+\infty)}$ is typically a step function, but for simplicity, we assume that it is continuous and differentiable. In a decentralized supply chain, the supplier's and retailer's profits are $\pi_{(+\infty),S} = \left(w_{(+\infty)} - c\right)q_{(+\infty)}$ and $\pi_{(+\infty),R} = p\min\left(q_{(+\infty)}, D\right) - w_{(+\infty)}q_{(+\infty)}$, respectively. Recall that the integrated (centralized) supply chain's profit is $\pi_{CSC} = p\min\left(q_{SC}, D\right) - cq_{SC}$, where the supplier and retailer are part of the same organization and managed by the same entity.

To analyze the possibility for a general supply chain coordination, one technique being used is to choose the payment schedule so that the profit of the retailer is an affine transformation of that of the supply chain (i.e., $\pi_{(+\infty),R}$ equals a constant fraction of π_{SC}). Thus, the following conditions are to be satisfied:

$$\pi_{(+\infty),R} = p\min\left(q^{*}_{(+\infty)}, D\right) - w_{(+\infty)}q^{*}_{(+\infty)} = \lambda\left(p\min\left(q_{SC}, D\right) - cq_{SC}\right) = \lambda\pi_{\mathrm{CSC}} \tag{5}$$

where $\lambda \in (0,1)$ is the fraction of the supply chain profit for the retailer. To be more specific, if the supplier sets the wholesale price as $w_{(+\infty)}\left(q_{(+\infty)}\right) = (1-\lambda)\,p\left(1-q_{(+\infty)}/4u\right) + \lambda c$, then the retailer will face with the following scenario. If she chooses $q_{(+\infty)} < q_{SC}$, she will have paid too much per unit seasonal product, and hence her marginal cost is increased. If she chooses $q_{(+\infty)} > q_{SC}$, then the unit price will decrease but the marginal revenue will decrease more, and hence it is unprofitable for the retailer. Thus, the optimal quantity choice for the retailer is the optimal centralized supply chain quantity $q_{SC} = 2u\left((p-c)/p\right)$. In this case, the supplier's profit is

$$\pi_{(+\infty),S} = \left((1-\lambda)\,p\left(1-q_{SC}/4u\right) + \lambda c - c\right)q_{SC} = (1-\lambda)\pi_{\mathrm{SC}} \tag{6}$$

On the basis of the analysis above, we shall characterize the relationship between infinite-price contract and supply chain coordination in the following proposition.

Proposition 1. *Consider a complex contract with an infinite number of price breaks (i.e., infinite-price contract) given by*

$$w_{(+\infty)}\left(q_{(+\infty)}\right) = (1-\lambda)\,p\left(1-q_{(+\infty)}/4u\right) + \lambda c \tag{7}$$

where $\lambda \in (0,1)$ denotes the fraction of supply chain profit for the retailer. Then, under the infinite-price contract, the retailer's optimal order quantity is q_{SC}, meaning that the supply chain can be coordinated. Furthermore, when $\lambda \in (1/4, 1/2)$, the infinite-price contract can achieve supply chain coordination and win-win outcome.

Under the infinite-price contract, the contract parameter λ plays the role of distributing the supply chain's profit between the retailer and the supplier. This mans that retailer's (and supplier's) profit is proportional to the centralized supply chain's profit, indication that a complex contract with an infinite number of price breaks can eliminate double marginalization. To implement this optimal contract, an infinite number of price breaks will have to be specified by the retailer. Clearly, it is not very practical for the supplier nor the retailer to enforce such an arbitrarily complex infinite-price contract. In this paper, we consider complex contracts, each with a low (limited) number of price breaks.

4.2.2. Complex contracts with a small number of price breaks

Recall that the complexity of a contract is measured by the number of wholesale price and price breaks. The focus is restricted to a menu of contracts $\left\{w_{(n),i}, Q_{(n),i-1}\right\}_{i=1,\cdots,n,\ n=1,2,\cdots}$ and the wholesale prices are assumed to be of equal proportion. Without loss of generality, it is assumed that $w_{(n),1} = p$, indicating that the supplier tries his best to extract the decentralized supply chain profit. Therefore, $c < w_{(n),n} < w_{(n),n-1} < \cdots < w_{(n),1} = p$, where n is a fixed finite positive integer .

To continue, we characterize the price discount parameter $\alpha_{(n)}$. Given an n-price contract ($n \geq 2$), suppose that the contract designer (the supplier) is completely rational, then the menu of contracts $\{w_{(n),i}, Q_{(n),i-1}\}_{i=1,\cdots,n,\ n=1,2,\cdots}$ is required to satisfy $\pi_{(n),S}\left(w_{(n),i}\right)-\pi_{(n),S}\left(w_{(n),i-1}\right) \geq 0$. Recalling the supplier's profit function $\pi_{(n),S}\left(w_{(n),i}\right)=\left(w_{(n),i}-c\right)q\left(w_{(n),i}\right)$ and the retailer's order quantity $q\left(w_{(n),i}\right)=2u\left(1-w_{(n),i}/p\right)$, we have

$$\left(w_{(n),i}-c\right)2u\left(1-\frac{w_{(n),i}}{p}\right)-\left(w_{(n),i-1}-c\right)2u\left(1-\frac{w_{(n),i-1}}{p}\right)\geq 0$$
$$\left(w_{(n),i}-c\right)\left(p-w_{(n),i}\right)-\left(w_{(n),i-1}-c\right)\left(p-w_{(n),i}\right)\geq 0$$
$$\left(w_{(n),i}-w_{(n),i-1}\right)\left(p+c\right)-\left(w_{(n),i}-w_{(n),i-1}\right)\left(w_{(n),i}+w_{(n),i-1}\right)\geq 0$$

Note that the price discount parameter is such that $0<\alpha_{(n)}<1$. Thus, $w_{(n),i}=\alpha_{(n)}w_{(n),i}<w_{(n),i-1}$. It is straightforward to show that $\left(w_{(n),i}+w_{(n),i-1}\right)-\left(p+c\right)\geq 0$. Hence, $w_{(n),i}$ is decreasing with respect to i, implying that it suffices to only consider the case of $\left(w_{(n),n}+w_{(n),n\text{-}1}\right)-\left(p+c\right)\geq 0$. That is, the supplier should at least receive his reservation profit. Specially, we assume that the supplier's reservation profit is normalized to 0. Hence, we get $\alpha_{(n)}^{n-2}+\alpha_{(n)}^{n-1}=\left(p+c\right)/p$. On the basis of the analysis above, the following proposition is obtained.

Proposition 2. *Under an n-price contract ($n\geq 2$), the price discount parameter $\alpha_{(n)}$ is given by*

$$\alpha_{(n)}^{n-2}\left(1+\alpha_{(n)}\right)=\frac{p+c}{p} \tag{8}$$

Specifically, the explicit expressions of $\alpha_{(2)}$, $\alpha_{(3)}$, $\alpha_{(4)}$, and $\alpha_{(5)}$ are in Table 2, which are sufficient for the contract designer under complete information. Next, we shall consider supplier's profit and retailer's profit. To find the supplier's profit under the menu of contracts $\{w_{(n),i}, Q_{(n),i-1}\}_{i=1,\cdots,n,\ n=1,2,\cdots}$, the choice of the contract for the retailer will be studied first. The retailer must choose one contract from the menu $\{w_{(n),i}, Q_{(n),i-1}\}_{i=1,\cdots,n,\ n=1,2,\cdots}$. She will prefer the contract i than the contract $i-1$ if and only if her profit is greater when the contract i is chosen. Based on the retailer's profit function $\pi_{(n),R}=\left(p-w_{(n),i}\right)q\left(w_{(n),i}\right)-\left(p/4u\right)q\left(w_{(n),i}\right)=up\left(1-\alpha_{(n)}^{i-1}\right)^2$ for $n\geq 2$ and $i=1,\cdots,n$, we can see that $\pi_{(n),R}$ is strictly increasing with respect to i because $0<\alpha_{(n)}<1$. Therefore, the retailer selects the contract $\{w_{(n),n}, Q_{(n),n-1}\}$. Next, given the retailer's optimal choice, supplier's optimal contract is $\{w_{(n),n}, q\left(w_{(n),n}\right)\}$ under complete information. Thus, under an n- price contract ($n\geq 2$), the supplier's and the retailer's profits are given by $\pi_{(n),S}=2u\left(1-\alpha_{(n)}^{n-1}\right)\left(\alpha_{(n)}^{n-1}p-c\right)$ and $\pi_{(n),R}=up\left(1-\alpha_{(n)}^{n-1}\right)^2$, respectively.

We shall look at the performance of a complex contract with $n-1$ price breaks in the worst-case scenario. The focus is on finding a bound on the optimality gap of the n-price contract, i.e., $\pi_{CSC}-\pi_{(n),DSC}$. Furthermore, we shall also find a bound on the double marginalization loss $\phi(n)$ for $n\geq 2$. Based on the analysis given above and

Definition 1, we obtain the following propositions.

Table 2: The menu of contracts $\left\{w_{(n),i}, Q_{(n),i-1}\right\}_{i=1,\cdots,n}$

	$w_{(n),1}$	$w_{(n),2}$	$w_{(n),3}$	$w_{(n),4}$	$w_{(n),5}$		$\alpha_{(n)}$	$Q_{(n),1}$	$Q_{(n),2}$	$Q_{(n),3}$	$Q_{(n),4}$		$\pi_{(n),S}$	$\pi_{(n),R}$	$\phi(n)$
$n=1$	$\frac{p+c}{2}$												$\frac{u}{2}\frac{(p-c)^2}{p}$	$\frac{u}{4}\frac{(p-c)^2}{p}$	25%
$n=2$	p	$\alpha_{(2)}p$					$\frac{c}{p}$	$\Psi(w_{(2),2})$					0	$u\frac{(p-c)^2}{p}$	0%
$n=3$	p	$\alpha_{(3)}p$	$\alpha_{(3)}^2 p$				$\frac{\left(\sqrt{5+\frac{4c}{p}}-1\right)}{2}$	$\Psi(w_{(3),2})$	$\Psi(w_{(3),3})$				$\Lambda\left(\alpha_{(3)}^2\right)$	$up\left(1-\alpha_{(3)}^2\right)^2$	$\phi(3)$
$n=4$	p	$\alpha_{(4)}p$	$\alpha_{(4)}^2 p$	$\alpha_{(4)}^3 p$			$\mathrm{Z}(p,c)$	$\Psi(w_{(4),2})$	$\Psi(w_{(4),3})$	$\Psi(w_{(4),4})$			$\Lambda\left(\alpha_{(4)}^3\right)$	$up\left(1-\alpha_{(4)}^3\right)^2$	$\phi(4)$
$n=5$	p	$\alpha_{(5)}p$	$\alpha_{(5)}^2 p$	$\alpha_{(5)}^3 p$	$\alpha_{(5)}^4 p$		$\Omega(p,c)$	$\Psi(w_{(5),2})$	$\Psi(w_{(5),3})$	$\Psi(w_{(5),4})$	$\Psi(w_{(5),5})$		$\Lambda\left(\alpha_{(5)}^4\right)$	$up\left(1-\alpha_{(5)}^4\right)^2$	$\phi(5)$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$		$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$		$\vdots$	$\vdots$	$\vdots$
$n\to+\infty$	p	$\alpha_{(+\infty)}p$	$\alpha_{(+\infty)}^2 p$	$\alpha_{(+\infty)}^3 p$	$\alpha_{(+\infty)}^4 p$	$\cdots$	$\alpha_{(+\infty)}\to 1$	$\Psi(w_{(+\infty),2})$	$\Psi(w_{(+\infty),3})$	$\Psi(w_{(+\infty),4})$	$\Psi(w_{(+\infty),5})$	$\cdots$	$(1-\lambda)u\frac{(p-c)^2}{p}$	$\lambda u\frac{(p-c)^2}{p}$	0%

Note. 1. $Q_{(n),i}=\Psi(w_{(n),i+1})=2u\frac{p-w_{(n),i+1}}{p}$, where $i=1,\cdots,n-1$ and $n=2,3,\cdots$.

2. $\Lambda\left(\alpha_{(\mathrm{n})}^{n-1}\right)=2u\left(1-\alpha_{(\mathrm{n})}^{n-1}\right)\left(\alpha_{(\mathrm{n})}^{n-1}p-c\right)$, where $n=3,4,\cdots$.

3. $\mathrm{Z}(p,c)=\frac{1}{6}\left(\frac{\Phi(p,c)}{p}+\frac{4p}{\Phi(p,c)}-2\right)$, where $\Phi(p,c)=\left(\left(12\sqrt{3}\sqrt{27c^2+50pc+23p^2}+108c+100p\right)p^2\right)^{\frac{1}{3}}$.

4. $\Omega(p,c)=-\frac{1}{4}+\frac{1}{12}\theta^{\frac{1}{2}}+\frac{1}{12}\sqrt{6}\left(4\theta 12^{\frac{2}{3}}p(p+c)-\frac{\theta 12^{\frac{1}{3}}\tau^2+3p\tau\theta-9p\tau}{p\tau\theta}\right)^{\frac{1}{2}}$,

where $\theta=-\frac{24\sqrt[3]{12^2}pc+24\sqrt[3]{12^2}p^2-6\sqrt[3]{12}\tau^2-9p\tau}{p\tau}$,

and $\tau=\left(\left(\sqrt{3}\sqrt{\frac{256c+283p}{p}}c+\sqrt{3}\sqrt{\frac{256c+283p}{p}}p-9c-9p\right)p^2\right)^{\frac{1}{3}}$

Proposition 3. *Under an* n*-price contract, the double marginalization loss is*

$$\phi(n)=\left(\frac{1-\alpha_{(n)}^{n-2}}{\left(1-\alpha_{(n)}^{n-1}\right)+\left(1-\alpha_{(n)}^{n-2}\right)}\right)^2\times 100\% \tag{9}$$

which is strictly increasing with respect to n *for* $n\geq 2$*, and satisfies* $0\leq\phi(n)<\phi(1)=1/4$.

Proof: According to formula (8) and noting that the centralized supply chain's profit is given by $\pi_{CSC}=\left(u(p-c)^2\right)/(p)$, it follows that $\pi_{CSC}=up\left(\left(1-\alpha_{(\mathrm{n})}^{n-2}\right)+\left(1-\alpha_{(\mathrm{n})}^{n-2}\right)\right)^2$. Next, we consider the optimality gap for the n-price contract, i.e., $\pi_{CSC}-\pi_{(n),DSC}$. Based on $\pi_{(n),S}=2u\left(1-\alpha_n^{n-1}\right)\left(\alpha_n^{n-1}p-c\right)$ and $\pi_{(n),R}=up\left(1-\alpha_n^{n-1}\right)^2$, it follows from (8) that

$$\begin{aligned}\pi_{CSC}-\pi_{(n),DSC} &= up\left(\left(1-\alpha_{(n)}^{n-2}\right)+\left(1-\alpha_{(n)}^{n-2}\right)\right)^2 - up\left(\left(1-\alpha_{(n)}^{n-1}\right)\left(3-2\alpha_{(n)}^{n-2}-\alpha_{(n)}^{n-1}\right)\right)\\ &= up\left(\left(1-\alpha_{(n)}^{n-2}\right)+\left(1-\alpha_{(n)}^{n-2}\right)\right)^2 - up\left(\left(1-\alpha_{(n)}^{n-1}\right)^2+2\left(1-\alpha_{(n)}^{n-1}\right)\left(1-\alpha_{(n)}^{n-2}\right)\right)\\ &= up\left(1-\alpha_{(n)}^{n-2}\right)^2\end{aligned} \tag{10}$$

From (4) and (10), we have $\phi(n)=\left(\left(1-\alpha_{(n)}^{n-2}\right)/\left(\left(1-\alpha_{(n)}^{n-1}\right)+\left(1-\alpha_{(n)}^{n-2}\right)\right)\right)^2$. Taking the first order differentiation of $\phi(n)$ with respect to n, we get

$$\begin{aligned}\frac{d\phi(n)}{dn} &= \frac{-2\ln\left(\alpha_{(n)}\right)\left(\alpha_{(n)}^{n-2}\left(\left(1-\alpha_{(n)}^{n-1}\right)+\left(1-\alpha_{(n)}^{n-2}\right)\right)-\left(1-\alpha_{(n)}^{n-2}\right)\left(\alpha_{(n)}^{n-1}+\alpha_{(n)}^{n-2}\right)\right)}{\left(\left(1-\alpha_{(n)}^{n-1}\right)+\left(1-\alpha_{(n)}^{n-2}\right)\right)^2}\\ &= -2\ln\left(\alpha_{(n)}\right)\frac{\left(\alpha_{(n)}^{n-2}-\alpha_{(n)}^{n-1}\right)}{\left(\left(1-\alpha_{(n)}^{n-1}\right)+\left(1-\alpha_{(n)}^{n-2}\right)\right)^2}\end{aligned} \tag{11}$$

Since $\ln\left(\alpha_{(n)}\right)<0$, $\alpha_{(n)}^{n-2}>\alpha_{(n)}^{n-1}$, we have $d\phi(n)/dn>0$, implying that $\phi(n)$ is strictly increasing with respect to n for $n\geq 2$. It is straightforward to show that $\phi(n)\geq 0$ as $0<\alpha_{(n)}^{n-1}<\alpha_{(n)}^{n-2}<1$. Next, we need to give an upper bound on the double marginalization loss. Under the n-price contract, we have $1-\alpha_{(n)}^{n-1}>1-\alpha_{(n)}^{n-2}$ as $0<\alpha_{(n)}^{n-1}<\alpha_{(n)}^{n-2}<1$. It is straightforward to show that $\left(1-\alpha_{(n)}^{n-1}\right)+\left(1-\alpha_{(n)}^{n-2}\right)>2\left(1-\alpha_{(n)}^{n-2}\right)$. Consequently, we obtain $\left(\left(1-\alpha_{(n)}^{n-1}\right)+\left(1-\alpha_{(n)}^{n-2}\right)\right)^2>\ 4\left(1-\alpha_{(n)}^{n-2}\right)^2$ as $1-\alpha_{(n)}^{n-1}>0, 1-\alpha_{(n)}^{n-2}>0$. This completes the proof.

The decentralized supply chain's profits $\pi_{(n),DSC}$ under complex contract with $n-1$ price breaks for $n\geq 2$ are higher than the supplier chain's profit π_{DSC} under simple contract. More importantly, the upper bound for the double marginalization loss can always be increased by adding one more price break. However, the worst-case is no more than $\phi(1)$. This result implies that a small number of price breaks in an all-quantity discount contract is sufficient for an n-price contract in practice. From Proposition 3, we have the following observations.

Observation 1. *Two-price contract can eliminate the double marginalization loss, i.e.,* $\phi(2)=0$, *but it cannot arbitrarily allocate the additional profits such that both the retailer and the supplier can benefit.*

Observation 2. *The* n*-price contracts reduce the double marginalization loss for* $n\geq 3$, *i.e.,* $0<\phi(n)<\phi(1)$, *and improve the performance of decentralized supply chain.*

In practice, the contract designer (the supplier) may actually prefer to offer complex contracts with small number of price breaks even though these contracts do not optimize the supply chain's performance. In Section 5, we analyze, through numerical simulation, the supply chain performances of the three-price contract, four-price contract, and five-price contract under different conditions.

5. The value of contract complexity

Two extreme types of contracts are covered: the simple contract that cannot coordinate the supply chain (Definition 1) and the complex contracts with an infinite number of price breaks that can coordinate the supply chain and achieve win-win outcome (Proposition 1). Observations 1 and 2 have shown that complex contracts with a small number of price breaks cannot coordinate the supply chain. However, these contracts are still valuable. In this section, we explore the value of contract complexity to supplier's profit, retailer's profit, and supply chain efficiency. In particular, this study investigates the value of contract complexity to the profits of supplier, retailer, and decentralized supply chain (the double marginalization) using the models considered in Section 4. The analysis is carried out through numerical simulation.

To better understand the value of contract complexity in reducing or eliminating supply chain efficiency loss, we define the normalized supplier's cost as $\bar{c} := c/p$. Our study is focused on 3 scenarios $\bar{c} := c/p = 1:2$, $\bar{c} := c/p = 1:3$, $\bar{c} := c/p = 1:4$, where the supplier's unit cost is $c = 10$ and the retailer's unit selling price is $p \in \{20, 30, 40\}$. In addition, we assume that the mean demand is $u = 1000$, and the range of the demand distribution of seasonal products is $2v = 2000$. Based on the setting of these parameters, the following proposition is obtained.

Proposition 4. *Under simple contract and complex contract, the supplier's and retailer's profits depend only on the contract parameters* c, p, *and* $\alpha_{(n)}$.

From the discussions above and Proposition 4, combined with analysis through numerical simulation (see Table 3), we have the following observation.

Observation 3. *For* $n \geq 2$, *it holds that*

(a) The more complex the contract, the higher the supplier's expected profit, but $\pi_{(n),S}$ *is always worse than* $\pi_{(1),S}$.

(b) The more complex the contract, the lower the retailer's expected profit, but $\pi_{(n),R}$ *is always better than* $\pi_{(1),R}$.

(c) The more complex the contract, the lower the decentralized supply chain's total profits, but $\pi_{(n),DSC}$ *is always better than* $\pi_{(1),DSC}$.

Consider the case when $\bar{c} := c/p = 1:2$, Observation 3 (a) (Observation 3 (b)) reveals that contract complexity can increase (decrease) the supplier's (retailer's) excepted profit. For example, there will be an increase (a decrease) of 60.94% (39.06%) when the supplier moves from a two-price contract to a five-price contract. Compared with the simple contract, as the contract complexity increases, the supplier's expected profit increases but always lower than $\pi_{(1),S}$, and the retailer's expected profit decreases but always higher than $\pi_{(1),R}$. More importantly, from Observation 3 (c), the claim that complex contracts with a finite number of price breaks can optimize the decentralized supply chain's profit is flawed. The decentralized supply chain's profit decreases by 19.20% when the supplier moves from a two-price contract to a five-price contract. A similar analysis can be carried out for the case when $\bar{c} := c/p = 1:3$ (or $\bar{c} := c/p = 1:4$).

It is interesting to observe the changes in the supply chain efficiency (i.e., $1-\phi(n)$) relative to the supplier's normalized cost $\bar{c}$. Under simple contract (one-price contract), the decentralized supply chain efficiency is a constant, i.e., $1-\phi(1)$=75%, for the cases when $\bar{c}:=c/p=1:2$, $1-\phi(1)$=75%; when $\bar{c}:=c/p=1:3$, and $1-\phi(1)$=75%; and when $\bar{c}:=c/p=1:4$ (see Table 3). Similarly, it is easy to see that the decentralized supply chain efficiency is a constant, which is 100%, under two-price contract. Under three-price contract, the decentralized supply chain efficiency exhibits an increase of 0.33% when the supplier's normalized cost moves from $\bar{c}:=c/p=1:4$ to $\bar{c}:=c/p=1:3$, and an increase of 0.59% when the supplier's normalization cost moves from $\bar{c}:=c/p=1:3$ to $\bar{c}:=c/p=1:2$ (see Table 3). The same observations under four-price contract (or five-price contract) are also revealed. As a result, the decentralized supply chain efficiency is strictly decreasing with reference to the supplier's normalized cost under complex contract with a finite number of price breaks ($n=3,4,5$). Proposition 5 characterizes the relationship between contract complexity and supply chain efficiency (see Table 3).

Proposition 5. *(a) The supply chain efficiency is independent of the supplier's normalized cost* $\bar{c}$ *under simple contract and two-price contract; and*

(b) under complex contract with a finite number of price breaks ($n=3,4,5$*), the supply chain efficiency* $1-\phi(n)$ *depends on the supplier's normalized cost* $\bar{c}$ *and increases with respect to* $\bar{c}$*.*

Next，by comparing simple contract with complex contracts, additional insights into the effect of the contract complexity on coordination contract are clearly observed from Proposition 4 and Proposition 5. Compared with simple contract, two-price contract can achieve supply chain coordination, but it is not feasible in practice because the retailer extracts all profit under coordination. Suppose that the supplier's normalized cost is $\bar{c}:=c/p=1:2$. Then, the coordination contract's efficiency is 87.45% (82.91%, 80.80%) under three (four, five)-price contract. This implies that coordination contract's efficiency decreases relative to the increase of contract complexity. We have the following observation.

Observation 4. *The increase of contract complexity does not increase the coordination contract's efficiency, but will reduce double marginalization.*

From Observation 4, both the supplier and the retailer, as a whole, are better off under the complex contract. However, from Table 3, It is also observed that the contract designer and the supply chain system do not necessarily benefit from a more complex contract. When $\bar{c}:=c/p=1:2$ ($\bar{c}:=c/p=1:3$, $\bar{c}:=c/p=1:4$), among the three-price contract, four-price contract and five-price contract, the three-price contract is better. It can capture 87.45% (86.88%, 86.53%) of the supply chain profit. We have the following observation.

Observation 5. *Three-price contract (complex contract with two price breaks) is sufficient for a general supply chain and it is preferred in practice despite being theoretically suboptimal.*

Table 3: Optimal contract parameters and supplier, retailer, and supply chain profits under different contracts

		$w_{(n),1}$	$w_{(n),2}$	$w_{(n),3}$	$w_{(n),4}$	$w_{(n),5}$	$\alpha_{(n)}$	$Q_{(n),1}$	$Q_{(n),2}$	$Q_{(n),3}$	$Q_{(n),4}$	$\pi_{(n),S}$	$\pi_{(n),R}$	$1-\phi(n)$
$\bar{c}=1:2$	$n=1$	15										2500	1250	75%
	$n=2$	20	10				0.5000	1000				0	5000	100%
	$n=3$	20	16.46	13.54			0.8228	354	646			2288	2085	87.45%
	$n=4$	20	17.81	15.87	14.13		0.8907	219	413	587		2425	1721	82.91%
	$n=5$	20	18.42	16.96	15.62	14.38	0.9209	158	304	438	562	2462	1578	80.80%
$\bar{c}=1:3$	$n=1$	20										6667	3333	75%
	$n=2$	30	10				0.3333	1333				0	13333	100%
	$n=3$	30	22.75	17.25			0.7583	483	850			6163	5418	86.86%
	$n=4$	30	25.47	21.63	18.37		0.8491	302	558	775		6489	4509	82.49%
	$n=5$	30	26.71	23.77	21.16	18.83	0.8902	220	415	589	744	6577	4153	80.47%
$\bar{c}=1:4$	$n=1$	25										11250	5620	75%
	$n=2$	40	10				0.2500	1500				0	22500	100%
	$n=3$	40	28.99	21.01			0.7247	550	949			10454	9015	86.53%
	$n=4$	40	33.08	27.36	22.63		0.8271	346	632	868		10970	7539	82.26%
	$n=5$	40	34.95	30.54	26.68	23.32	0.8737	252	473	666	834	11108	6959	80.30%

6. Conclusions and managerial insights

This study analyzes a two-tier supply chain consisting of one supplier and one retailer. The retailer procures seasonal products from the supplier and sells these products to its end consumers under Newsvendor environment. The supplier offers contracts with different price breaks and determines the wholesale prices and price blocks. Among these contracts, the retailer chooses one and decides the quantity of items to buy from the supplier. To the best of our knowledge, this study is the first to investigate the influence of contract complexity on general supply chain coordination in operation management. It is clearly important to consider contract complexity as a factor in the coordination of the contract design. From the study being carried out in this paper, it was found that the claim that complex contracts can optimize the supplier's profit in a decentralized supply chain is flawed. This finding is consistent with the finding of Kalkanci et al. (2011). On the other hand, although complex contracts do not optimize the decentralized supply chain's profit, they are better than simple contracts in the context of improving decentralized supply chain's performance, such as reducing the double marginalization loss.

The finding in this paper reveals the role that contract complexity plays in the coordination of the contract design, i.e., the value of the contract complexity in the general supply chain coordination. The results show that complex contracts with an infinite number of price breaks can coordinate the general supply chain and achieve the win-win outcome under mild conditions. The finding is consistent with that obtained by Cachon (2003). It is also shown that the simple contract and complex contracts with a small number of price breaks do not coordinate the supply chain, but the latter can improve the performance of a decentralized supply chain. These analyses show that an increase in contract complexity does not increase the efficiency of coordination contract. This also analyzes the impact of supplier's normalized cost on supply chain efficiency, showing that the supply chain efficiency is independent of the normalized cost of the supplier under the simple contract and the double-price contract setting. However, it depends on the normalized cost of the supplier under under complex contract with n-price breaks for $n = 3,4,5$. Finally, combined with theoretical predictions and the analysis being carried out through numerical simulation, it is found that the contract designers and supply chain systems do not necessarily benefit from relatively more complex contracts. It also shows that the three-price contract (complex contract with two price breaks) is sufficient for a general supply chain and it is preferred in practice, although theoretically suboptimal.

The research in this paper still has some limitations. If the supplier and retailer in this study are risk-neutral decision makers, it would be of interest to consider their behavioral preferences, including risk aversion and loss aversion. Therefore, studying the impact of contract complexity on supply chain coordination with behavioral preference will be potentially an important issue. Secondly, this study does not consider the impact of information asymmetry on the value of contract complexity. Thus, the role of information asymmetry in contract design will also be a promising research direction. Thirdly, this paper only studies the coordination problem between a single supplier and a single retailer. In practice, a supplier often works with multiple retailers simultaneously. Thus, the impact of contract complexity on coordination of a general supply chain consisting of one supplier and multiple retailers will be an important topic for future research.

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