J. Management and Humanity Research
Vol. 8, 2022, 1-9
ISSN: 2582-7766 *(online)*
Published on 20 September 2022
www.researchmathsci.org
DOI:http://dx.doi.org/10.22457/jmhr.v08a012249

Journal of Management and Humanity Research

A Discussion on "Financing Newsvendor Inventory"

***Jinqi Wu*[1*] and *Xiaolan Huang*[2]**

[1]College of Business Administration
Chongqing University of science and technology,
Chongqing, 401331, P.R.China
[2]College of Economics and Management
Chongqing University of Posts and Telecommunications,
Chongqing, 400065, P.R.China
[*]Corresponding author. Email: 286354196@qq.com

Received 19 July 2022; Accepted 2 September 2022

Abstract. Maqbool Dada's capital-constrained newsvendor model implies that a newsvendor does not need to pay off the loan fully when his revenue is not sufficient. This requires the newsvendor to declare bankruptcy in that case, which we call the possible bankruptcy situation. Therefore, the situation of the Maqbool Dada's model applied is very limited. We modify and improve the Maqbool Dada's model by assuming that the newsvendor must fully pay off the loan in any instance so that it can be applied widely; We give two examples and show that even in the possible bankruptcy situation, the increase of the Maqbool Dada model's expected profit for the newsvendor is very less and the bankruptcy risk increases more compared with the revised Maqbool Dada's model; And in the condition of the social optimal order, the social efficiency of the revised model is better than the Maqbool Dada's model.

Keywords: **C**apital constraint; newsvendor financing problem; expected profit; bankruptcy risk.

1. Introduction

There have been a large number of literature on the newsvendor problem, such as [1], but until recently studies with capital-constrained considerations are scarce.

Several earlier studies started to focus on issues related to the capital-constrained newsvendor (CCNV) [2-4]. Vairaktarakis [2] developed a series of minimax regret formulations for the multi-product newsvendor problem (MPNP) with a budget constraint by describing certain uncertainties using interval and discrete demand scenarios. Abdel-Malek et al. [3] presented an exact solution formula to the MPNP with budget constraints when the demand probability density function is uniform and a generic iterative method (GIM). The formula yields a near optimum solution for general continuous density functions of the demand. Later Abdel-Malek and Montanari [4] noted that when the budget is tight, the methods used in the previous study [3] could lead to negative optimum order quantities. This is because they divided the solution space into three distinct regions by two thresholds. But in the third region， the tight budget is not enough to order all the products. Therefore they suggested an alternate approach which deletes some products until the remaining ones can fit within the available budget.

More recently, work on capital-constrained newsvendors became increasingly popular [5-9]. One work proposed an improved solution procedure for the fuzzy EOQ model with fuzzy budget and storage capacity constraints by using the max-min operator [5]. Another developed a solution algorithm for the constrained MPNP, which was binary in nature and applicable to general types of demand distribution functions, both in discrete and in continuous [6]. For the MPNP, Chen and Chen [7] considered a reservation policy by providing discount rate to those customers who are willing to make a reservation. Ref. [8] formulates the MPNP with both supplier quantity discounts and a budget constraint as a mixed integer nonlinear programming model due to price discounts. Zheng [9] investigated a portfolio approach to the MPNP with budget constraints, in which the procurement strategy for each newsvendor product was designed as a portfolio contract. It also gave an efficient solution procedure and provided a sensitivity analysis to the model.

Maqbool Dada [1] was the first one who developed a new approach to CCNV that the newsvendor could overcome the capital constraint by borrowing money from a financial institution. It also illustrated how the institution determined the interest rate according to the newsvendor's financial situation.

However, we noted that the situation of the Maqbool Dada's model applied is very limited (we call it the possible bankruptcy situation): The debtor (newsvendor) does not have any other revenue sources, and does not have the fixed assets that can be mortgaged, too. Once the revenue the debtor gets from the goods sold cannot pay off the loan principal and interest fully, the debtor will declare bankruptcy. We revise the model, broader its applications; We give two examples and point out: Even in the possible bankruptcy situation, the increase of the Maqbool Dada model's expected profit for the newsvendor is very less and the bankruptcy risk increases more compared with the revised Maqbool Dada's model; And in the condition of the social optimal order, the social efficiency of the revised model is better than the Maqbool Dada's model. The mathematical symbols used in this article are as table 1.

Table 1: The mathematical symbols used in this article

Q_0	the EOQ for the classical newsvendor problem	$F(\cdot)$	cumulative probability function of customer demand	m	discounted income of units of goods
Q	the EOQ for newsvendor	$f(\cdot)$	probability density function of market demand	y	the quantity of goods the newsvendor needs sell to repay loan principal and interest
c	the order price of unit goods	$\eta(\eta < cQ_0)$	the funds of the newsvendor owns	π_a	the profit function for the newsvendor
p(p> c)	the price of unit goods sold	$B = cQ - \eta$	the principle the newsvendor finance	π_b	the profit function for the bank
D	the customer demand	r	the bank rates for the newsvendor finances	π_0	the profit of the newsvendor when no loans

A Discussion on "Financing Newsvendor Inventory"

Note: Subscript notation 1 is for the Maqbool Dada's model, and subscript notation 2 is for the revised Maqbool Dada's model. Some symbols which are not included in this table will be described in the following text and we would not repeat them again here.

2. Maqbool dada model and its issues

2.1. Maqbool Dada model

The newsvendor purchase goods before selling. It is not enough for him to purchase goods with his own funds, so he loans from the bank, and then he sells the goods. The customer demand probability distribution function is random; the goods unsold cannot be refunded. After the sales, the sales revenue for the newsvendor will be repaid to the bank's principal and interest preferentially; the newsvendor optimizes his profit through the changes of the order quantity. The classical newsvendor's optimal order quantity, denoted by Q_0, satisfies $\bar{F}(Q_0) = c/p$. Thus the newsvendor's optimization objective function in Maqbool Dada's model is written as [1]:

$$\max_{Q>\eta/c} \pi_n = -[\eta + B(1+r)\bar{F}(y) + p\int_0^y xdF(x)] + [p\int_0^y xdF(x) + p\int_y^Q xdF(x) + pQ\bar{F}(Q)] \quad (1)$$

$$\text{where } y = \frac{(cQ-\eta)(1+r)}{p} = \frac{B}{m},\ m = \frac{p}{1+r} \quad (2)$$

$\eta + B(1+r)\bar{F}(y) + p\int_0^y xdF(x)$ denotes the newsvendor's cost and $p\int_0^y xdF(x) + p\int_y^Q xdF(x) + pQ\bar{F}(Q)$ is the revenue. We substitute formula (1) with formula (2) obtaining [1]:

$$\pi_1 = -\eta + p\int_y^Q \bar{F}(x)dx$$

The EOQ Q_1^* for the newsvendor in Maqbool Dada model satisfies:

$$m\bar{F}(Q_1^*) = c\bar{F}(y) \quad (3)$$

2.2. The revision of the newsvendor problem

The reason to revise the Maqbool Dada model will be explained in the following section, we will not interpret it again here. First of all, we change the formula (1) into:

$$\max_{Q>\eta/c} \pi_a = -\eta - B(1+r) + p\int_0^Q xdF(x) + pQ\bar{F}(Q) \quad (4)$$

where $\eta + B(1+r)$ denotes the costs coming from the newsvendor's capital and he should fully pay off the loan, $p\int_0^Q xdF(x) + pQ\bar{F}(Q)$ denotes the revenues from the newsvendor sales.

Since $B = cQ - \eta$, formula (4) can be changed into:

$$\pi_a = -\eta - (cQ-\eta)(1+r) + p\int_0^Q xdF(x) + pQ\bar{F}(Q) = -\eta - (cQ-\eta)(1+r) + p\int_0^Q \bar{F}(x)dx$$

The EOQ Q_2^* for the newsvendor can be fully characterized by the Karush–Kuhn–Tucker situation:

$$\bar{F}(Q_2^*) = \frac{c}{m}. \quad (5)$$

Generally, we note that $\bar{F}(y) > 0$, so $\bar{F}(Q_2^*) = \frac{c}{m} > \frac{c}{m}\bar{F}(y_1) = \bar{F}(Q_1^*)$, $Q_2^* < Q_1^*$. This means that the newsvendor can order more quantity in the Maqbool Dada model. $\bar{F}(Q_2^*) = \frac{c}{m} > \frac{c}{p} = \bar{F}(Q_0)$, so $Q_2^* < Q_0$, the order quantity of the revised model is slightly less than the classical newsvendor order quantity. If $\frac{r}{1+r} < F(y)$ then $\bar{F}(Q_1^*) = \frac{c}{m}\bar{F}(y) < \frac{c}{m(1+r)} = \bar{F}(Q_0)$, $Q_1^* > Q_0$; on the contrary, turn over. In general, interest r is a very small number, so $Q_1^* > Q_0$, this means that the newsvendor order more quantity in the Maqbool Dada model than in the classical newsvendor model.

2.3. Maqbool Dada model's issues

Formula (1) implies that the newsvendor can only pay off the loan for $p\int_0^y xdF(x) = B(1+r)F(y) - p\int_0^y F(x)dx < B(1+r)F(y)$ when his sales is less than y. It means that the newsvendor does not fully pay off the loan, and the remaining loan $p\int_0^y F(x)dx$ is untreated. So $F(y)$ is the probability of not repaying the loan fully for the newsvendor.

Here, the author of the Maqbool Dada model may argue: if the newsvendor's realized sales revenue falls short of the loan principle and the associated interests, the newsvendor declares bankruptcy, and the bank seizes the sales proceedings. By declaring bankruptcy, the newsvendor walks way without having to repay the shortfalls. This is the typical form of bankruptcy for the limited liability company under which the debtor has limited liability. And in reality, most of the company is the limited liability company, the firm owners do not have to pay off corporate loans by using personal property.

However, we divide the loans into two cases by different debtors: Personal loans and corporate loans. (i) When the loans are personal, newsvendor needs to pay off the loan fully with his other income or property in case his revenue is not sufficient, so the newsvendor's cost is $\eta + B(1+r)$, as the description of formula (4), so the formula (1) does not apply. (ii) When the loans are corporate, and the corporate revenue is not sufficient: i) the corporate will pay off the loans with other sources of business income if the corporate has; otherwise the corporate will pay off the loans with the fixed assets if it has. At this time, the newsvendor's cost is $\eta + B(1+r)$ as the description of formula (4), so the formula (1) does not apply. ii) Therefore, formula (1) is suitable only under very strict preconditions: the corporate has no resource of income from other business and no fixed assets which can be mortgaged (otherwise, the corporate can auction these assets to pay off its loan), and once the corporate revenue of this business is not sufficient to repay the bank, it will declare bankruptcy (we call it possible bankruptcy situation).

As mentioned above in 2.2, the author of Maqbool Dada model may also argue the optimization order quantity in formula (1) is more, and maybe better. This is because the newsvendor doesn't pay off his loan fully in the assumption of the formula (1) and this decreases his costs, thus, he will order more goods with initiative. However, the formula (1) is infeasible and the optimization order quantity in formula (1) is not the best in most situations. Only under very strict condition (the possible bankruptcy situation), this kind of order quantity will be right. But in this situation, some of these funds are not paid back, so the risk is transferred to the bank.

Although the conditions are very restricted in formula (1), there are some cases that may exist in reality (these cases require the corporate declare bankruptcy if this business revenue is not sufficient to pay off its loan. We call it possible bankruptcy situation) where formula (1) is suitable. We will use two numerical examples to discuss this situation in the following paragraphs. We will see that the optimal profit gotten from formula (1) is only a little more than that gotten from formula (4), but the risk in formula (1) increases sharply. Thus, the order quantity gotten from formula (4) is also a good choice.

2.4. The revision of the bank's problem

Since the newsvendor repays the loan from the bank fully in any condition, the bank determines r or m to maximize its expected profit.. This leads:

$$\max_{r} \pi_b = Br \tag{6}$$

Using (2), formula (6) can be rewritten as

$$\max_{m} \pi_b = (p-m)y \tag{7}$$

The first-order condition π_b respect to m is:

$$\frac{d\pi_b}{dm} = -y^*(m) + (p-m)\frac{dy^*}{dm} = 0 \tag{8}$$

where $\frac{dy^*}{dm} = \frac{c\bar{F}[(\eta+my)/c] - myf[(\eta+my)/c]}{m^2 f[(\eta+my)/c]}$,. Hence, $c/p = \bar{F}(Q_0) < \bar{F}(Q_2^*) = c/m$. This allows us to conclude that the bank will select m such that $y^*(m) > 0$. As a consequence, we can conclude immediately that at equilibrium $\frac{dy^*}{dm} = \frac{y^*(m)}{p-m} > 0$, which ensures the uniqueness of the equilibrium point since the best response function $y^*(m)$ is strictly monotone, These conclusions are summarized as follows.

Proposition 2. If F (·) is IFR, (a) the Stackelberg game played out between the bank and the newsvendor has a unique equilibrium point (y^*, m^*), $\bar{F}^{-1}(c/m^*) = Q_2^* < Q_0 = \bar{F}^{-1}(c/p)$ and $\frac{dy^*}{dm^*} > 0$.

2.5. Some comparative statics

Having established that the equilibrium is unique, we are in a position to perform comparative statics about the revision of the newsvendor and bank's problem. This yields the following comparative statics with respect to η, p and c .

Proposition 3. (a) As η increases, y^*, B^* and r^* decrease, m^* increases. (b) For a given η, as p increases, $y^*, B^*, \hat{Q}, r^*$ and m^* increase. And (c) for a given η, as c increases, y^* increases, but $\hat{Q}, m^*, B^*$ decrease.

Applying the Implicit Function Theorem to (5) yields：

$$\frac{dm}{d\eta} = \frac{mf[\hat{Q}]}{c\bar{F}(\hat{Q}) - my^* f(\hat{Q})}$$

Since：

$$\frac{dy^*}{dm^*} = \frac{c\bar{F}(\hat{Q}) - m^* y^* f(\hat{Q})}{m^{*2} f(\hat{Q})} > 0$$

At equilibrium $c\bar{F}(\hat{Q}) - m^* y^* f(\hat{Q}) > 0$,

Therefore, $\frac{dm^*}{d\eta} > 0$ Consequently, $\frac{dy^*}{d\eta} < 0$ and $\frac{dB^*}{d\eta} < 0$ .A

pplying the Implicit Function Theorem (2) yields

$$\frac{dy^*}{dp} = \frac{1}{1+r}\frac{dy^*}{dm^*} > 0, \quad \frac{dm^*}{dp} = \frac{1}{1+r} > 0。$$

Because $\frac{dy^*}{dm^*} > 0$, $B^* = m^* y^*$, and $c\hat{Q} = \eta + B^*$, the monotone property of $\hat{Q}$ with respect to p is immediate.

For part (c) substituting $p/(1+r^*)$ for m^* and applying the Implicit Function Theorem yields:

$$[1 - \frac{1}{1+r^*}]\frac{dy^*}{dm}dp - \frac{p}{(1+r^*)^2}\frac{dy^*}{dm}dr = 0$$

Hence, $\frac{dr^*}{dp} = \frac{r^*(1+r^*)}{p} > 0$。The monotone properties of $y^*, \hat{Q}, B^*, m^*$ respect to c can be proved in the same fashion as Maqbool Dada model.

3. The comparison of numerical examples

Because the formula (1) requires the corporate declare bankruptcy if its revenue is not sufficient to pay off its loan, $F(y)$ (the probability for the newsvendor does not fully pay off the loan) is also the probability of bankruptcy. We use $\beta := F(y)$ to denote the risk coefficient of the corporate declare bankruptcy.

3.1. The demand is a uniform distribution

Supposing $c = 1, p = 2, a = 0, b = 1$, and assuming the demand D is a uniform distribution which is defined in region $[0,1]$, then,

$$f(x) = \begin{cases} 1, x \in [0,1]; \\ \text{otherwise} \end{cases} \text{ and } F(x) = \begin{cases} 0, x < 0; \\ x, x \in [0,1]. \\ 1, x > 1 \end{cases}$$

3.1.1. The comparison of Q_1^* and Q_2^* in possible bankruptcy situation

Using formula (3), we can get $Q_1^* = \frac{cm + \eta c - m^2}{c^2 - m^2} = 0.5416$.

Substituting Q_1^* into formula (1), the optimal expected profit of the newsvendor $\pi_1(Q_1^*) = 0.2523$ can be obtained.

Then, substituting Q_1^* into formula (2), we get the newsvendor's risk coefficient $\beta(Q_1^*) = F(y_1) = 0.126833$.

Furthermore, using formula (5), we obtain $Q_2{*} = \frac{m-c}{m} = 0.4750$.

Substituting $Q_2{*}$ into formula (1), we get the newsvendor’s optimal expected profit $\pi_1(Q_2{*}) = 0.2491$.

And substituting $Q_2{*}$ into formula (2), the newsvendor’s risk coefficient $\beta(Q_2{*}) = F(y_2) = 0.091875$ can be gotten.

In the means of π_1, the newsvendor’s profit increases by 1.29% under the order quantity $Q_1{*}$ comparing with $Q_2{*}$, but at the same time, the risk coefficient increases by 38.05%. This tells us that although the profit increases slightly in $Q_1{*}$, the risk associated with it increases more. So we can conclude that $Q_2{*}$ is better than $Q_1{*}$ with taking the risk into consideration.

3.1.2. The comparison of $Q_1{*}$ and $Q_2{*}$ in the situation of social optimal order quantity

When the newsvendor has enough funds, his order quantity $Q_0 = 1 - \frac{c}{p} = 0.5$ is also the social optimal order.

The social optimal profit objective function is:

$$\pi_0 = -cQ + p\int_0^Q \bar{F}(x)dx \tag{9}$$

Substituting Q_0, $Q_1{*}$, $Q_2{*}$ into formula (9), we can obtain $\pi_0(Q_1{*}) = 0.24827 < \pi_0(Q_2{*}) = 0.249375 < \pi_0(Q_0) = 0.25$, respectively. Obviously, the order quantity $Q_2{*}$’s social efficiency is better than that of $Q_1{*}$.

3.2. The demand is a negative exponential distribution

Let’s suppose $c = 1, p = 2, a = 0, b = 1$ and the demand D is a negative exponential distribution which is defined in the region $[0, \infty]$ and the mean value is 1, then $f(x) = e^{-x}$, $F(x) = 1 - e^{-x}, \bar{F}(x) = e^{-x}$.

3.2.1. The comparison of Q_1^* and Q_2^* in possible bankruptcy situation

$Q_1{*} = \frac{m \ln \frac{m}{c} - \eta}{m-c} = 1.02496$ can be derived from the formula (3). The newsvendor’s optimal expected profit $\pi_1(Q_1{*}) = 0.349275$ can be yielded from the formula (1) and $Q_1{*}$.and substituting $Q_1{*}$ into formula (2) yields the newsvendor’s risk coefficient

$$\beta(Q_1{*}) = F(y_1) = 0.380605.$$

Using formula (5), we can get $Q_2{*} = \frac{m-c}{m} = 0.644357$. In this case, we can get

$$\pi_1(Q_2{*}) = 0.319226, \beta(Q_2{*}) = F(y_2) = 0.180787$$

when we substitute $Q_1{*}$ into formula (1) and (2).

In the means of π_1, the newsvendor’s profit increases by 9.41% with $Q_1{*}$ comparing with Q_2*, and the risk coefficient increases by 110.53%. It is obvious that the risk associated with Q_1* increases much more than the increase of profit. Thus we can conclude that Q_2* is better than Q_1* with taking the risk into consideration.

3.2.2. The comparison of Q_1^* and Q_2^* in the situation of social optimal order quantity

When the newsvendor's fund is sufficient, his order quantity

$$Q_0 = -\ln(\frac{c}{p}) = 0.693147$$

is the social optimal order quantity.

Substituting Q_0, Q_1*, Q_2* into formula (9), we can get

$$\pi_0(Q_1*) = 0.257418 < \pi_0(Q_2*) = 0.305643 < \pi_0(Q_0) = 0.306853,$$

respectively.. It is clear that the social efficiency associated with Q_2* is better than that of Q_1*.

4. Conclusion

In this paper, we revise the Maqbool dada model so that it can be applied to personal loan and most of the company loans. Comparing with the revised model, The Maqbool model plays a quite limited role on increasing profit even in the possible bankruptcy conditions which is extremely strict, meanwhile, the risk of bankruptcy is sharply increased. As a result, our revised model is very useful and its social efficiency is better than Maqbool model as the numerical examples show.

Acknowledgement. This research is supported by the Science and Technology Research Program of Chongqing Municipal Education Commission (Grant No. KJQN202001542). Also, the authors are thankful to the reviewers for their constructive comments.

Conflict of interest. The authors declare that they have no conflict of interest.

Authors' Contributions. All the authors have equal contribution.

REFERENCES

1. Maqbool Dada and Qiaohai Hu, Financing newsvendor inventory, *Operations Research Letters*, 36 (2008) 569–573.
2. George L. Vairaktarakis, Robust multi-item newsboy models with a budget constraint, *Int. J. Production Economics*, 66 (2000) 213-226.
3. Layek Abdel-Malek, Roberto Montanari and Libia Cristina Morales, Exact, approximate, and generic iterative models for the multi-product Newsboy problem with budget constraint, *Int. J. Production Economics*, 91 (2004) 189–198.
4. Layek L. Abdel-Malek and Roberto Montanari, An analysis of the multi-product newsboy problem with a budget constraint, *Int. J. Production Economics*, 97 (2005) 296–307.
5. Shuo-Yan Chou, Peterson C. Julian and Kuo-Chen Hung, A note on fuzzy inventory model with storage space and budget constraints, *Applied Mathematical Modelling*, 33 (2009) 4069–4077.
6. Bin Zhang, XiaoyanXu and Zhongsheng Hua, A binary solution method for the multi-product newsboy problem with budget constraint, *Int. J. Production Economics*, 117

(2009) 136–141.

7. Liang-Hsuan Chen, Ying-Che Chen, A multiple-item budget-constraint newsboy problem with a reservation policy, *Omega*, 38 (2010) 431–439.
8. Guoqing Zhang, The multi-product newsboy problem with supplier quantity discounts and a budget constraint, *European Journal of Operational Research*, 206 (2010) 350–360.
9. Bin Zhang and Zhongsheng Hua, A portfolio approach to multi-product newsboy problem with budget constraint, *Computers & Industrial Engineering,* 58 (2010) 759–765.