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Optimisation of Water Loss Management Strategies: Multi-Criteria Decision Analysis Approaches

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Abstract. Water loss in the water distribution systems (WDS) is a challenge to many water authorities in the world but the problem is crucial in the less developed countries. The effect of water losses in the WDS includes the reduction in the revenue and availability of water, interruption in the quality of water, and inflation of the operation and maintenance cost of the water authorities. Using data from the Moshi Urban Water Supply and Sanitation Authority (MUWSA) Tanzania, an assessment of strategies used for water loss management (WLM) was carried out through an integrated model of Multi-Criteria Decision Making (MCDM) and Integer Linear Programming (ILP) which is an optimisation technique. The family of MCDM methods, Multi-Attribute Value Theory (MAVT), Simple Multi-Attribute Rating Technique Exploiting Ranks (SMARTER), and Simple Additive Weighting (SAW) were employed to assess and prioritise the strategies while the ILP was used to formulate a decision model. The model was used to select a portfolio of the best strategies. Sixteen strategies were identified. The results show that the comparison between the bulk meter and customers' meter on detecting the physical or apparent losses was ranked as the best strategy in managing the loss while the network zoning was ranked as the worst strategy. The model selected thirteen out of sixteen strategies to form the portfolio of the best strategies to be employed by the MUWSA for water loss management. Furthermore, the model was found to be robust as the selected portfolio of strategies remained the same even when the weights of the criteria were changed. The developed model in this study will assist the decision-makers to assess, prioritise and choose the best strategies for reducing or controlling water loss in the distribution system.

Keywords: Alternatives; Criteria; Integer Linear Programming; Multi-Criteria Decision Making methods; Water Loss Management; Water Distribution System.

AMS Mathematics Subject Classification (2010): 76B75

1. Introduction

Water shortage is a growing problem worldwide and getting worse due to the effects of climate change. Water demand is anticipated to grow significantly throughout the period 2010 - 2050 in the industrial sector and domestic use in Africa, Asia, and Central and South America due to the population growth, industrialisation policies and expansion of water distribution services in urban areas [1, 2]. This situation has forced the water authorities to protect the water sources and utilise the water resources creatively.

Water loss in the WDS is another challenge facing most of Urban Water Supply Authorities (UWSA) in the less developed countries, as much water is lost on the way to the consumers. The loss is caused by leakage, theft of infrastructures, illegal connections, tampering of meters, among others [3, 4]. Water loss in WDS in the less developed countries is about 45% to 50% of the total water produced and leakage contributes more than 70% of the total loss [3, 5]. However, the water loss in WDS contributes to the nonrevenue water (NRW) problem to many water authorities. The World Bank (WB) set the NRW target for a well-performing UWSSA to be less than 23%, while the target for the Southern Africa Development Community (SADC) is 20% [3]. In Tanzania, the NRW average for all the UWSA was recorded to be 43.6% for year 2014/2015, 41.6% for year 2015/2016, 38.4% for year 2016/2017, 36% for year 2017/2018 and 32.3% for year 2018/2019 (Ministry of Water and Irrigation, Tanzania (MoWI) report, 2014), (Energy and Water Utilities Regulatory Authority (EWURA) Water Utilities Performance Review Report, 2017) and (MoWI budget reports for year 2018/2019 and 2019/2020). Although the trend of five consecutive years shows the improvement of NRW in Tanzania's UWSA, the latest average (32.3%) is above the recommended targets by the WB (23%) and SADC (20%). Measures should be taken to meet the recommended targets.

Organisations such as the International Water Association (IWA) through its Water Loss Task Force (WLTF) have proposed and applied numerous strategies, methodologies, and procedures which are now used globally by water management authorities aimed to reduce or control water loss in the WDS and to increase income from water sales. Emphasis is placed on ensuring the accurateness of the water meters, the stability of water distribution, and the management of apparent and physical water losses. Real water loss reduction or control is achieved through approaches such as the control of active leaks, pressure management, quick repairs, quality repairs, infrastructure management, and assets management. While apparent water loss is reduced or controlled by methods that reduce the following: customers' meter errors, meter reading errors, billing system errors, and illicit use of water [4, 6].

Despite the recommended strategies, studies on water loss problems in the WDS have attracted researchers and practitioners to improve the services by reducing water loss, improving the design, and improving the operations of the WDS. Most of the studies carried out are from science, engineering, and mathematics. Studies on mathematics especially the mathematics branch of operational research have used MCDM methods, optimisation techniques, and other approaches. To improve the results, researchers have combined the MCDM methods and mathematical optimisation techniques to solve problems in decision making. The hybrid of these methods has been employed by [7] who used the Preference Ranking Organisation Method for Enrichment Evaluation (PROMETHEE) and ILP methods to assess the strategies used to balance water supply and demand in WDS. Also, [8] used the Analytic Hierarchy Process (AHP) and ILP to

study the District Metering Areas (DMAs) in water supply networks. This study has used the MAVT, SMARTER, and SAW methods integrated with the ILP technique to form a model that was used to assess, prioritise and select the best strategies for WLM to help decision-makers (DMs) in their planning.

2. Materials and methods

The model developed for this study is comprised of two parts; the first part was concerned with deriving the family of the MCDM methods and the second part was concerned with deriving the ILP models.

2.1. MCDM methods

These are techniques that unify several and contradicting criteria in the decision process. They are tools developed in the arena of decision concepts to solve problems in operational research [9]. The methods form a restricted number of decision alternatives in which the DMs have to assess and rank or prioritise the alternatives basing on the weights of the limited set of evaluation criteria [9, 10]. Reference [11, 12] discussed two major categories of the MCDM weighting methods. The first category is concerned with compensatory weighting methods such as MAVT, AHP, SMARTER, SAW, Simple Multi-Attribute Rating Technique Swings (SMARTS), Measuring Attractiveness by a Categorical Based Evaluation Technique (MACBETH), Multi-Attribute Utility Theory (MAUT). Complex Proportional Assessment (COPRAS) and Technique for Order Preference by Similarity to Ideal Solution (TOPSIS) which are used in Multi-Attribute Utility (MAU). These methods aggregate different ideas into a single function for optimisation. The methods use numbers to represent the preference of the considered action. The second category is concerned with non-compensatory weighting methods such as Elimination Et Choix Traduisant la Realite (ELECTRE) (French) (whose English translation is Elimination and Choice Translating Reality), PROMETHEE and their families. These are used in outranking methods which represent DMs' preferences to solve their problems. In a compensatory approach, the evaluation of alternatives considers trade-offs between criteria while in non-compensatory methods, a loss of a strategy on a criterion can't be compensated by other criteria [11, 13].

Weighting the evaluation criteria in MCDM methods is vital since the final result of decision making mainly depends on the weights. Reference [12, 14] defined three groups of rank-order weighting methods. The first group is subjective weighting methods; it assigns the weights of the criteria according to the choices of the DMs. The elicitation process in subjective methods are explained clearly and mostly used for MCDM in water resource management. The common methods in this group are AHP. SMARTS/SMARTER. The second group is formed by objective weighting methods. The criteria weights in these methods are obtained through mathematical approaches in which DMs have no part in the determination of the importance of a criterion. Popular objective weighting methods include: TOPSIS, least mean square (LMS), Statistical variance method, Criteria Importance Through Inter-criteria Correlation (CRITIC), and Multi-Objective Optimisation. The last group is a combination of weighting methods; these are hybrid techniques that are comprised of a mixture of multiplicative and additive techniques.

Based on [7, 15], the compensatory weighting methods MAVT, SMARTER and SAW were used in this study. The SMARTER method was used to allocate weights to the evaluation criteria according to DMs' ranking. The SMARTER method was chosen in the process because it uses a swing procedure to attain a constant scale, also it uses linear function values in the evaluation. Furthermore, the SMARTER method is more precise in generating the weights than the weights assigned by the DM. Studies by [16-18] identified various techniques used by the SMARTER for generating weights to the evaluation criteria. The common techniques are rank-sum (RS), rank reciprocal (RR), rank exponent (RE) and rank order centroid (ROC). This study has used the ROC technique because it generates weights that represent the centre of mass of all weights of the rank order of the criteria. Besides, the method has much less error for ranked criteria and has a clear statistical basis [18].

The ROC weights are given by Eq. (1);

$$W_i(ROC) = \frac{1}{n} \sum_{j=1}^n \frac{1}{j}$$
; $i = 1, 2, 3... n$ (1)

where **n** is the number of criteria and **i** represent the rank The criteria are assigned to weights with vector $W = [W_1, W_2, \dots, W_n]$, where $W_1 > W_2 > W_3 > W_4 > \dots > W_n$, which satisfies Eq. (2). $\sum_{j=1}^n W_j = 1$ (2)

The MAVT method was used to aggregate the performance of strategies through all the criteria to obtain a cumulative evaluation value. The weighted normalised value $(V_{ii}(a))$ for the alternatives over each criterion is given by Eq. 3.

$$V_{ii}(a) = w_i a_{ii}$$

(3)

The performance of each strategy through all criteria is the weighted normalised sum of functions of each criterion given by Eq. (4).

$$V_{ij}(a) = \sum_{i=1}^{n} w_j a_{ij} \quad j=1,2,\dots n$$
where \mathbf{w}_j is the weight of \mathbf{j} criterion and a_{ij} is the normalised value of strategy \mathbf{i} in respect of criterion \mathbf{i} .
$$(4)$$

The SAW method was used to rank the strategies according to the sum of the weighted value of the strategy as discussed by [19-21]. Based on the studies by [22-25], the SAW method uses the linear – sum method (Eq. 5), the linear – max method Eq. (6), the linear – MaxMin method Eq.(7), and the vector normalization Eq.(8) for normalisation of data [23].

i) *Linear scale transformation – sum method*

$$a_{ij} = \frac{x_{ij}}{\sum_{i=1}^{m} x_{ij}} \qquad Benefit \ criteria$$
$$a_{ij} = \frac{\frac{1}{x_{ij}}}{\sum_{i=1}^{m} \frac{1}{x_{ij}}} \qquad Cost \ criteria \qquad (5)$$

ii) Linear scale transformation - max method

$$a_{ij} = \frac{x_{ij}}{\max x_j}$$
 Benefit criteria

$$a_{ij} = \frac{\min x_j}{x_{ij}} \qquad Cost \ criteria \tag{6}$$

iii)

$$Linear scale transformation - MaxMin method$$

$$a_{ij} = \frac{x_{ij} - \min x_j}{\max x_j - \min x_j} \qquad Benefit \ criteria$$

$$a_{ij} = \frac{\max x_j - x_{ij}}{\max x_i - \min x_i} \qquad Cost \ criteria \qquad (7)$$

$$a_{ij} = \frac{x_{ij}}{\sqrt{(\sum_{i=1}^{m} x_{ij}^2)}} \qquad Benefit \ criteria$$
$$a_{ij} = 1 - \frac{x_{ij}}{\sqrt{(\sum_{i=1}^{m} x_{ij}^2)}} \qquad Cost \ criteria \qquad (8)$$

where x_{ij} represents the score of *i*-th strategy in respect of *j*-th criterion before normalization, a_{ii} represents the normalised value.

The ranking of strategies through the SAW method is done by considering only benefit criteria after transforming the cost into the benefit criteria [20, 21, 23, 26].

The sum (Q_i) of the weighted normalised values of all criteria over a strategy is computed using Eq. (9).

$$Q_i = V_{ij}(a) = \sum_{j=1}^{n} W_j a_{ij} \qquad i = 1, 2, \dots, n$$
(9)

where w_j represents the weight of criterion j and a_{ij} is the normalised value of strategy i in respect of criterion j.

2.2. The ILP technique

In this study, the ILP and the Binary Integer Programming (BIP) were used to formulate the decision model that has been used to select or reject strategies used for WLM. The models were solved using a free Software package LINDO 6.1. The general ILP equations for the model are shown in Eq. (10) to Eq. (14).

a) Objective function: This denotes the maximisation of the sum of the weighted normalised values which must be optimised so as to obtain the maximum number of optimal strategies. This is given by;

$$Max Z = \sum_{i=1}^{m} Q_i S_i$$
where the index i = 1, 2, 3... m
$$S_i \text{ - denotes strategy for index } i$$
(10)

 Q_i – is the sum of weighted/ranking values for the strategy i

b) The constraints are;

i) Implementation cost in million Tanzanian Shillings (TZS) $\sum_{i=1}^{m} b_i S_i \leq c_i \qquad (11)$ where $b_i = amount \ of \ resources \ (funds) \ used \ to \ implement \ the \ strategy \ i.$

 $c_i = cost or budget limit for implementation of strategies i.$

ii) Conflicting strategies found in one category

$$\sum_{i=1}^{m} S_i \ge 1 \tag{12}$$

iii)	The optimal number of strategies to be selected	
	$\sum_{i=1}^{m} S_i \le 16$	(13)
iv)	Binary/decision variable	
	$S_i = (0 \text{ or } 1)$	(14)

2.3. Tools for data collection

The data used were obtained from government and other reports by reviewing the documents and content analysis. The primary data were collected from the knowledgeable and experienced DM of the MUWSA, through a questionnaire and face to face interview.

3. Results and discussion

3.1. Description of problem and objectives

The first stage in decision-making is to define explicitly the problem and objectives of what is needed in the study to identify the root causes and thereby understand the context of the company under study. Based on MUWSA's information, the problem of the study was on how the company manages the strategies used in controlling or reducing water loss in the WDS. The objectives are to identify, prioritize and select the portfolio of the best strategies used at MUWSA for WLM.

3.1.1. Identification of Alternatives and evaluation criteria

The surveyed alternatives for WLM in this study were:

- 1. Alternative 1: Education. This is comprised of four strategies used for education to the community on the effective usage of water to facilitate saving of water at home and outside, and to inspire people to report the visible leakages and faults to the water authority to speed up the repairs and avoid unnecessary wastage of water:
 - S_1 Advertising campaigns
 - S_2 Educational campaigns in schools
 - S_3 Ward meetings with the society
 - S_4 Meeting with local leaders
- 2. *Alternative 2: Illegal use control.* This has one strategy intended to control losses that are caused by illegal use of water by the consumers (illicit connection, a setback of the meter and damage or theft of the infrastructure).

- S₅ Illegal use control
- 3. *Alternative 3: Network zones and metering areas.* This has one strategy:
 - *S*₆ *Network zoning and establishment of District Metering Areas (DMA).*
- 4. Alternative 4: indicators to quantify the losses. This has one Strategy concerned with the use of the meters to quantify the losses, it gives essential data for planning the actions needed to control losses
 - *S*₇ *The use of the indicators to quantify the losses.*
- 5. *Alternative 5: Strategies used to control inaccuracy meter*. This has two strategies:
 - S_8 Calibration of the meters
 - *S*₉ *Replacement of the defect meters*
- 6. Alternative 6: Detection of apparent/physical losses. This has three strategies:
 - S_{10} Visual inspection of the WDS
 - S_{11} Comparison between the bulk meter and customer meter readings
 - S_{12} Report from the community on the detected leak via a toll-free phone
- 7. Alternative 7: Pipes replacement. This has one strategy
 - *S*₁₃ *Replacement of dilapidated pipes*
- 8. Alternative 8: Quality Pipes. This has one strategy
 - *S*₁₄ *Installation of quality pipes*
- 9. Alternative 9: Repairs. This has one strategy
 - *S*₁₅ *Timely repair of pipe leaks (active leakage control)*
- 10. Alternative 10: Pressure. This has one strategy
 - S_{16} Pressure management

The identified evaluation criteria in this study were;

- 1. C_1 : Income generation. This criterion is used to evaluate the capacity of a given strategy to improve income. The highest score value of C_1 is the best of the alternatives.
- 2. C_2 : *Investment cost*. This criterion evaluates the cost required to execute a given strategy. The lowest score value of C_2 is the best strategy.
- 3. C_3 : Operation & Maintenance cost. This criterion evaluates the cost related to the implementation of a given strategy. The lowest score value of C_3 is the best strategy.

- 4. C_4 : Saving of Water. This criterion evaluates the capacity of a given strategy to reduce water losses. The highest the score value of C_4 is the best strategy.
- 5. C_5 : Quality of Water. This criterion evaluates the capability of a given strategy for retaining the quality of water. The highest score value of C_5 is the best strategy.
- 6. *C*₆: *Water supply reliability.* This criterion evaluates the capacity of a given strategy to reduce flow disruptions. The least number of disruptions (burst, leaks, and illegal uses) measured by the value of C_6 represents the best strategy.
- 7. C_7 : *Efficiency*. This criterion evaluates the efficiency of a given strategy to minimize water losses. The highest score value of C_7 is the best strategy.

3.1.2. Identification of the DM

This study considered only one DM who is knowledgeable about both the technical and managerial issues of the company.

3.2. Evaluation of strategies

The DM filled the questionnaire, evaluated the strategies against the criteria, and responded to the interview questions. The Likert scale of 5 = excellent; 4 = very good; 3 = good; 2 = fair; and 1 = unsatisfactory was used to score each criterion over the strategies. Table 1 shows the scores of strategies given by the DM against each Criterion.

Strategies	Criteria							
	<i>c</i> ₁	<i>C</i> ₂	<i>C</i> ₃	C_4	C_5	<i>C</i> ₆	C ₇	
S_1	4	3	2	4	3	3	4	
S_2	3	1	1	3	3	3	3	
S_3	5	2	2	5	3	3	4	
S_4	4	2	2	4	3	3	4	
S_{5}	4	3	2	4	2	3	3	
S_6	4	3	2	4	2	2	3	
$\tilde{S_7}$	4	2	1	4	2	3	3	
S_8	5	3	2	5	1	2	4	
S_9	5	4	2	5	1	2	4	
S_{10}	4	2	1	4	3	3	4	
S_{11}^{10}	4	1	1	4	3	3	4	
S_{12}^{11}	4	2	1	4	3	3	4	
S_{13}^{12}	5	4	2	5	4	4	4	
S_{14}^{10}	4	4	2	4	4	4	4	
S_{15}	4	3	3	4	4	4	4	
S_{16}^{10}	4	3	1	4	3	3	3	
Benefit/Cost	Ben	Cost	Cost	Ben	Ben	Ben	Ben	

Table 1: Score evaluation matrix: Strategies versus criteria

3.3. Normalisation of data

The purpose of carrying out the normalisation process is to convert the values of strategies to the 0 to 1 scale, where 0 is the least strategy value and 1 the highest strategy value in every criterion if its objective is to do maximization (benefit) and/or minimization (cost) [11]. This study carried out the normalisation process using the linear scale transformation- sum method Eq. (5) which transforms the cost (minimum) criteria to benefit (maximum) criteria. Table 2 shows the normalised matrix which has been carried out in this study.

Table 2: Normalised value matrix: Strategies versus Criteria

Strate	Criteria						
gies	<i>c</i> ₁	<i>c</i> ₂	<i>c</i> ₃	<i>c</i> ₄	<i>c</i> ₅	<i>c</i> ₆	<i>c</i> ₇
S_1	0.05970	0.04598	0.04615	0.05970	0.06818	0.06250	0.06780
S_2	0.04478	0.13793	0.09231	0.04478	0.06818	0.06250	0.05085
S_3	0.07463	0.06897	0.04615	0.07463	0.06818	0.06250	0.06780
S_4	0.05970	0.06897	0.04615	0.05970	0.06818	0.06250	0.06780
S_5	0.05970	0.04598	0.04615	0.05970	0.04545	0.06250	0.05085
S_6	0.05970	0.04598	0.04615	0.05970	0.04545	0.04167	0.05085
S_7	0.05970	0.06897	0.09231	0.05970	0.04545	0.06250	0.05085
S_8	0.07463	0.04598	0.04615	0.07463	0.02273	0.04167	0.06780
S_9	0.07463	0.03448	0.04615	0.07463	0.02273	0.04167	0.06780
S_{10}	0.05970	0.06897	0.09231	0.05970	0.06818	0.06250	0.06780
S_{11}	0.05970	0.13793	0.09231	0.05970	0.06818	0.06250	0.06780
$S_{12}^{}$	0.05970	0.06897	0.09231	0.05970	0.06818	0.06250	0.06780
S_{13}	0.07463	0.03448	0.04615	0.07463	0.09091	0.08333	0.06780
S_{14}^{-1}	0.05970	0.03448	0.04615	0.05970	0.09091	0.08333	0.06780
S_{15}	0.05970	0.04598	0.03077	0.05970	0.09091	0.08333	0.06780
S_{16}	0.05970	0.04598	0.09231	0.05970	0.06818	0.06250	0.05085
Sum	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000

3.4. Elicitation of weights

The weights computed by the SMARTER – ROC technique Eq. (1) for the seven criteria have the values given in Eq. (15);

$$W_{1} = 0.3704, W_{2} = 0.2276, W_{3} = 0.1561, W_{4} = 0.1085, W_{5} = 0.0728, W_{6} = 0.0442, W_{7} = 0.0204$$
(15)
The DM's ranking of the evaluation criteria is as given in the relation (16):
 $C_{1} > C_{4} > C_{7} > C_{3} > C_{2} > C_{6} > C_{5}.$
(16)

This means that; $C_1 \rightarrow W_1$, $C_4 \rightarrow W_2$, $C_7 \rightarrow W_3$, $C_3 \rightarrow W_4$, $C_2 \rightarrow W_5$, $C_6 \rightarrow W_6$, and $C_5 \rightarrow W_7$.

The weighted normalised values calculated by Eq. 3 are given in Table 3.

Strategi	Criteria						
es	C_1	C_2	C_3	C_4	C_5	C_6	C_7
	0.3704	0.0728	0.1085	0.2276	0.0204	0.0442	0.1561
S_1	0.02211	0.00335	0.00501	0.01359	0.00139	0.00276	0.01058
S_2	0.01659	0.01004	0.01002	0.01019	0.00139	0.00276	0.00794
S_3	0.02764	0.00502	0.00501	0.01699	0.00139	0.00276	0.01058
S_4	0.02211	0.00502	0.00501	0.01359	0.00139	0.00276	0.01058
S_5	0.02211	0.00335	0.00501	0.01359	0.00093	0.00276	0.00794
S_6	0.02211	0.00335	0.00501	0.01359	0.00093	0.00184	0.00794
S_7	0.02211	0.00502	0.01002	0.01359	0.00093	0.00276	0.00794
S_8	0.02764	0.00335	0.00501	0.01699	0.00046	0.00184	0.01058
S_9	0.02764	0.00251	0.00501	0.01699	0.00046	0.00184	0.01058
S_{10}	0.02211	0.00502	0.01002	0.01359	0.00139	0.00276	0.01058
<i>S</i> ₁₁	0.02211	0.01004	0.01002	0.01359	0.00139	0.00276	0.01058
<i>S</i> ₁₂	0.02211	0.00502	0.01002	0.01359	0.00139	0.00276	0.01058
<i>S</i> ₁₃	0.02764	0.00251	0.00501	0.01699	0.00185	0.00368	0.01058
S_{14}	0.02211	0.00251	0.00501	0.01359	0.00185	0.00368	0.01058
S_{15}	0.02211	0.00335	0.00334	0.01359	0.00185	0.00368	0.01058
<i>S</i> ₁₆	0.02211	0.00335	0.01002	0.01359	0.00139	0.00276	0.00794

Maselle Joseph Kadenge, Verdiana Grace Masanja and Mashaka James Mkandawile **Table 3:** Weighted normalised matrix: Strategies versus Criteria

3.5. Ranking of strategies

The highest value of Q_i represents the best ranked or the most prioritised strategy. Table 4 shows the computed values of Q_i Eq. (9) and ranking of the strategies.

Rank	Strategies (S_i)	Value (Q_i)	Rank	Strategies (S_i)	Value (Q_i)
1.	S ₁₁	0.07049	9.	S_{16}	0.06115
2.	S_3	0.06939	10.	\mathbf{S}_4	0.06047
3.	S ₁₃	0.06827	11.	S_{14}	0.05934
4.	S_8	0.06587	12.	\mathbf{S}_2	0.05892
5.	\mathbf{S}_{10}	0.06547	13.	\mathbf{S}_1	0.05879
5.	S_{12}	0.06547	14.	S_{15}	0.05851
7.	S_9	0.06503	15.	S_5	0.05568
8.	S_7	0.06236	16.	S_6	0.05476

Table 4: Value of Q_i and ranking of Strategies

Based on the information in Table 4, the complete ranking of strategies for WLM is obtained as shown in relation (17).

 $S_{11} \succ S_3 \succ S_{13} \succ S_8 \succ S_{10}, S_{12} \succ S_9 \succ S_7 \rightarrow S_{16} \succ S_4 \succ S_1 \rightarrow S_2 \succ S_1 \succ S_5 \succ S_6 \quad (17)$ $S_{11} \text{ indicates the best strategy and } S_6 \text{ indicates the worst strategy.}$

3.6. Assessment of the operational restrictions and selection of the strategies

In this study, the ILP was used to assess the operational restrictions of the company and to select the portfolio of strategies and using the BIP technique the numbers 0 or 1 have been used to represent the selection choice of the strategies instead of their arbitrary values. For the studied problem, the developed ILP model is as follows:

Objective function:

 $\begin{array}{ll} \textit{Maximize} & Z = 0.05879S_1 + 0.05892S_2 + 0.06939S_3 + \\ 0.06047S_4 + 0.05568S_5 + 0.05476S_6 + 0.06236S_7 + 0.06587S_8 + \\ 0.06503S_9 + 0.06547S_{10} + 0.07049S_{11} + 0.06547S_{12} + 0.06827S_{13} + \\ 0.05934S_{14} + 0.05851S_{15} + 0.06115S_{16} \end{array} \tag{18}$

Subject to constraints

 $\begin{array}{ll} 14.58S_1 + 14.58S_2 + 14.58S_3 + 14.58S_4 + 31.59S_5 + 29.16S_6 & + \\ 2.43S_7 + & 75.33S_8 + & 75.33S_9 + & 4.86S_{10} + & 4.86S_{11} + & 4.86S_{12} + \\ 29.16S_{13} + & 51.03S_{15} + & 4.86S_{16} \leq & 243 \\ S_1 + & S_2 + S_3 + & S_4 \geq 1 \\ S_{10} + & S_{11} + & S_{12} \geq 1 \\ S_{10} + & S_{11} + & S_{12} \geq 1 \\ S_1 + & S_2 + & S_3 + & S_4 + & S_5 + & S_6 + & S_7 + & S_8 + & S_9 + & S_{10} + & S_{11} + & S_{12} + & S_{13} + \\ S_{14} + & S_{15} + & S_{16} \leq 1 \\ S_{10} + & S_{11} + & S_{12} \leq 1 \\ S_{14} + & S_{15} + & S_{16} \leq 1 \\ S_{16} + & S_{16} = \\ S_{16} + & S_{16} + \\ S_{16} + & S_{16} = \\ S_{16} + & S_{16} + \\ S_{16} + & S_{16} = \\ S_{16} + & S_{16} + \\ S_{16} + & S_{16} = \\ S_{16} + & S_{16} + \\$

 $S_1, S_2, S_3, S_4, S_5, S_6, S_7, S_8, S_9, S_{10}, S_{11}, S_{12}, S_{13}, S_{14}, S_{15}, S_{16} = [0 \text{ or } 1]$ (24) where: Eq. (18) is the objective function in which the coefficients are the Q_i values. Eq. (19) is the budget restriction constraint for implementation of preventive actions which are in 1,000,000/= TZS. Eq. (20) to Eq. (22) represent the multiple-choice strategies found in one category. The constraints ensure that at least one strategy can be adopted in WLM. Eq. (23) is the constraint that represents the optimal number of strategies to be selected in a portfolio. Eq. (24) represents the binary constraints, i.e. the decision variables, S_i) whose values are either 0 or 1; where 1 means that S_i is the selected strategy and 0 otherwise.

The ILP model Eq. (18) subject to Eq. (19) to Eq. (24) was solved using the LINDO 6.1 software package, the variables, S_1 , S_2 , S_3 , S_4 , S_7 , S_8 , S_{10} , S_{11} , S_{12} , S_{13} , S_{14} , S_{15} , and S_{16} yielded the value 1, while variables, S_5 , S_6 , and S_9 yielded the 0 value. The variables whose value is 1 are alternatives selected to form a portfolio of best strategies and those whose value is 0 are eliminated. The eliminated strategies are of less importance and their roles can be performed by the remaining strategies. The selected strategies represent a total cost of TZS 235.71 million which is 97% of the total cost budgeted by the water authority. This means the authority will save 3% (TZS 7.29 million) if the strategies of this portfolio are implemented. The maximum sum of the weighted normalised values of the selected strategies is Z = 0.82456.

4. Sensitivity analysis of the model

This is an important concept for the successive use and execution of quantitative decision models. Sensitivity analysis intends to assess the stability or robustness of the optimum

result by changing some parameters [27]. For that, the ranking of the criteria was changed after doing mathematical calculations which lead to the change of weights of the criteria using mathematical formulae as explained by [28]. Two important definitions were considered when carrying out the sensitivity analysis of the model.

i) The criticality degree of the criterion C_k , (D'_k) : This is the smallest percentage amount that causes the current value of the weight (w_k) to change, and results in the change of the existing ranking of alternatives. The D'_k is calculated by Eq. (25).

$$D'_{k} = \min |\delta'_{k,i,j}|, \forall n \ge k \ge 1 \text{ and } 1 \le i < j \le m$$

$$(25)$$

ii) The sensitivity coefficient of criterion C_k , $(sens(C_k))$: This is the inverse of criticality degree. The Sens (C_k) is calculated by Eq. (26).

sens
$$(C_k) = \frac{1}{D'_k}$$
, for any $n \ge k \ge 1$ (26)

The decision criterion with the highest sensitivity coefficient is considered to be the most important one. The minimum relative change (min $\delta'_{k,i,j}$) in criteria weights are given by the formulae (27) to (30):

$$\min \delta'_{k,i,j} = \frac{\min \delta_{k,i,j}}{w_k} \ge 100 \text{, for } l \le i < j \le m \text{ and } l \le k \le n$$
(27)

 $min\delta_{k,i,j}$ represents the minimum change in the weight W_k and W_k is the current weight of criterion C_k . And,

$$\delta_{k,i,j} < \left(\frac{Q_j - Q_i}{a_{j,k} - a_{i,k}}\right) \quad \text{if } a_{j,k} > a_{i,k} \quad \text{or}$$

$$\delta_{k,i,j} > \left(\frac{Q_j - Q_i}{a_{j,k} - a_{i,k}}\right) \quad \text{if } a_{j,k} < a_{i,k} \quad (28)$$

For $\delta_{k,i,j}$ to be achievable the condition in (inequality 23) must be met.

$$\left(\frac{Q_j - Q_i}{a_{j,k} - a_{i,k}}\right) \le w_k \tag{29}$$

where Q_i , Q_j , and a_{ik} , a_{jk} are the values of weighted sum and normalised values of strategies S_i and S_j respectively on weight w_k . Therefore,

$$\delta'_{k,i,j} < \left(\frac{Q_j - Q_i}{a_{j,k} - a_{i,k}}\right) \times \frac{100}{w_k} , \text{ if } a_{j,k} > a_{i,k} \quad \text{or}$$

$$\delta'_{k,i,j} > \left(\frac{Q_j - Q_i}{a_{j,k} - a_{i,k}}\right) \times \frac{100}{w_k} , \text{ if } a_{j,k} < a_{i,k} \quad (30)$$

In the analysis, the new ranking of decision criteria is given in relation (31).

 $C_2 > C_1 > C_3 > C_4 > C_7 > C_6 > C_5$ (31) Using the weights of criteria generated by SMARTER- ROC method in subsection (3.4) implies that $C_2 \rightarrow W_1, C_1 \rightarrow W_2, C_3 \rightarrow W_3, C_4 \rightarrow W_4, C_7 \rightarrow W_5, C_6 \rightarrow W_6, C_5 \rightarrow W_7$. The new ranking of strategies is given by relation (32).

$$S_{11} \succ S_2 \succ S_{10}, S_{12} \succ S_7 \succ S_3 \succ S_4 \rightarrow S_{16} \succ S_8 \succ S_{13} \succ S_1 \succ S_{15} \succ S_9 \succ S_5 \succ S_6 \succ S_{14}$$
(32)

With S_{11} (the comparison between the bulk meter and customers' meter on detection of physical or apparent losses) indicating the best strategy and A_{14} (installation of quality pipes) indicating the worst strategy.

Furthermore, by using the values of the new sum of weighted normalised (Q_i) of the strategies, the new ILP model was formulated and when solved the same portfolio of strategies S_1 , S_2 , S_3 , S_4 , S_7 , S_8 , S_{10} , S_{11} , S_{12} , S_{13} , S_{14} , S_{15} , and S_{16} was selected. Three strategies S_5 , S_6 , and S_9 were eliminated from the list as well. This implies that the ILP model for selecting the portfolio of the best strategies is robust regardless of the ranking of strategies and the change of weights of evaluation criteria.

5. Conclusion

This study aimed to optimise the strategies used in WLM in the WDS using MCDM and ILP techniques. The MCDM methods were used to assess and prioritise the strategies while the ILP technique was used to select the portfolio of the best strategies to be used in WLM in the WDS.

In assessing the alternatives, the result showed that the Q_i values for all strategies were above 50% when compared with the value of the best strategy (Table 4), meaning that all the investigated alternatives were important for water loss management. The MCDM methods rank the *comparison between the bulk meter and customers' meter on the detection of physical or apparent losses* (S_{11}) as the best strategy for WLM while *network zoning and establishing DMA* (S_6) as the worst strategy.

In selection of strategies, the ILP model selected thirteen strategies: advertising campaigns (S_1) , education campaign in schools (S_2) , ward meeting with the society (S_3) , meeting with local leaders (S_4) , Indicators for quantifying the losses (S_7) , Calibration of meters (S_8) , visual inspection (S_{10}) , comparison between the readings of bulk meter and customers' meter (S_{11}) , report from the community (S_{12}) , Replacement of dilapidated (decay) pipes (S_{13}) , Installation of quality pipes (S_{14}) , Timely repair of pipe and fitting leaks (S_{15}) , and Pressure management (S_{16}) . The model eliminated three strategies: Illegal control (S_5) , Network zoning and establishing DMA (S_6) , and replacement of defect meters (S_9) . It was established that the selected strategies cost 97% (TZS 235.71 million) of the total budgets set for WLM by MUWSA, meaning that the authority will save 3% (TZS 7.3 million) of its budget which can be allocated to other operational activities.

Furthermore, the sensitivity analysis of the model was done by altering the ranking of criteria. The model selected the same strategies as those selected before altering the ranking of the criteria. This implies that the model is robust for selecting the best strategies applicable in WLM in the WDS.

Basing on the results, the combination of MAVT-SMARTER- SAW the MCDM methods and the numerical optimisation techniques (ILP and BIP) are the appropriate approaches for decision making, especially in water resource management.

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