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Robustness Assessment of Urban Rail Transit Network Based on the Interdependency Analysis: Chongqing Rail Transit in Jiangbei and Yuzhong as an Example

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Abstract. As transportation demand develops, urban rail transit networks become increasingly crowded, making them more linked and harder to manage. Furthermore, system failures and climate change are expected to increase the number of disruptions. Consequently, evaluating system robustness is critical to ensure it's operating safely. Complex network theory is used to investigate the overall topological structure of the urban rail transportation network (URTN). Attack strategies for URTN are designed when the topological and robustness analysis parameters have been calculated. According to the research, malicious attacks have a more significant effect on system robustness than random attacks. The proposed methodology is applied to the Chongqing rail transit in the Jiangbei and Yuzhong districts.

Keywords: Robustness assessment; urban rail transit system; Chongqing rail transit network.

1. Introduction

Urban rail transit establishes itself as a viable mode of public transportation and plays an increasingly important part in people's everyday commute. By significantly boosting public transportation capacity, urban rail transit systems offer an effective alternative for resolving transportation difficulties in cities. This advantage has accelerated the building of metro systems in many significant towns [1]. As a result of this benefit, several metropolitan cities have hastened the construction and operation of metro networks. As the number of lines rises, metro systems usually expand to vast and complicated network sizes [2]. The urban rail transit system is under threat from various risk incidents, so it's critical to look into its robustness and ensure it operates safely. Many metropolises and nations, such as Shanghai, Beijing, and Singapore, have developed many rail transportation lines, resulting in complex subway networks with intricate inter-station coupling arrangements and high station densities [2]. The significance of urban rail transit is demonstrated by its high passenger capacity, excellent transportation efficiency, low environmental effect, and various other features [3]. However, the systematic risk of insufficient planning, insufficient collaborative management procedures, malicious assaults, and natural calamities has aroused governments' and academia' attention. Given that a rail transit system's principal role is to permit the mass movement of people and products throughout a network, any interruption in the rail transit network's components might affect the system's regular operation [4]. Electrical problems, train breakdowns, and engineering failures can result in the temporary shutdown of functions in one or both directions, and incidents such as suicides, strikes, or demonstrations can also disrupt the rail transport network's functioning. More dramatic occurrences, such as terrorist attacks, would have a disproportionately significant and long-lasting impact on the network [5]. In particular, because the network's many stations are placed in various locations, station disruptions might affect the overall rail transport network. To guarantee that the rail transportation network runs smoothly, it is necessary to examine its robustness and identify specific critical nodes with a high susceptibility, which is the purpose of this research.

Robustness is a term that refers to the property of being robust and healthy or of being unlikely to break or fail. The robustness of a rail transportation network is determined in several unintentional behaviors, most of which are related to system dependability and variability, and is a design criterion for the system. A rail transportation network can resist unexpected risk events with a minimal drop in operational performance and retain partial characteristics in the face of network component failures. The amount of stress that a system can sustain before failing is called its capability [6]. Considering the many types of hazards that exist in the urban rail transit network (URTN), this study investigates the rail transportation system's resiliency. Understanding a rail transport network's topological properties is crucial for increasing the network's resilience to internal interruptions and external threats [7]. Numerous studies have examined the safety management of rail transportation systems using complex network theory. The following qualities make complex network theory applicable: (1) the rail transportation network has precise characteristics of a scale-free network, with most nodes in the scale-free system. Networks have exponents between 2 and 3, and there is no accurate degree or size. Still, only a few nodes, known as hubs, have a considerable degree of Connectivity; (2) Conventional applications of complex networks to a variety of infrastructures, including grid networks, aviation networks, pipeline networks, and the Internet, have demonstrated the theory's applicability at a variety of network sizes; (3) Recent risk events in the rail transport system have revealed that risk events in hub stations can result in systemic malfunction or operational changes [8].

The preceding three characteristics give a reasonable basis for examining rail transportation networks via the lens of complex network theory. The complex network theory is used in this research, and the case study is the Chongqing rail Transit System (CRT) in Jiangbei and Yuzhong districts. The following is a description of the study. The literature review is the second section. The methodology for the URTN is shown in the third section, which includes the complex Network, topological parameters, robustness analysis parameters, and attack strategies. The fourth section presents the case study and results, and the discussion and conclusion are shown in the last two sections.

2. Literature review

Many studies have been conducted to investigate the robustness of subways networks based on Network topological characteristics. The world's most extensive subway systems were studied, and the findings revealed that the networks are resistant to random attacks due to their high connectivity and low maximum vertex degree [9]. Several studies have been undertaken on the safety of metro systems by studying networks, notably using topographical mapping models. A metro network may be represented in a topological graph by reducing metro tunnels and metro stations as connections and nodes in the topology. A network's topological analysis, which includes nodes and connections, route length, and cluster coefficient, offers a practical and logical foundation for determining the safety of a transportation network. For complicated network analysis, the SMALL-WORLD network model was proposed. The nodes of a small-world network system are generally heavily clustered while having low characteristic route lengths, meaning that even if the network is disrupted, the nodes are still linked. Large-scale networks were investigated, and it was observed that node connection, or network connectivity, typically follows a scale-free power-law distribution. This means that the Network's Connectivity is resilient in the face of random failure but susceptible in the event of a deliberate assault. The primary network evaluation methods, on the other hand, are essentially qualitative, using conceived metrics that do not need a thorough comparison of network safety levels. In this respect, network analysis combined with quantitative measurements of resilience and vulnerability aids in determining a network's degree of safety [10]. The robustness of metro networks in an accident was studied statistically. The robustness of a metro network is measured in terms of network connection remaining after a node failure. The topological features of the rail transit network were quantitatively examined, and the resilience of urban rail transit was assessed using a complex network [11]. Ten theoretical and four numerical resilience measures and their efficacy in determining the robustness of existing metro networks were explored utilizing network science and graph theory.

The resilience of a complex network refers to its capacity to resist random or targeted assaults. Most network robustness tests assume that network nodes are homogenous and abstract. The majority of real-world networks, on the other hand, are made up of nodes [12]. The robustness of urban rail transportation was assessed using complex network theory. The topological properties of the Beijing rail transit system were quantitatively analyzed using a mathematical, statistical model, with the results demonstrating the system's typical scale-free network characteristics. Some cases, such as the Shanghai metro system and the Beijing rail transit system, were used in this research study. Similarly, the statistical topological characteristics of the Beijing rail transport network were quantitatively studied using complex network theory. The findings indicated that the loop line's damage threshold is lower than that of a straight line when assaulted and that assaulting the loop line is harder to regulate. Topological metrics were used to assess the Shanghai rail transit network's dependability and resilience, and two new measures, functionality loss and subway line connectivity were developed [13,14]. A vulnerability analysis of the Shanghai metro network was also performed, and it revealed that stations with a high degree of Betweenness are more critical in maintaining network size. In contrast, stations with a high degree of Betweenness are essential to network efficiency and Connectivity [15]. Under daily operational risk events, a resilience method was demonstrated integrating the network topological and passenger volume characteristics of a rail transit network. The results revealed that the identification of critical stations depends on the duration time of different risk events and the characteristics of the stations [16].

The metro network vulnerability was proposed from line operation and indicated that (1) Metro lines with a high passenger volume significantly impact network vulnerability. And (2) in an emergency, the circle line could significantly impact passenger flow re-distribution. In addition, the networked characteristics of two metro networks were analyzed, and two malicious attacks were used to investigate the vulnerability of metro networks [17].

3. Research methodology

3.1. Urban rail transit network (URTN) construction

Several networks have been built using complex network theory for the world's most extensive subways. The components of a system are often defined as vertices in a complex network model, and the interactions between vertices are represented as edges. The network properties are shown, including high connectivity, low maximum vertex degree, and typical small-world and scale-free features.

According to complex network theory, stations in the URTN could be virtualized as nodes, and tracks connecting two stations directly could be virtualized as edges. The URTN could be an undirected graph because urban rail transit generally has two-way traffic.

The undirected graph is defined as $G = \langle V, \mathrm{E}, A \rangle$, where

$V = \left\{ v_i \middle| i = I \triangleq \{1, 2..., N\} \right\}$ Is the set of network nodes. And

$E = \left\{ a_{ij=(vi,vj)} \middle| i, j \in I \right\} \subseteq V \times V$ Is the set of network edges between two nodes. And

$A = [\mathrm{a}_{ij}]_{N \times N}$ is the network adjacency matrix. Where a_{ij} is defined as:

$$a_{ij} = \begin{pmatrix} 1 & (\mathrm{v}_i, \mathrm{v}_j \in E \\ 0 & (\mathrm{v}_i, \mathrm{v}_j \notin E \end{pmatrix} \qquad (1)$$

Furthermore, assume $a_{ij} = 0$ for all $i \in I$. Because the graph is undirected. And thus, the adjacency matrix A is symmetric and nonnegative. The network adjacency matrix is established according to the operation situation of the URTN. The network could be constructed by inputting the adjacency matrix into the software UCINET.

3.2. Topological Parameters of URTN

(1) Degree and Degree Distribution

In the context of URTN, degree K_i refers to the number of lines connected to the station and represents the local property of the station. The higher the degree, the more lines the station connects to, and the higher the interactive ability of the station. The degree distribution p_k the probability of a node with a degree k is a probability distribution function over the whole node.

(2) Betweenness

The Betweenness B_i of the node V_i is the number of shortest paths among all pairs of nodes passing through the node, which measures the nodes as a "bridge" and reflects the node's load.

3.3. Robustness analysis parameters of URTN

(1) Network efficiency

Network efficiency demonstrates the average closeness of every node in the network. The higher the closeness, the shorter the distance between nodes, further the higher the efficiency. Network efficiency is defined as

$$E = \frac{1}{N(N-1} \sum_{i \neq j \in I} \frac{1}{d_{ij}} \tag{2}$$

where N is the number of nodes in the network and d_{ij} Denotes the length of the shortest path between node i and node j. The value of E ranges from 0 to 1. E =1 represents the URTN being an overall coupled network. In this condition, the network has the highest connectivity and highest ability to provide alternative paths when attacked. E = 0 represents all nodes in the network being isolated. In this condition, the Network's Connectivity is the lowest, and the ability to provide alternative paths is zero. Thus, the network is the least robust.

(2) Connectivity

Connectivity indicates the network's degree of connectivity. The greater the network's intensity, the greater its connectivity, and the less the effect of risk occurrences on the network. Connectivity is defined as

$$\partial = \frac{TE}{N(N-1)} \tag{3}$$

where TE is the number of true edges. The (∂) value ranges from 0 to 1. Similar to the network efficiency, E. $\partial = 1$ Represents the URTN being an overall coupled network. In this condition, the network has the highest connectivity and highest ability to provide alternative paths when attacked. σ = 0 represents all the nodes in the network being isolated. The Network's Connectivity is the lowest, and the ability to provide alternative paths is zero. In this condition, the network is the least robust.

(3) Relative size of the maximal connected subgraph (RSMCS)

H is a maximal connected subgraph of G. The relative size of the maximal connected subgraph is defined as

$$S = \frac{N'}{N} \tag{4}$$

where N' Is the number of nodes of H. The value of S ranges from 0 to 1. S = 1 indicates that the network has not been attacked, and S = 0 indicates that the network crashes after being struck. For the value of S, the closer to 1, the higher the ability to provide alternative paths of the network after risk event attacks; conversely, the closer to 0, the lower the ability to do so.

3.4. Attack strategies for the URTN

The rail transit network may have two kinds of attacks: random and malicious. The common actions of the two types of attacks are shown in Table1.

Table 1: The common actions of random attacks and malicious attacks

Attack types	
Random attacks	Malicious attacks
1-Technical malfunctions; Track damages, shattered wheels, failure of the brakes, failing to signal, Train collision, power outage, broken rail, line failure, speed-related crash 2-Passengers' actions; Overcrowding, platform suicide, falls onto the track, drops on the escalators, gang battle, subconsciously destruction due to drunkenness, smoke in the subway, wrongly service by the driver, passenger-carrying dangerous materials, passenger-carrying animals, wild sparked by rumors, caught in the train door.	- Group fighting. -carrying dangerous things like weapons in the tracks. -carrying poison and hazardous goods. -kidnapping. -burglary. -manual damage on the tracks. -manual threat on the train. -intentional explosions. -set flames. -shooting. -Human-caused disaster, and so on.

Both categories of attack could lead to the disruption of operation. In the network simulation, the process of the attacked station is disrupted, and the station will be an isolated node in the network.

4. Research case study and results

4.1. Case study

4.1.1. Basic network information of Chongqing rail transit network

In Chongqing, China, the Chongqing Rail Transport (CRT) is a rapid transit system. Since 2005, it has served the city's important commercial and leisure downtown areas and inner suburbs. The CRT has eight lines and 198 stations as of July 2021, with a total track length of 370 kilometers (230 miles). Lines 1, 4, 5, 6, 10, and the Loop are heavy-rail subways, whereas Lines 2 and 3 are high-capacity monorails. To keep up with urban expansion, work is being done on the Lines 9, 18, and Jiangtiao lines and extensions to Lines 1, 4, 5, 6, 10, and the Loop line. There will be an 18-line network. The Chongqing Rail Transit is a unique transit system in China because of the geography of Chongqing being a densely-populated but mountainous city with multiple river valleys. Heavy-monorail technology is used on two lines, allowing steep gradients, tight turns, and quick transit capacity. They have the ability to carry 32,000 people per hour in each direction. The two monorail lines in the system comprise 98 kilometers (61 miles), making it the world's most extended monorail system. Even if the 11.0 kilometer (6.8 miles) Airport branch is removed, Line 3 is 56.1 kilometers (34.9 miles) long. Its monorail network's length and capacity make it the world's busiest monorail system, with 94 million and 250 million rides on Lines 2 and 3, respectively, in 2015. According to the most recent data, line 3 is also the busiest single monorail line. Chongqing Rail Transit is also constructing numerous very long metro-only suspension bridges. The 1,650 m (5,410 ft) long Egongyan Rail Transit Bridge, with the main suspension span of 600 m (2,000 ft), carries the southern arc of the Loop line over the Yangtze River, making it the world's longest cable-supported metro-only bridge by the main span. Line 10 trains will go over a 1,225-meter (4,019-foot) cable-stayed bridge with a 480-meter (1,570-foot) main span, making it the world's longest metro-only cable-stayed bridge by the main span. The Gaojiahuayuan Rail Transit Bridge crosses the Jialing River and carries the Loop line's western arc. It is a 594 m (1,949 ft) long bridge with a 340 m main span (1,120 ft). Finally, the Chongqing Rail Transit system has double-deck bridges that accommodate both automotive and metro traffic, notably the Chaotianmen Bridge, the longest arch bridge.

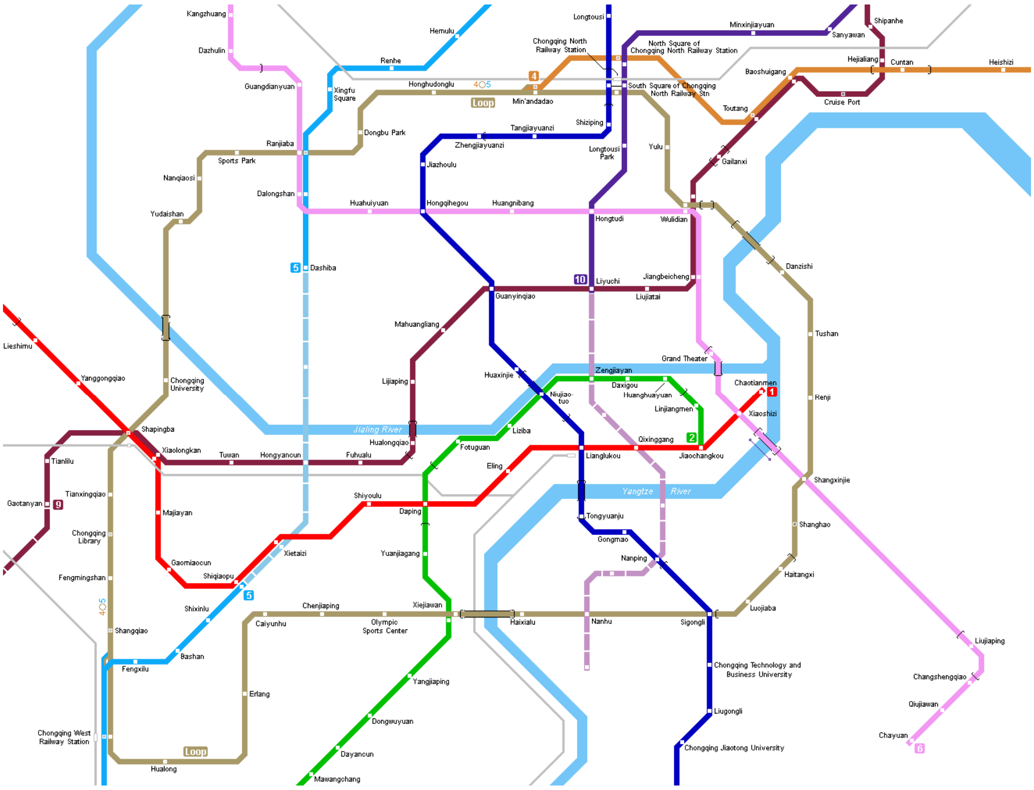

Figure 1: Chongqing rail transit system map

4.1.2. Information and Topological Characteristics of Jiangbei and Yuzhong districts network

Jiangbei and Yuzhong districts are located in the middle of the city of Chongqing. Jiangbei and Yuzhong districts enjoy a high vitality, as most of the railways of Chongqing city pass through them.

There are about 32 metro stations in Jiangbei and Yuzhong districts, including eight distribution stations and 24 separate stations. All the stations are distributed between (line1, line2, line3, line4, line5, line6, line10 and the loop line), while this study included 13 additional stations are located within the neighboring districts. Still, they are connected to the stations of Jiangbei and Yuzhong districts to include 45 stations in the study. Figure 3 shows the topological map of the network that the blue nodes represent the main nodes in the network while the red nodes represent the nodes located in the neighboring districts.

Figure 2: Chongqing map

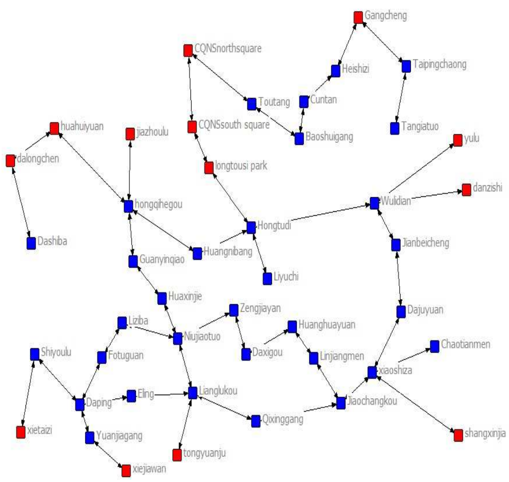

Figure 3: Topological map of the Network

4.2 Topological Characteristics of the Chongqing rail transit (in Jiangbei and Yuzhong districts) network and results

The topological characteristics of the network containing data collection, degree and degree distribution, and the betweenness and robustness analysis are calculated using the UCINET program.

4.2.1. Degree and degree distribution

The degrees of nodes in the Chongqing rail transit system in the Jiangbei and Yuzhong districts network were calculated using UCINET, as shown in Figures 4&5. The highest node degree is 4, as can be observed.

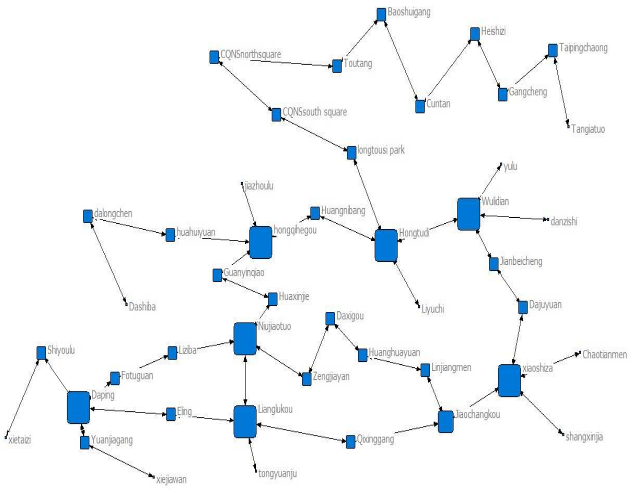

Figure 4: Stations with high degrees

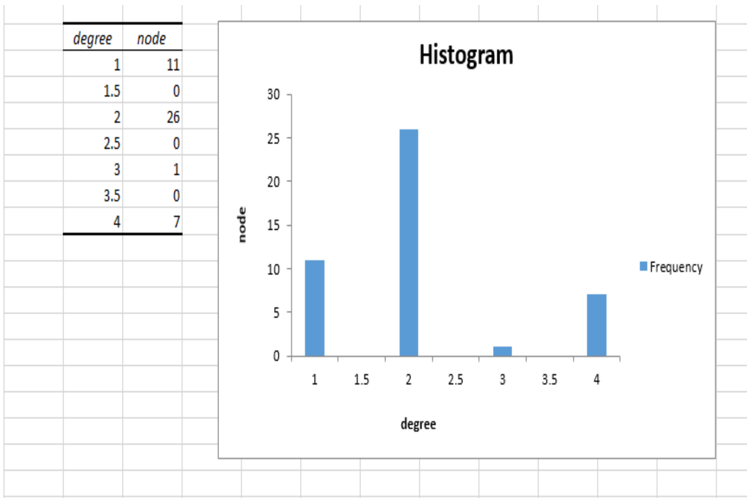

Figure 5: Histogram of the nodes degrees

4.2.2. Betweenness

The Betweenness of nodes in the Chongqing rail transit system in the Jiangbei and Yuzhong districts network was calculated using UCINET. The nodes with the top 10 betweenness values are shown in Table 2. The station Hongtudi possesses the largest betweenness value, 444.

Table 2: Betweenness of the nodes

Betweenness					
No	node	betweenness	no	node	Betweenness
1	Hongtudi	444.000	6	Guanyinqiao	290.000
2	Hongqihegou	381.000	7	Huaxinjie	288.000
3	Niujiaotuo	333.000	8	CQNSsouth Square	288.000
4	Longtousi park	315.000	9	CQNSnorth square	259.000
5	Huangnibang	301.000	10	Lianglukou	247.000

4.3. Robustness analysis for the network

4.3.1. Random attack simulation

The function "= RANDBETWEEN (1, 45)" was first used to generate the network's random attack nodes. Five node numbers were generated: 14, 26, 43, 29, and 3. the five nodes' representativeness is 14 (Liziba), 24 (Dajuyuan), 43 (huahuiyuan), 29 (cuntan), and 3 (Jiachankou). The simulations for the independent attack and continuous attack were then implemented. The three robustness assessment parameters for URTN, network efficiency, connectivity, and RSMCS, were calculated following the random attack simulation. The calculation results and the decrease rates are shown in Table 3 (network efficiency), Table 4 (connectivity), and Table 5 (RSMCS).

4.3.2. Malicious attack simulation

Malicious attacks were implemented for the nodes with high degree values and betweenness values. According to the calculation results of the two parameters, five nodes with the top values of degree and Betweenness are the targets of attack. The five nodes are 21 (Hongtudi, degree = 4, betweenness = 444), 19 (Hongqihegou, degree = 4, betweenness = 381), 13 (Niujiaotuo, degree = 4, betweenness = 333), 5 (Lianglukou, degree = 4, betweenness = 247) and 23 (Wulidian, degree = 4, betweenness = 228). After the malicious attack simulation, the three robustness assessment parameters for URTN, network efficiency, connectivity, and RSMCS were calculated. The calculation results and the decrease rates are shown in Table 3 (network efficiency), Table 4 (connectivity), and Table 5 (RSMCS).

5. Discussion

5.1. Network efficiency analysis

The network efficiency and the decrease rates after the random attack and malicious attack are shown in Table 3. As shown in Table 3, after the independent random attack, the highest decrease rate of the network efficiency is 6.61% (from 0.22693 to 0.21193). In contrast, after the continuous attack, the highest decrease rate of the continuous attack is 28.4% (from 0.22693 to 0.16228). For the malicious attack, after the independent attack, the highest decrease rate of the network efficiency is 25.6 %(from 0.22693 to 0.16868). While after the continuous attack, the highest decrease rate of the continuous attack is 64.8%

It can be concluded that the (TRT in Jiangbei and Yuzhong) network presents high robustness for independent random attacks. Among the five random attack nodes, the four nodes (14 (Liziba), 26 (Dajuyuan), 43 (Huahuiyuan) and, 29 (Cuntan)) are independent nodes, and node 3 (Jiaochangkou) is transfer stations and connect to two rail lines each. For the separate random attack, all the network efficiency decrease rates are below 7%, demonstrating the network's good robustness for independent random attacks. As for the continuous attack, the network efficiencies decrease continuously while the attack nodes increase. When there are five attack nodes, the network efficiency decrease rates reach their largest extent, which is 28.4%.

Comparatively, the malicious attack presents more impacts on the network efficiency. The network efficiency decrease rate for the independent node 21 (Hongtudi) is as much as 25.6%, the largest decrease rate for the separate attack after the malicious attack. The Hongtudi station is located in Jiangbei district –Chongqing city and is one of Line Six and Ten transfer stations. Due to the unique location of the nodes in the Network in Jiangbei and Yuzhong districts, the attack of this station separates the nodes inline Six and Line Ten, as well as eleven nodes in line ten, will be separated from the main network, which means that the whole network has been divided into two parts. Under this condition, the trains in the main network cannot reach the separated nodes and vice versa. Thus, the separation of the network heavily decreases the network efficiency. While after the continuous attack, the highest rate of the continuous attack is 64.8% which is the largest decrease rate for the continuous attack after the malicious attack. In Figure 6, the node 21 (Hongtudi), 19 (Hongqihegou), 13 (Niujiaotuo), 5 (Lianglukou), and 23 (Wulidian). It is highlighted with a red circle. The stations are located in Jiangbei and Yuzhong districts –Chongqing city and are in the lines (one, two, three, six, and ten). Due to the unique location of the nodes in the network, these stations' continuous attacks could separate the whole network, which means that the entire network has been divided into several parts. Under this condition, the trains in the main network cannot reach the separated nodes and vice versa. Thus, the separation of the network heavily decreases the network efficiency.

5.2. Network connectivity analysis

The Connectivity of the original Network is 0.0237. The decreases in connectivity after the random attack and malicious attack are shown in Table 4.

Table 4 shows that the malicious attack presents more influences on network connectivity than the random attack for the independent node attack. The connectivity calculation focuses on the number of true edges of the network. The nodes in the malicious attack have high degree values or betweenness values, and they connect to more edges than other nodes. For example, the nodes 21 (Hongtudi), 19 (Hongqihegou),

Table 3: The network efficiency after attacks.

Network efficiency										
Random attack nodes	Original Net work	14	26	43	29	3	14→26	14→26→43	14→26→43→29	14→26→43→29→3
Network efficiency	o.22693	0.22481	0.21865	0.21193	0.21269	0.21408	0.21397	0.20101	0.18501	0.16228
Network efficiency decrease rate	0.00%	-0.93%	-3.65%	-6.61%	-6.27%	-5.66%	-5.71%	-11.4%	-18.4%	-28.4%
Malicious attack nodes	Original Net work	21	19	13	5	23	21→19	21→19→13	21→19→13→5	21→19→13→5→23
Network efficiency	0.22693	0.16868	0.18811	0.20554	0.1998	0.20044	0.13975	0.12078	0.08980	0.07980
Network efficiency decrease rate	0.00%	-25.6%	-17.1%	-9.42%	-11.9%	-11.6%	-38.4%	-46.7%	-60.4%	-64.8%

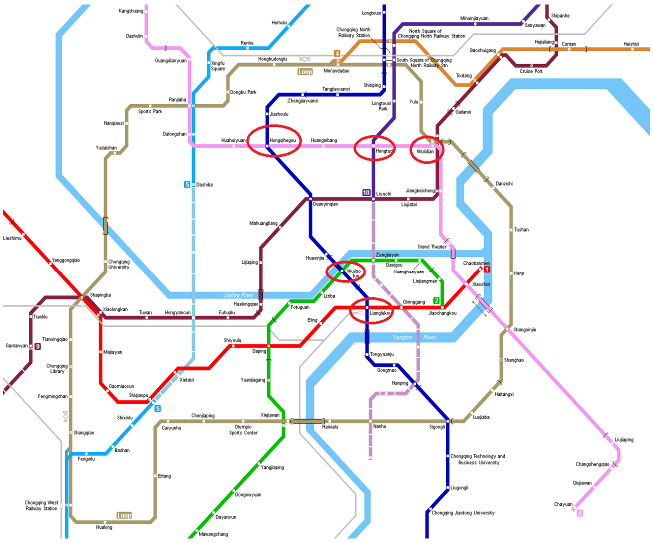

Figure 6: Highlights the five important stations in the network.

13 (Niujiaotuo), 5 (Linglukou), and 23 (Wulidian). Are all transfer stations for two lines and connect to 4 edges. Thus, the attacks of the nodes decrease the connections of these edges. Comparatively, in the random attacks, the selected five nodes connect two edges or three edges. Therefore, the independent malicious attack affects the network connectivity more than the separate random attack. The network connectivity decreases continuously while the isolated nodes increase in the continuous random attack. After the continuous attacks by five nodes (14→26→43→29→3), the decrease rate of the network connectivity reaches its maximum of 13.5% while in the continuous malicious attack; the network connectivity decreases continuously while the isolated nodes increase. After the continuous attack by five nodes (21→19→13→5→23), the network connectivity decrease rate reaches 59.9%.

Table 4: The connectivity after attacks

Network connectivity										
random attack nodes	Original Network	14	26	43	29	3	14→26	14→26→43	14→26→43→29	14→26→43→29→3
Connectivity	0.0237	0.0237	0.0237	0.0232	0.0221	0.0232	0.0237	0.0232	0.0213	0.0205
Connectivity decrease rate	0.00%	0.00%	0.00%	-2.10%	-6.75%	-2.10%	0.00%	-2.10%	-10.1%	-13.5%
Malicious attack nodes	Original network	21	19	13	5	23	21→19	21→19→13	21→19→13→5	21→19→13→5→23
Connectivity	0.0237	0.0179	0.0216	0.0227	0.0227	0.0227	0.0155	0.0139	0.0121	0.0102
Connectivity decrease rate	0.00%	-24.4%	-8.86%	-4.21%	-4.21%	-4.21%	-34.5%	-41.3%	-48.9%	-56.9%

5.3. Relative size of the maximal connected subgraph (RSMCS) analysis

Table 5 demonstrates how the random attack affects the RSMCS parameter. The number of nodes in the largest connected subgraph is the focus of RSMCS. After the unexpected attack of the five nodes ((14 (Liziba), 26 (Dajuyuan), 43 (Huahuiyuan), 29 (Cuntan), and 3 (Jiaochangkou)). which ranges from 75.55% to 97.77. including the independent and continuous attacks, the attacked nodes become the isolated nodes, and the rest of the nodes in the network are still connected. Therefore, the RSMCS after the random attack presents less variation. Comparatively, the malicious attack of the nodes 21 (Hongtudi), 19 (Hongqihegou), 13 (Niujiaotuo), 5 (Linglukou), and 23 (Wulidian). It seriously impacts the parameter RSMCS, ranging from 48.88% to 97.77%. The illustration of the RSMCS decreases is similar to the illustration of the network efficiency decreases in Section 5.1.1. The above five nodes in the important location of the network are the necessary nodes that link to different sections of the network or connect a cluster of nodes with the main network. The attack of these nodes will separate the network into several sections, which will significantly influence the scale of the subgraph. Therefore, the maximal connected subgraph in the network is the rest of the network, excluding the separated nodes. The separation of the network decreases the RSMCS obviously. Malicious attacks of the nodes with important locations will significantly impact the robustness of the network.

Table 5: The RSMCS after attacks

RSMCS										
Random attack nodes	Original Network	14	26	43	29	3	14→26	14→26→43	14→26→43→29	14→26→43→29→3
RSMCS	100.00%	97.77%	97.77%	93.33%	88.88%	97.77%	95.55%	88.88%	77.77%	75.55%
Malicious attack nodes	Original network	21	19	13	5	23	21→19	21→19→13	21→19→13→5	21→19→13→5→23
RSMCS	100.00%	73.33%	88.88%	97.77%	95.55%	93.33%	62.22%	60.00%	55.55%	48.88%

6. Conclusion

Urban rail transit has grown dramatically in recent years as a primary mode of public transportation. Its functioning should be assumed to be safe. This study discusses the robustness of the urban rail transportation system (URTN). The complex network theory was employed in this research due to the topological properties of the rail transportation network. The topological parameters, the URTN's robustness analysis parameters, and defining attack plans for the URTN are all part of the robustness analysis process. The case study used the Chongqing rail transit system in the Jiangbei and Yuzhong districts, and the recommended approach was executed.

Malicious attacks have a more negative effect on the URTN than random attacks, as shown by the robustness evaluation metrics network efficiency and relative size of the maximum connected subgraph (RSMCS). The decrease in robustness is produced by impacting a few edge nodes, which are nodes that connect various areas of the network or connect a cluster of nodes to the main network. This indicates that if certain types of nodes are attacked, the network will be divided into many parts, reducing the network's robustness.

As a consequence of the findings, protection for these nodes should be increased. To a considerable extent, the network transfer stations attacks significantly impact the network's robustness than nodes on an individual rail transport network. Thus, more attention should be devoted to the safety of transfer stations for multi-rail transport lines.

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