

A Financing Newsvendor Inventory Problem with Supplier Buyback Contract Based on Biased Game

Jinqi Wu¹ and Xiaolan Huang^{2}*

¹School of Business Administration, Chongqing University of Science and Technology, Chongqing, 401331, P.R.China.

²School of Economics and Management, Chongqing University of Posts and Telecommunications, Chongqing, 400065, P.R.China

*Corresponding author. Email: 409185377@qq.com

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Abstract. In this paper, we assume that the supplier will buy-back the retailer's unsold goods when the retailer order more than the market demand in the capital-constrained newsvendor problem, so the retailer and supplier can take on the hazard together. This system forms the biased game through the retailer and supplier making the order quantity, the supply price and buy-back price for their max-profits respectively. Furthermore, we studied the current satiation of the supply chain integration between the suppliers and the retailers. At last, a numerical example was included to illustrate the effect of the supply chain integration.

Keywords: Newsvendor problem; Stackelberg biased game; Supply chain Integration

1. Introduction

The newsvendor problem has become a multi-aspect model and is full of vitality because of its important application value in economic life and its wide coverage with a large number of literatures. Erlebacher has given a method of solving the newsvendor problem with constraint and multi-product [1]. Vairaktarakis further studied multi-product newsvendor problem with a budget constraint, by describing two kinds of demand scenarios (interval and discrete) and proposed an algorithm accordingly [2]. Certain uncertainties using interval and discrete demand scenarios was described where a series of mini-max regret formulations were developed for the multi-product newsvendor problem (MPNP) with a budget constraint. An exact solution formula to the MPNP with budget constraint was presented when the demand probability density function is uniform with Generic Iterative Method (GIM) [3]. Layek et al. (2005) presented the research in [3] was analyzed and proved, which could lead to negative optimum to negative optimum order quantities and proposed region calculation to divide starting value and extended extant method [4]. Abdel-Malek et al. (2004) presented an exact solution formula, an approximate model and an Iterative Model of the MPNP with budget constraint [5]. Furthermore they researched dual of the multi-product newsvendor problem (MPNP) with budget constraint [6]. The solution space of the MPNP with a budget constraint was discussed where a threshold which has been used to divide the solution space into three distinct regions was

obtained [4]. Abdel-Malek et al. (2007) tried to get the solution in the method of quadratic programming, and the method of approximation was used to solve the MPNP [7]. A multiple objective programming model was proposed by discussing two-product newsvendor problem with constraint and the two products can substitute one another [8-9]. An improved solution procedure for the fuzzy EOQ model with fuzzy budget and storage capacity constraints was presented by using the max-min operator [10]. A solution algorithm was developed for the constrained MPNP while applying general types of demand distribution function, discrete and continuous types of demand distribution functions to natural binomial [11].

Maqbool Dada (2008) was the first one who developed a new approach to the retailer (the newsvendor) problem with capital-constrained (CCNV) that the retailer could overcome the capital-constrain by borrowing money from a financial institution [12]. The Maqbool model also illustrates how the institution determines the interest rate according to the retailer's financial situation. However, Maqbool data model implies that the retailer does not pay off the loan fully when his revenue is not sufficient. This is infeasible practically because the retailer may suffer some legal disputes or he may face the risk of bankruptcy.

In the Maqbool model [12], the retailer takes all risks but the upstream supplier doesn't share any of these risks. In this paper, we supposed that the supplier would repurchase these unsold products to share market risk with retailer if the retailer's purchase quantity exceeds the market demand. Through a buyback contract, the supplier helps to lower the risks token by the retailer. What' more, it can help to meet clients' needs, preserve brand image, promote new products and competition between retailers.

Usually, retailers can maximize his profile through controlling the purchase quality and the suppliers can set selling price and buy-back price to realize maximum profit; they constitute a Stackelberg game model. However, in this game, one of them has only one kind of strategy (the purchase quality); the other has two kinds of strategies (the selling price and the buyback price). This asymmetric game was called biased game and the game model developed in this paper was called the Stackelberg biased game model.

2. The Stackelberg biased game model of retailer and supplier

In this paper, we considered the capital-constrained newsvendor problem (CCNV) with buy-back contract and guarantee: the retailer purchases first before he sells his goods, but his capital is not sufficient so he need loan from a financial institution. At this time, the supplier provides financing guarantee to the retailer. Customer demand is a probability distribution function with random occurrence and the products un-disposed would be repurchased by the supplier in a price which is lower than the selling price fixed by the supplier. The retailer would pay back the loan first when he gets his revenue. If the retailer's income is sufficient to pay off his loan, he repays the loan by himself, but if his revenue is not enough to pay off the principal and interest, the insufficient part should be undertaken by the warrantor (the supplier). The retailer can optimize his profit by adjusting his order quantity while the supplier can also get an optimal interest with fixing the sales price and the repurchase price. The retailer and the supplier compose a Stackelberg biased game model. The focused model parameters include:

Q_0 : The classical newsvendor's optimal order quantity

Q : The retailer's order quantity

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c : Unit cost of production

r : The interest rate of bank

$w(w > c > 0)$: Unit purchasing price

$p(p > w)$: Unit sales price

$m = \frac{p}{1+r}$: The discounted revenue from one unit of sales

$b(c > b > 0)$: Unit repurchase price

$n = \frac{b}{1+r}$: The discounted revenue from one unit of buy-back products

D : The customer demand

$F(\cdot)$: The cumulative probability distribution of customer demand

$f(\cdot)$: The density probability distribution of market demand

$\eta(\eta < wQ_0)$: The retailer's entity capital

$B = wQ - \eta$: The loan amount borrowed from bank

π_1 : The profit function of retailer

π_2 : The profit function of supplier

π_0 : The retailer's profit without borrowing from bank when his entity capital is sufficient

$\pi_T = \pi_1 + \pi_2$: the entire supply chain profit

2.1. The description of retailer's problem

For simplicity, we assume a zero salvage value for any excess inventory and zero goodwill costs for lost sales and defined $\bar{F}(x) = 1 - F(x)$, the classical newsvendor's optimal order quantity Q_0 satisfies $F(Q_0) = \frac{p-w}{p-b}$.

The retailer's optimal objective function was described in the following form:

$$\max_Q \pi_1 = -\eta - B(1+r) + p \int_0^Q x dF(x) + pQ\bar{F}(Q) + b \int_0^Q (Q-x) dF(x) \quad (1)$$

where:

$\eta + B(1+r)$: The newsvendor's cost of entity capital and loan

$p \int_0^Q x dF(x) + pQ\bar{F}(Q)$: The sales income of the newsvendor

$b \int_0^Q (Q-x) dF(x)$: The buy-back revenue of the unsold products

Furthermore, from the formula (1) and using the Karush–Kuhn–Tucker conditions, we can get:

$$\frac{d\pi_1}{dQ} = -(p-b) \frac{w-n}{m-n} + (p-b)\bar{F}(Q)$$

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The retailer's optimal ordering quantity Q^* is determined as follows:

$$F(Q^*) = \frac{m-w}{m-n} \quad (2)$$

We have given the retailer's best reaction to w and b . Then, let's consider of the supplier's problems. As a dominant player in the Stackelberg game model, the supplier will take the retailer's reaction to the change of w and $b(or n)$ into consideration. And this reaction can be observed by applying the Implicit Function Theorem to the formula (2):

$$\frac{dQ}{dw} = -\frac{1}{(m-n)f(Q)} \quad (3)$$

$$\frac{dQ}{dn} = -\frac{m-w}{(m-n)^2 f(Q)} \quad (4)$$

where $Q = F^{-1}\left(\frac{m-w}{m-n}\right)$.

2.2. The description of supplier's problem

The supplier can determine w and $b(or n)$ to maximize his profit and his optimization objective function can be obtained by the formula below:

$$\max_{w,b} \pi_2 = (w-c)Q - b \int_0^Q (Q-x)dF(x) \quad (5)$$

The first-order condition π_2 respect to w, n is:

$$\frac{\partial \pi_2}{\partial w} = Q + (w-c) \frac{dQ}{dw} - bF(Q) \frac{dQ}{dw} = 0 \quad (6)$$

$$\frac{\partial \pi_2}{\partial n} = (w-c) \frac{dQ}{dn} - bF(Q) \frac{dQ}{dn} = 0 \quad (7)$$

where $Q = F^{-1}\left(\frac{m-w}{m-n}\right)$, $\frac{dQ}{dw}, \frac{dQ}{dn}$ is given by (3) and (4) respectively.

From the formula (6) and (7), the only equilibrium point (w^*, n^*) of the Stackelberg biased game between the supplier and the retailer can be gotten.

3. The model of the supply chain integration

In this part, the optimal objective of supply chain coordination is supply chain integration and pursuing the optimization of entire supply chain to gain the maximum profit. The integration of supply chain is also known as supply and marketing integration which means the supplier and the retailer belongs to the same economic entity or they form a strategic alliance. So the optimization of the entire supply chain which is represented by $\pi_T = \pi_1 + \pi_2$ in our model is the goal of this business model. Thus

$$\begin{aligned} \max_Q \pi_T &= -cQ - Br + p \int_0^Q x dF(x) + pQ\bar{F}(Q) \\ &= -cQ - (wQ - \eta)r + m(1+r) \int_0^Q \bar{F}(x) dx \end{aligned} \quad (8)$$

Through the first-order condition π_T respect to Q , we can get:

$$F(Q) = (p-c-rw)/p \quad (9)$$

Thus, the optimal order quantity of the entire supply chain is

$$Q^* = F^{-1}[(p-c-rw)/p]$$

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and the entire profit of supply chain is determined by the formula (8).

From the formula (8), we know the buy-back revenue of the unsold products $b \int_0^Q (Q - x) dF(x)$ is disappeared that because of it is the only exchange between retailer and supplier in the interior supply chain, so (8) is independent of parameter b .

The first-order condition π_T respect to W is:

$$\frac{d\pi_T}{dw} = -c \frac{dQ}{dw} - Qr - w \frac{dQ}{dw} + p\bar{F}(Q) \frac{dQ}{dw} = 0 \quad (10)$$

Applying the Implicit Function Theorem to the formula (9), we get:

$$\frac{dQ}{dw} = -\frac{r}{pf(Q)} \quad (11)$$

We pay attention to the formula (9), (10) and (11) the only optimization point (Q^*, w^*) of the entire supply chain can be gotten.

4. Examples

We assume that the demand D is an uniform distribution which is defined in the region $[a, b]$ thus,

$$f(x) = \frac{1}{b-a}, F(x) = \frac{x-a}{b-a}, \bar{F}(x) = \frac{b-x}{b-a},$$

the standard deviation is $\sigma = \frac{b-a}{\sqrt{12}}$, the mean value is $\mu = \frac{b+a}{2}$.

We suppose $c=1, w=2, p=3, a=0, b=0.5$ and set these basic parameters values $\eta=0.5, r=0.05$

(a) For the model in which the retailer is the dominant player, the optimal quantity can be derived from the formula (3):

$$Q^* = \frac{m-w}{m-n} = 0.36 < Q_0 = 0.4 \quad (11)$$

Using formula (11) and (1), the optimal expected profit of the retailer can be obtained:

$$\pi_1(Q^*) = \eta r - wQ(1+r) + pQ - \frac{1}{2}(p-b)Q^2 = 0.187 > \pi_1(Q_0) = 0.185$$

Here, the numerical value is increased by 1.08% comparing with the best order quality Q_0 in the classical newsvendor model.

(b) The integration of supply chain requires the optimization of the entire profit $\pi_T = \pi_1 + \pi_2$ of the supply chain.

Considering of the classic optimal order quality $Q_0 = (p-c)/p$, the optimal order quality satisfies:

$$Q^* = (p-c-rw)/p = 0.63 < Q_0 = 0.67 \quad (12)$$

Substituting the formula (12) into formula (10) yields the optimal expected profit of the retailer

$$\pi_T(Q^*) = \eta r - cQ - wQr + p(Q - \frac{1}{2}Q^2) = 0.6267 > \pi_1(Q_0) = 0.6250$$

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which increased by 0.27% comparing with the best order quality Q_0 in the classical newsvendor model.

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