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Modeling and Optimization of Clean Water Distribution Networks

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Abstract. In this study, an optimal clean water distribution system cost model has been developed to find the minimum cost to distribute clean water. The model was then tested with real data collected from Ihumwa water distribution network of Dodoma city and other treatment cost data from literature to test the workability of the model. Hydraulic parameters such as head losses of the pipes, flow velocity and pipe pressure are calculated using water flow software. The resulted model was solved using LINGO software and the optimal cost of clean water distribution system was found by testing the different maximum and minimum velocity and pressure that give an optimal cost.

Keywords: Modeling, optimization model, clean water distribution

AMS Mathematics Subject Classification (2010): 46N10

1. Introduction

The first and most important public service that people demand is a consistent supply of clean and safe water. A network of pipes, tanks, pumps, valves and other hydraulic elements consists of the water distribution system. The goal is to supply quality water under specific pressure conditions and a range of specifications to customers.

The UN Water Development Report of 2018 shows that many people will be affected by drinking water shortages by 2050. This is due to increased demand for water, reduced water resources and increasing water pollution driven by spectacular population and economic increase [1].

It is complex to manage and allocate water from the multi-reservoir systems and thus requires dynamic modeling systems to obtain optimal performance [2].

The water distribution network consists of pumps, pipes, valves and node sets of a reservoir and pipe connections. A set of stationary points, some of which are nonlinear, determines the flow pressure in the network. The experimentally determined relationship between the pressure and the flow rate is associated with nonlinear conditions (i.e. the discrete pressure from one point of the pipe to the other is a flow rate/time nonlinear function).

Based on [3], the problem of classical pipe network analysis is based on finding a set of flows and pressures in the water distribution pipe network when inputs and withdrawals are known. New water systems (NWAs) are difficult to manage due to increased urbanization, changing consumer needs, old infrastructure, operating costs and lack of water resources.

The study of [4] contends that in a wide range of industrial processes and urban centers, WDNs are present. WDNs are formed by reservoirs, pipes, nodes, loops and pumps and their design can be formulated as an optimization problem. The primary goal is to minimize the cost of distributing water, which depends on pipe diameters and flow directions, in the given network.

In the study of [5] used an integrated model of Multi-Criteria Decision Making (MCDM) and Integer Linear Programming (ILP) to optimize of water loss management strategies.

The study of [6], shows a design method for a fixed flow speed, where the entire cost corresponds to that expenses of the demand flow variable. The method is built on the Granados method, which is an instinctive and practical gradient based technique. To familiarize it to regular demand, the idea of similar flow speed and volume is presented and used in a simple case study.

Effective decision-making when it comes to water and wastewater services requires a comprehensive approach that ensures the best return at an acceptable level of risk, taking into account the costs of constructing, operating, maintaining and disposing of capital assets over their lifetime [7].

In the study of [8] the optimization model known as deterministic mathematical programming proposed to determine the minimum cost of looped WDNs. The model optimization taking into account pipe lengths and a discrete set of commercially available diameters and the constraints is mass balances in nodes, energy balances in loops and hydraulic equations. The discrete optimization problem is reformulated by generalized disjunctive programming to a non-linear integer-blended programming problem (MINLP). The problem is solved by General Algebraic Modeling System (GAMS) environment.

Moreover, to exercise the modeling and optimization of water distribution one needs to have the best model that will optimize the cost of clean water distribution networks in the selected area or data of a given area. In many literatures, the available models for optimization of water distribution network have been used to optimize the cost based on hydraulic parameters, example in the study of [9] which took place in the southern area of Italy (Crotone), it uses nonlinear optimization model to optimize drinking water distribution systems in relation to the effects of climate change. PSO method was used in the study of optimization of tree Pipe networks layout and size, by [10]. [11] Used mixed integer Linear Programming (MLP) to optimize the allocation of water and the location of one more reservoirs.

Although many studies have been conducted on various cost optimizations for water distribution networks, some of these studies do not consider the cost of water treatment parameters and hydraulic parameters in the same model while formulating water cost optimization models.

The aim of this study is to formulate an optimization model to optimize the cost of clean water distribution network using [12] model for the hydraulic parameter used, i.e. pressure, velocity and flow rate and model by [13] for the case of water treatment parameters. With a slight modification of the parameters for these two models, a new model has been developed in this study to optimize the cost of a clean water distribution system.

2. Optimization model

The LP model is based on the papers of [12] and [13]. The objective function to be minimized is the price of clean water distribution networks cost, composed of pipes diameters cost and the associated water treatment parameter. The constraints are pressures and velocity limits, maintenance cost, energy cost, chemical for water treatment cost and personnel cost.

2.1. Definition of the model parameters and variables

Table 1 defines parameters and variables of the model:

Table 1: Description model parameters and variables

Symbols	Definition
L_n	The length of pipe n
CP_n	The cost of unit length of pipe n
$C(d_n)$	Represents the cost of the pipes
d_n	Diameter of pipe n
M_c	Maintenance costs
E_c	Energy cost
C_c	Chemical cost
P_C	Personnel cost
β	Maintenance cost coefficient
α	Energy cost coefficient
δ	Chemical cost coefficient
μ	Personnel cost coefficient
Y1	Number of Maintenances
Y2	Average quantity of energy

Y3	Average quantity of chemicals
Y4	Number of personnel.
P_i	Represent pressure head at node i
P_j	Represent reference node pressure
Prmax	Maximum pressure
Prmin	Minimum pressure
V_{min}	Minimum velocity
V_{max}	Maximum velocity
Q_n	flow discharge
APC	Average person cost
ACT	Average chemical cost
AQC	Average quantity of chemicals
AQE	Average Quantity of energy in KWh

2.2. Objective function.

The sum of all tube diameters and their costs and the cost of treatment must be considered in the objective function.

$$Min(C(dn) + T(m_c + E_c + C_c + P_c)) \quad (1)$$

where $C(d_n)$ represents the cost of the pipes which includes transportation and installation cost and $T(m_c, E_c, C_c, P_c)$ is the treatment cost which include maintenance cost, energy cost, chemical cost and personnel cost.

The cost of the pipes is given as in equation (2)

$$C(d_n) = \sum_{n=1}^{NP} L_n CP_n\,(d_n) \quad (2)$$

The treatment cost is given as in equation (3)

$$T(m_c + E_c + C_c + P_c) = \beta Y_1 + \alpha Y_2 + \sigma Y_3 + \mu Y_4 \quad (3)$$

Now the objective function which is the total cost of distributing clean water is given as equation (4)

$$Min(\sum_{n=1}^{NP} L_n CP_n\,(d_n) + \beta Y_1 + \alpha Y_2 + \delta Y_3 + \mu Y_4 \quad (4)$$

Maintenance cost coefficient (β) is given as in equation (5)

$$\beta = \frac{AMC}{NM} \quad (5)$$

Energy cost coefficient(α) is given as in equation (6)

$$\alpha = \frac{AEC}{AOF} \tag{6}$$

Chemical cost coefficient (δ) is given as in equation (7)

$$\delta = \frac{ACT}{AQC} \tag{7}$$

Personnel cost coefficient (μ) is given as in equation (8)

$$\mu = \frac{APC}{NP} \tag{8}$$

The first term of the objective function has the non-linearity property therefore is multiplied by the summations and non-zero unit variables such as $X_{N.}$ The addition of all commercially available pipes gives the general objective function as in equation (9).

$$Min(\sum_{j=1}^{NPA}\sum_{n=1}^{NP} L_n CP_n(d_n) + \beta Y_1 + \alpha Y_2 + \delta Y_3 + \mu Y_4) \tag{9}$$

where NPA is the number of tube sizes available on the marketplace.

2.3. Constraints of the objective function

The following constraints apply to the objective function:

2.3.1. Pressure constraint

The pressure constraint for this study is upper and lower pressure that gives the optimal cost for clean water distribution which is given by equation (10) and equation (11).

$$P_i \geq Pr_{min} \tag{10}$$

$$P_i \leq Pr_{max} \tag{11}$$

where P_i is the pressure head at node i, which is given by equation (12)

$$P_i = P_j + \Delta Z - HL_j \tag{12}$$

where HL_j are head-losses from reference node and end at node i, which are calculated using Hazen-Williams formula for this study and they are given by equation (13) below.

$$HL_j = \frac{10.67 * L_n * Q_n^{1.85}}{C^{1.85} * d_n^{4.87}} \tag{13}$$

Equation (12) substituted into equation (10) and (11), respectively, to give equations (14) and (15).

$$P_j + \Delta Z - HL_j \geq Pr_{min} \tag{14}$$

$$P_j + \Delta Z - HL_j \leq Pr_{max} \tag{15}$$

The equations (14) and (15) are multiplied by summation and non-zero unit variable in the head loss to make them linear constraints as in equation (16) and (17)

$$P_j + \Delta Z - \sum_{j=1}^{NPR} HL_j \, X_{NJ} \geq Pr_{min} \tag{16}$$

$$P_j = \Delta Z - \sum_{j=1}^{NPR} HL_J \, X_{NJ} \leq Pr_{max} \tag{17}$$

Therefore, equation (16) and (17) are the model pressure constraints. Where NPR is the number of pipes connected to the reference node.

2.3.2. Velocity constraint

The Flow velocity constraint is given as in equation (18).

$$V_{min} \leq V_n \leq V_{max} \tag{18}$$

$V_{\min}$ is the minimum allowable flow speed in the pipe, $V_{\max}$ is the maximum allowable flow speed in the pipe and V_n is the pipe flow speed which is given by equation (19).

$$V_n = \frac{4Q_n}{\pi d_n^2} \tag{19}$$

Substituting equation (19) in equation (18) and multiplying by summation and non-zero unit variable results in equation (20) which is a velocity model constraint.

$$V_{min} \leq \sum_{j=1}^{NPA} \frac{4Q_n}{\pi d_n^2} X_{NJ} \leq V_{max} \tag{20}$$

2.3.3. Maintenance constraint

The products of maintenance coefficient cost and the number of maintenance in a month is greater or equal to the average maintenance cost and it is given by equation (21).

$$\beta Y_1 \geq M \tag{21}$$

2.3.4. Energy constraint

The product of energy coefficient cost and the average quantity of electricity used is greater or equal to the average cost of electricity and it is given by equation (22).

$$\alpha Y_2 \geq E \tag{22}$$

2.3.5. Chemical constraint

The product of chemical coefficient cost and the average quantity of chemical is greater or equal to the average cost in chemical and it is given by equation (23).

$$\delta Y_3 \geq Z \tag{23}$$

2.3.6. Personnel constraint

The product of personnel coefficient cost and the number of personnel is greater or equal to average personnel cost and it is given by equation (24).

$$\mu Y_4 \geq P \tag{24}$$

2.4. Developed optimization model

The optimization model developed in this study

Objective function

$$Minimize(\sum_{j=1}^{NPA}\sum_{n=1}^{NP} L_n CP_n(d_n) + \beta Y_1 + \alpha Y_2 + \delta Y_3 + \mu Y_4 \tag{25}$$

Subject to the constraints

$$P_j + \Delta Z - \sum_{j=1}^{NPR} HL_j X_{Nj} \geq Pr_{min} \tag{26}$$

$$P_j + \Delta Z - \sum_{j=1}^{NPR} HL_J X_{Nj} \leq Pr_{max} \tag{27}$$

$$V_{min} \leq \sum_{j=1}^{NPA} \frac{4Q_n}{\pi d_n^2} X_{Nj} \leq V_{max} \tag{28}$$

$$\beta Y_1 \geq M \tag{29}$$

$$\alpha Y_2 \geq E \tag{30}$$

$$\delta Y_3 \geq Z \tag{31}$$

$$\mu Y_4 \geq P \tag{32}$$

$$Y_1, Y_2, Y_3, Y_4 \geq 0 \tag{33}$$

$$X_1, X_2, X_3, X_4, X_5 \geq 0 \tag{34}$$

3. Model application

The model developed in this study for analyzing the clean water distribution cost has been applied to the water treatment data for DUWASA (these are secondary data) and hydraulic data (raw data) collected from ongoing water projects in Dodoma city.

Table 2: Details of the pipe laid, IHUMWA, DODOMA

Junction	pipe size	Length(m)	Discharge(l/s)	Head-loss (m)	Velocity(m/s)
Tank- J1	300	279.8	48.9	0.37	0.69
J1-J2	300	643.9	48.9	0.85	0.69
J1-J11	250	450.5	48.9	1.44	0.99
J2-J17-J30	200	774.7	48.9	6.95	1.5
J10-J12-J16	160	768.1	48.9	21.66	2.42
J11-J10-J9	160	224.2	48.9	6.32	2.42
J9-J15-J8	160	265.3	48.9	7.48	2.42
J8-J7-J6	160	355	48.9	10.01	2.42
J6-J2	160	277.8	48.9	7.84	2.42
J2-J3	160	271.8	48.9	7.67	2.42
J19-J20	160	259	48.9	7.3	2.42
J19-J30	160	293.4	48.9	8.28	2.42
J21-J22-J20	160	634.1	48.9	17.88	2.42
J16-J14-J5	75	1353.6	48.9	1533.33	11
J5-J4	75	332.7	48.9	376.88	11
J4-J3	75	335.6	48.9	380.16	11
J5-J6	75	240.1	48.9	271.87	11
J9-J13	75	414.6	48.9	469.65	11
J27-J28	75	639.9	48.9	724.86	11
J21-J23	75	333.1	48.9	377.33	11
J23-J24	75	175.7	48.9	199.03	11
J24-J25-J26	75	345.7	48.9	391.7	11
J19-J25	75	341.4	48.9	386.73	11
J26-J27	75	411.4	49.9	466.02	11
J28-J29	75	367.4	48.9	416.18	11
TOTAL	LENGTH	10788.8			

Table 2. Shows the details of pipes laid at Ihumwa water supply network in Dodoma region. Discharges head-lose and velocity of the pipes from table 2 is calculated using computer water software.

Table 3: Average Discharge and Head-loss (HL) for the pipe used

Diameter(mm)	Pressure(N/m^2)	Discharge(Q)(l/s)	Headloss(HL)(m)	Elevation(m)
300	12	49	3.22	0
250	12	48.9	1.44	0
200	10	47.4	6.95	0
160	10	48.9	10.5	0
75	10	48.9	499.4	0

Table 3. Shows the averages of pipe discharges, head-loss; elevation of pipes calculated from table 2.

Table 4: Cost of pipe used

Diameter(mm)	length(m)	Cost per unit length(TZS)	Total Cost(TZS)
300	924	78260	72312240
250	451	52440	23650440
200	751	44230	33349420
160	3283	41190	13945310
75	5380	12680	67089880
Total length	10788	Total Cost	334347290

Table 4. Shows the cost of pipe used in the n**e**twork.

Table 5: Quantities of chemicals and electricity

MONTH	CHEMICALS (Kg)	ELECTRICITY (KWh)
January	676390	4179.68
February	598725	3370.76
March	721710	4447. 21
April	595075	3895.38
May	776845	4323.99
June	678925	3903.20
July	780265	5017.82
August	731695	4794.49
September	725530	4538.19
October	647515	4960.54
November	690345	4856.01
December	669420	4838.86
Total	8292440	53126.04
Average	691036.67	4427.17

Table 5. Shows the quantity of chemicals utilized for water treatment and the quantity of electricity utilized annually. The Average Quantity of energy (AQE) is in kWh.

Table 6: Cost of energy, treatment and maintenances/ operation, personnel cost

Month	Energy cost(TZS)	Treatment cost(TZS)	Maintenance cost(TZS)	Personnel cost(TZS)
January	1437812.58	550154.3	142815.66	2739500
February	1159542.47	482400.45	138948.17	3353560
March	1529840	482400.45	162890.08	2758905
April	1340012.36	592573	162861.16	2258900
May	1487451.4	770491.33	164424.94	3590680
June	1342699.26	770491.33	164795.29	930800

July	1726131.58	779536.98	167046.38	3442000
August	1649303.26	736698.3	157902.79	2852770
September	1561136.88	933843.9	170776.23	5234190
October	1706424.45	836497.95	171417.747	4538500
November	1324500	1670468.16	922190.85	172909.84
December	1664566.2	887812.35	169732.44	3426700
Total	16765388.6	8745091.19	1946520.72	33024305
Average	1397115.717	728757.6	162210	2752025.417

Table 6. Shows the cost of energy, chemicals, maintenance and personnel cost together with their total and averages which are used to calculate cost coefficients for energy, chemicals, maintenance and personnel cost.

Table 7: Calculated cost coefficients

Maintenance cost coefficient (β)	27035
Energy cost coefficient (α)	315.58
Chemical cost coefficient (δ)	1.05
Personnel cost coefficient (μ)	34400.3

Table 7. Shows the Calculated water treatment cost coefficients.

3.1. Resulting model

Objective function.

Minimize

$$72312240X_1 + 23650440X_2 + 33349420X_3 + 13945310X_4 + 67089880X_5 + 27035Y_1 + 315.58Y_2 + 1.05Y_3 + 34400.3Y_4 \quad (35)$$

Subject to the constraints

$$3.22X_1 + 1.44X_2 + 6.95X_3 + 10.95X_4 + 499.4X_5 \leq 750 \quad (36)$$

$$3.22X_1 + 1.44X_2 + 6.95X_3 + 10.95X_4 + 499.4X_5 \geq 690 \quad (37)$$

$$0.69X_1 + 0.993X_2 - 1.56X_3 + 2.44X_4 + 11.1X_5 \geq 0.5 \quad (38)$$

$$0.69X_1 + 0.993X_2 - 1.56X_3 + 2.44X_4 + 11.1X_5 \leq 3 \quad (39)$$

$$27035Y_1 \geq 16210.06 \quad (40)$$

$$315.58Y_2 \geq 1397115.717 \quad (41)$$

$$1.05Y_3 \geq 728757.6 \quad (42)$$

$$34400.3Y_4 \geq 2752025.417 \quad (43)$$

$$Y_1, Y_2, Y_3, Y_4 \geq 0 \quad (44)$$

$$X_1, X_2, X_3, X_4, X_5, \geq 0 \quad (45)$$

4. Discussion

The resulting optimisation model produced an optimisation problem which was solved with the help of LINGO (linear, interactive, discrete optimizer) software. The model shows that the unknowns for pipes and water treatment give optimal cost for clean water distribution networks. The unknown for pipes give the optimal solution under controlled minimum and maximum Pressure and velocity of water in the pipes.

4.1. Costs comparison

From table 4.1 the cost of the pipes is reduced from TZS 334,347,290 to TZS 322,664,634.4 which equals 1.5% of the total pipes cost, while the cost of treatment is reduced from TZS 5,040,138.734 to TZS 4,894,084.659 which equals 2.9% of the total treatment cost.
The total cost of distributing clean water is reduced from TZS 339,387,428.7 to TZS 327,558,700 which equals 4.4% of the total cost of distributing clean water in the given network.

Table 8: Comparison of the optimal and original costs of the decision variables of the model

	OPTIMAL COST	ORIGINAL COST	DIFFERENCE
PIPE COST	322664634.4	334347290	11682655.6
TREATMENT COST	4894084.659	5040138.734	146054.075
TOTAL COST	327558700	339387428.7	11828709.68

4.2. Maximum and Minimum pressure and velocity for optimal cost.

From the model the minimum and maximum pressure that gives the optimal solution are 690N/m^2 and 750N/m^2, respectively, while the minimum and maximum velocities are 0.5m/s and 3m/s, respectively, as shown in Table 9.

Table 9: Maximum and Minimum velocity pressure for optimal cost.

	velocity(m/s)	pressure(N/m^2)
Maximum	3	750
Minimum	0.5	690

5. Conclusion

The model developed in this study was used to optimize the cost of the clean water distribution network. Hydraulic data from the Dodoma region under DUWASA (Dodoma Urban water supply, Sanitation Authority) and treatment cost data from other literature are used to test the capabilities of the developed model.
The developed optimization model is characterized by non-linearity in the first term and it is linear in the second term. The non-zero unit variable is multiplied in the first term and its associated constrains in order to make the model linear which can be solved as a linear programming problem to find the optimal cost of distributing clean water.
The model representation of the delivery system for clean water was solved using LINGO software by testing different maximum and minimum pressure and velocities that gives minimum cost of distributing clean water in a given system. The maximum and

minimum pressure that gives an optimal cost for distributing clean water are 700N/m^2 and 650N/m^2, respectively, while the maximum and minimum velocity are 3m/s and 0.5m/s, respectively.

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