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Insurance Companies Portfolio Optimization with Possibilities of Recovery after Ruin: A Case of Exponential Utility Function

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Abstract. In this paper, we propose and analyze the perturbed mathematical model for modeling the portfolio of insurance companies with possibilities of recovery after ruin. Return on investment and refinancing are used as approaches for overcoming ruin. The model is analyzed for different cases of possibilities of recovery after ruin within [0, 1]. The results indicate that the return on investment plays an important role in reducing the ultimate ruin and that as the possibility of recovery for insurance companies increases the return on investment reduces the ruin at a fast rate. Finally, the study recommends that all insurance companies should have well trained staff in risk management who can study the company's portfolio and gives suggestions to managers on how to avoid or minimize ruin and how to recover in case ruin occurs.

Keywords: Ruin probability, Value function, Refinancing, Jump-diffusion, Possibilities of recovery

AMS Mathematics Subject Classification (2010): 37N40

1. Introduction

Risks affect many aspects of human life and in some cases may even result in financial loss. Therefore it is important to secure expensive property and insurance provides that security. In [1], Kasumo observed that the provision of insurance requires competent management as poor management may lead to the eventual ruin of the insurance company, resulting in its failure to fulfill its obligations. This happens when the insurer's surplus level falls below zero, thus making the company bankrupt.

According to [2] insurance refers to a contract that is represented by a policy where individuals or entities receive financial protections from a given insurance company against losses. In [3], Kozmenko and Oliynyk have observed and suggested that in fulfilling its obligations, an insurance company will have a collection of investments that generate income to cover clients' claims and this collection of investments for an insurance company is known as an insurance portfolio. In addition, an insurance company holding a portfolio with many liquid investments reduces the investor's risk since these investments enables the company to fulfill claims whenever they arise.

One of the best measures that an insurance company should take is risk management on its portfolio. Several measures are available for managing risk in an insurance company. Refinancing and investment are some of the measures to overcome the risks of the insurance company and give optimal returns to the shareholders. [4] reveals that using investment, the insurer distributes part of those risks to the paying investments which in turn can save a company to cover clients' benefits during ruin. [5] worked under the martingale invariance hypothesis and assumed the existence of conditional density to study the optimal reinsurance and investment of the insurer where the surplus process of the insurer was assumed to satisfy a jump-diffusion process and the dynamics of the risky stock price followed a Heston model. They considered proportional reinsurance and also investment optimization problems for insurers existing in financial markets depending on risky stock assets, a savings account, and corporate bonds, while [6] studied ruin probability based on a dual risk model having risk-free investments.

Through investment and refinancing strategies, insurers may protect themselves against any potentially big losses or at least ensure that their earnings will remain relatively stable when there is a possibility of recovery after ruin. In the literature (see, for example, [7,8,9]) many optimization problems have arisen as part of the risk management process to study how insurance companies can control ultimate ruin. In [10], studied how to optimize the control in investments and reinsurance problems for an insurer using a jump-diffusion risk process but with the independence of the Brownian motions while [4] studied the optimal investment and reinsurance problem for an insurer and a reinsurer using jump-diffusion processes. This paper seeks to establish the ways of minimizing ruin through portfolio management by maximizing insurance portfolio for the case of exponential utility function when there are possibilities of recovery after ruin.

This paper is organized as follows: In Section 2, we propose a model for maximizing insurance portfolio for a case of exponential utility function when there are possibilities of recovery after ruin. We investigated the behaviour of the value function for the proposed model. In Section 3 the numerical simulations were carried out and their results are presented. In the last section, we present the conclusion recommendation and possible extension of the paper.

2. Model formulation and analysis

We now present the model formulation and its analysis. In this work, we consider continuous time stochastic processes in the time interval $[0,T]$ where $0<T<\infty$. A stochastic process is a family of random variables $X=(X_t)_{t\in[0,T]}$ defined on the probability space (Ω, F, P) and valued in a measurable space (Ω, F) and indexed by time t. For each $\omega \in \Omega$, the mapping $X(\omega): t \in [0,T] \to X(t;\omega)$ is called the path of the process for the event ω. All stochastic quantities and random variables are defined on a large enough stochastic basis $(\Omega, F, (F)_{t\in[0,T]}, P)$ satisfying the usual conditions, that is to say, $(F)_{t\in[0,T]}$ is right continuous and P-complete, P is the probability measure defined on F and $(F)_{t\in[0,T]}$ is an augmented filtration.

In reality, the income of the insurer is not deterministic, there exist fluctuations in the number of customers, claim arrivals, and premium income [11]. If both refinancing (capital injection) and investment are absent, then the basic model for the insurance process can be given by a perturbed risk process X_t defined by

$$X_t = p + ct + \sigma_X W_{X,t} - \sum_{i=1}^{N_{X,t}} Y_{X,i} \quad , t \geq 0 \tag{1}$$

In this case, c is the premium rate, that is the insurer's premium income per unit time assumed to be received continuously and is calculated by the expected value principle, that is $c = (1+\theta)\lambda_X \mu_X$ where $\theta > 0$ is the relative safety loading of the insurer. Also $X_0 = p$ is the initial capital of the insurance company and W_X is a standard Brownian motion independent of the compound Poisson process $\sum_{i=1}^{N_{X,t}} Y_{X,i}$. Here λ_X is the intensity of the counting process $N_{X,t}$ for the claims and let F_X be the distribution function of the claims $Y_{X,i}$. It is assumed that F_X is continuous and concentrated on $[0,\infty)$. We interpret equation (1) above as follows: ct is the premium income received by an insurance company up to time t. The Brownian term $\sigma_X W_X$ is meant to take care of small perturbations in premium income and claim sizes, $N_{X,t}$ is the claim number process and $Y_{X,i}$ are claim sizes. It is assumed that $F_X(0) = 0$ and at least one of σ_X or λ_X is non-zero. A vast number of researchers have studied this classical risk process perturbed by diffusion in the insurance industry, some of them being [1,4,11,12,13,14].

For refinancing process we let M be an increasing process with $M_{0^-} = 0$. The process with refinancing (capital injections) is denoted by $X_t^M = X_t + M_t$ with X_t being the surplus process and $X_0 = p$. The capital injection process M has to be chosen such that $X_t^M \geq 0$ for all t (almost surely); it could then be optimal to inject capital already before the process reaches zero. Therefore, by using equation (1) the model with refinancing will be given by equation (2) below;

$$X_t^M = p + ct + \sigma_X W_{X,t} - \sum_{i=1}^{N_{X,t}} Y_{X,i} + M_t \quad , t \geq 0 \tag{2}$$

For the investment process, we assume the risk-free (bond) price process is given by

$$dB_t = r_0 B_t dt \tag{3}$$

where $r_0 \geq 0$ is the risk-free interest rate, which is assumed to be constant. B_t is the price of the risk-free bond at a time t.

According to [15] stocks are an important way for the company to raise funds. Let us also describe the risky asset (stock) price process by the Geometric Brownian Motion (GBM) given by

$$dS_t = rS_t dt + \sigma_S S_t dW_{S,t} \tag{4}$$

where S_t is the price of the stock at time t, $r \geq 0$ is the expected instantaneous rate of stock return, $\sigma_S \geq 0$ is the volatility of the stock price and $W_{S,t} : t \geq 0$ is a standard Brownian motion defined on the complete probability space $(\Omega, F, (F)_{t\in[0,T]}, P)$. [16] gave the generalized return on investment process R_t as

$$R_t = rt + \sigma_R W_{R,t} + \sum_{i=1}^{N_{R,t}} S_{R,i} \;\;, t \geq 0,\; R_0 = 0 \tag{5}$$

where $W_{R,t}$ is a Brownian motion independent of the surplus process R_t, also $\sum_{i=1}^{N_{R,t}} S_{R,i}$ is a compound Poisson process with intensity λ_R which represents the sudden changes in income (jumps), the term $\sigma_R W_{R,t}$ represents the fluctuation in income of an insurance company and the rt is the non-risky part of the investment process. If we assume $\lambda_R = 0$, that is, there are no jumps, the resulting model which was also discussed by [17] is the Black-Scholes option pricing formula given by

$$R_t = rt + \sigma_R W_{R,t} \;\;, t \geq 0,\; R_0 = 0 \tag{6}$$

Equation (6) above is the return on investment model, where r is the risk-free part, hence $R_t = rt$ means that one unit invested at time zero will be worth e^{rt} at time t.

2.1. Stochastic differential equation for the wealth

In this section, a basic insurance process with investment, which is expressed by the stochastic differential equation for the wealth after refinancing, is formulated. Now let us consider the investment problem of an insurance company that seeks to transfer current wealth into the bond and stock. The company's preference is to choose a dynamic portfolio strategy in order to maximize the expected utility of wealth at some future time *T*. Therefore in order to describe the company's actions the portfolio strategy is formulated.

Assume that the joint distribution of the $W_{X,t}$ and $W_{S,t}$ that are used is bivariate normal and we denote their correlation coefficient by ρ, that is $E[W_{X,t} W_{S,t}] = \rho t$. The company needs to monitor its wealth, let the amount of money invested in risky asset (stock) at time t under investment policy π be denoted by π_t, where $\{\pi_t\}$ is a portfolio strategy suitable and admissible control process, that is to say π_t satisfies $\int_0^T \pi_t^2 dt < \infty$ a.s., for all $T < \infty$. Let $\{Z_t, t \geq 0\}$ denote the corresponding wealth process, then the dynamic of Z_t is given by

$$dZ_t = \pi_t \frac{dS_t}{S_t} + \left(X_t^M - \pi_t\right)\frac{dB_t}{B_t} + dX_t^M \tag{7}$$

with $Z_0 = z > 0$ being the initial wealth of the company.

By substituting equations (3) and (4) above into equation (7) then the wealth process with investment and refinancing will follow the stochastic differential equation

$$dZ_t = \left(\pi_t r + \left(X_t^M - \pi_t\right) r_0\right)dt + \sigma_S \pi_t dW_{S,t} + dX_t^M \tag{8}$$

where X_t^M is given by equation (2).

2.2. Optimal control problem for maximizing the expected utility of terminal wealth

In [18], the authors studied the problem of the expected utility of wealth in the discrete time for a given investor. In the study it was conjectured that minimizing the ruin probability is strictly related to maximizing the exponential utility of terminal wealth of the investor, the assumption behind the conjecture was that the investor is allowed to borrow an unlimited amount of money and without risk-free interest rate.

Let a strategy α describe the stochastic process $\{\pi_t, M_t\}$, where π_t the amount invested in the risky asset at time t and M_t is the capital refinanced/injected at time t and denote the set of all admissible strategies by α_S . Suppose now that the insurer is interested in maximizing the utility function of its terminal wealth, say at time T . The utility function $u(z)$ is typically increasing and concave $(u''(z) < 0)$. For a strategy α let's define the utility attained by the insurer from state z at time t as follows;

$$V_\alpha(t,z) = \mathrm{E}[u(Z(T))/Z(t) = z] \tag{9}$$

Therefore the objective is to find the optimal value function

$$V(t,z) = \sup_{\alpha \in \alpha_S} V_\alpha(t,z) \tag{10}$$

and the optimal strategy $\alpha^*\{\pi_t^*, M_t^*\}$ such that $V_{\alpha^*}(t,z) = V(t,z)$.

2.3. Maximizing the exponential utility of terminal wealth

An ordinary investor under discrete time and space was studied by [18] where it was found that when the investor had an exponential utility function such as $u(z) = -e^{-\theta z}$ and aiming at maximizing the utility of terminal wealth at fixed terminal time, then the optimal policy was an investment of a fixed constant amount. The conclusion given by a strategy was in general optimal for minimizing the probability of ruin or maximizing the probability of survival.

Using equations (9) and (10) above, let π_t^* denote the optimal policy and suppose that the company is now having an exponential utility of the form (11) below, where $\gamma > 0$ and $\theta > 0$.

$$u(z) = \lambda - \frac{\gamma}{\theta} e^{-\theta z} \tag{11}$$

This kind of utility function has constant absolute risk aversion (ARA) since $-u''(z)/u'(z)=\theta$, it plays a very important role in actuarial and insurance mathematics at large.

Theorem 2.1. The optimal policy of maximizing expected utility at a terminal time T is investing at each time $t \le T$ a constant amount given by

$$\pi_t^* = \frac{r}{\sigma_S^2\theta} - \frac{\rho}{\sigma_S} \tag{12}$$

Then the optimal value function becomes

$$V(t,z) = \lambda - \frac{\gamma}{\theta}\exp\{-\theta z + (T-t)Q(\theta)\} \tag{13}$$

where $Q(.)$ is the quadratic function defined by

$$Q(\theta) = \frac{1}{2}(1-\rho^2)\theta^2 - \left(X_t^M r_0 - \rho\left(\frac{r-r_0}{\sigma_S}\right)\right)\theta - \frac{1}{2}\left(\frac{r-r_0}{\sigma_S}\right)^2 \tag{14}$$

Proof: For our problem of maximizing utility of terminal wealth at a fixed terminal time T. Then the HJB equations for $t < T$ can be obtained as follows

$$\begin{cases} \sup_{\pi_t}\left\{\ell^{\pi_t}V(t,z)\right\}=0 \\ V(T,z)=u(z) \end{cases} \tag{15}$$

where $V(t,z) = \sup_{\pi_t} \mathrm{E}^{t,z}\left[u\left(Z_T^{\pi_t}\right)\right]$ this is the same as saying for each (t, z) we need to solve the nonlinear PDE of (15) and there after find the value of π_t that will maximize the function (16) below

$$f(\pi_t) = V_t + \left[\pi_t r - \pi_t r_0 + X_t^M r_0\right]V_z + \frac{1}{2}\left[\sigma_S^2\pi_t^2 + 2\rho\sigma_S\pi_t + 1\right]V_{zz} \tag{16}$$

Suppose we assume that the HJB equation (15) consists of a classical solution V that satisfies $V_z > 0$ and $V_{zz} < 0$ now differentiating with respect to π_t and equating to zero in (16) the following optimizer is obtained

$$\pi_t = -\frac{\rho}{\sigma_S} - \left(\frac{r-r_0}{\sigma_S^2}\right)\left(\frac{V_z}{V_{zz}}\right) \tag{17}$$

Substituting equation (17) back into equation (16) then after some simplifications equation (15) become

$$\begin{cases} V_t + \left[X_t^M r_0 - \rho\left(\frac{r-r_0}{\sigma_S}\right)\right]V_z - \frac{1}{2}\left(\frac{r-r_0}{\sigma_S}\right)^2\frac{V_z^2}{V_{zz}} + \frac{1}{2}(1-\rho^2)V_{zz} = 0 \quad \text{for } t<T \\ V(T,z)=u(z) \end{cases} \tag{18}$$

The PDEs obtained in equation (18) above are quite different from those obtained in other studies of utility maximization such as those by [19, 20]. Since we want to solve the

PDE under a given case when $u(z)=\lambda-\frac{\gamma}{\theta}e^{-\theta z}$. To solve the PDE in equation (18) above under this case let's assume that it has the solution of the following form

$$u(z)=\lambda-\frac{\gamma}{\theta}e^{-\theta z+g(T-t)} \tag{19}$$

where $g(.)$ is a given suitable function, with this assumption, then

$$\begin{cases} V_t(t,z)=[V(t,z)-\lambda][-g'(T-t)] \\ V_z(t,z)=[V(t,z)-\lambda][-\theta] \\ V_{zz}(t,z)=[V(t,z)-\lambda][\theta^2] \end{cases} \tag{20}$$

Since the boundary condition is $V(T,z)=\lambda-\frac{\gamma}{\theta}e^{-\theta z}$ this mean that $g(0)=0$ now let us insert (20) into (18) and simplify to get

$$-g'(T-t)+\frac{1}{2}(1-\rho^2)\theta^2-\left(X_t^M r_0-\rho\left(\frac{r-r_0}{\sigma_S}\right)\right)\theta-\frac{1}{2}\left(\frac{r-r_0}{\sigma_S}\right)^2=0 \tag{21}$$

Now letting $Q(\theta)=\frac{1}{2}(1-\rho^2)\theta^2-\left(X_t^M r_0-\rho\left(\frac{r-r_0}{\sigma_S}\right)\right)\theta-\frac{1}{2}\left(\frac{r-r_0}{\sigma_S}\right)^2$ gives

$$g'(T-t)=Q(\theta) \tag{22}$$

Integrating equation (22) and using $g(0)=0$ gives the value function (13). Since the value function is known, we can now obtain the control (12) by substituting the values of V_z and V_{zz} from equation (20) into equation (17).

Finally we need to show that the value function and the control obtained above are optimal. This is revealed upon checking the value function (13) since it is seen to be twice continuously differentiable thus we conclude that it satisfies the conditions of classical verification theorems as stated by [21], therefore these are the optimal value function and controls.

2.4. Treating possibility of recovery after ruin for insurance companies

In this section, an approach on how to handle the possibility of recovery after ruin for insurance companies is suggested and developed for the first time in insurance mathematics. We assume that an insurance company had wealth $X_{t_{\tau^-}}$ before the time of ruin suppose an insurance company has a possibility $\gamma\in[0,1]$ of recovery after ruin, where $\gamma=0$ means that the company has no possibility of recovery at all and $\gamma=1$ means the company has a possibility of recovering a full wealth after ruin. Now, this study suggests that the new wealth or capital of the company for running the insurance business be given by the following formula

$$X_t^N=\gamma X_{t_{\tau^-}} \tag{23}$$

Therefore, this approach is used in performing the simulation on different cases of possibilities γ under different situations of wealth or capital X_t when compounded with investment and/or refinancing.

3. Numerical experiments and discussion of results

We numerically observe the mathematical characteristics of the optimal value function given by equation (13) above. Mainly, the target here was to maximize the exponential utility of the terminal wealth. All the model simulations in this paper were performed in an HP ENVY 17 with an Intel(R) Core(TM) i7-8550U CPU processor at 1.80GHz to 1.99GHz and 16.0GB of RAM and the figures were constructed by using MATLAB R2020a. Values of the parameters are presented in Table 1 below, some were estimated and some were obtained from other studies.

Table 1: Model parameters and their values

Symbol	Definition	Value(s)	Source
σ_S	The volatility of the stock price	0.2, 1.3	[22]
λ	Number of claims received per unit time	10, 20, 200	Estimated
θ	Safety loading of the insurer	0.8, 2, 3, 5	[1]
X_t^M	Refinanced surplus process	50, 70, 100, 1000	Estimated
r	Instantaneous rate of stock return	0.05	[13]
r_0	Risk-free interest rate for the bond	0.02, 0.04	[23]
ρ	Correlation coefficient	0.03	[4]
γ	Possibility of recovery	2%, 25%, 80%	Estimated

In Figure 1 we observe an increase in optimal value function at the terminal time, this increase is observed to be irrespective of the initial wealth. Finally the value function attains its maximum value and this value is maintained for all time and initial wealth.

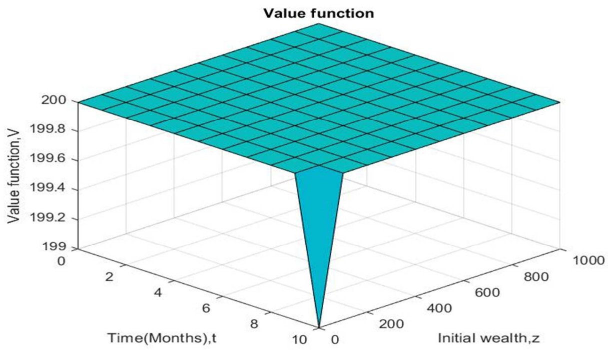


Figure 1: Dependence of the value function over the initial wealth and time.

In Figure 2 we observe that an insurance company has increasing absolute risk aversion (IARA) but also as wealth increases the value function also increases very rapidly at the initial time. According to [24] this implies that as wealth increases an insurance company is advised to hold fewer investments in risky assets likewise this paper recommends that an insurance company should hold as few investments in risky assets as possible.

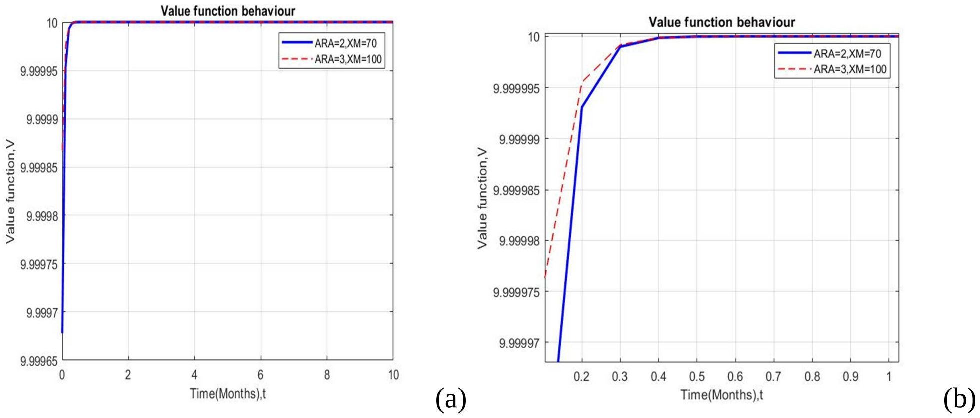


Figure 2: Behaviour of the value function with respect to absolute risk aversion and capital

In Figure 3(a) it appears as if the two graphs coincide but a zoomed graph given in figure 3(b) shows that the value function increase with the increase in the volatility of the stock price. It is clearly observed from Figure 3(b) that when volatility was increased from 0.2 to 1.3 the value function also increased its value, this in turn indicates that taking a relatively higher risk on risky assets will give the insurance company a much better expected wealth utility. These results confirm the results of [4] since the insurance company's utility maximization can be realized for large volatility of the stock price.

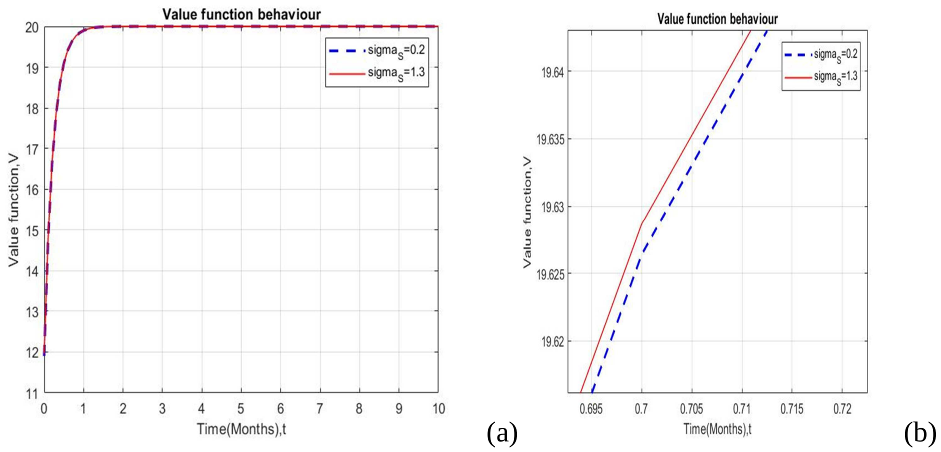
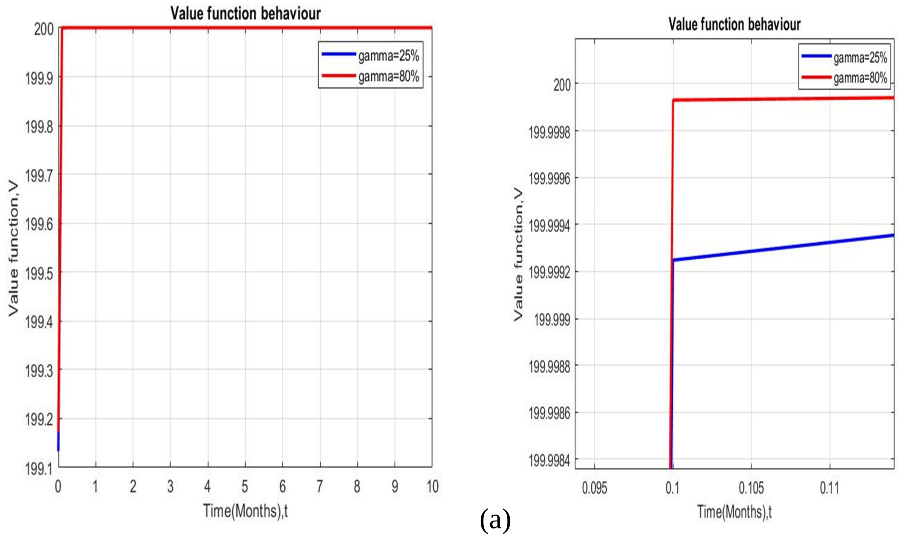


Figure 3: Behaviour of the value function with respect to the volatility of the stock price

Also in Figure 4(a) we observe like two graphs coincides but a zoomed graph given in Figure 4(b) clearly shows that the value function increases with the increase in the possibility of recovery of the insurance company since we see that with the possibility of recovery of 25% the value function has small value as compared to 80% of possibility of recovery. In turn, this indicates that when an insurance company has a higher possibility of recovery it will have a much better expected wealth utility since its value function is much larger.



(a) (b)

Figure 4: Behaviour of the value function with respect to the possibility of recovery of the insurance company.

4. Conclusion

In this study, we have proposed a way of maximizing the exponential utility of terminal wealth. We also formulated a risk process compounded by refinancing and investment thereafter a stochastic differential equation for the wealth was derived and an optimal control problem for maximizing the expected utility of terminal wealth was formulated and solved. An approach to treating the possibility of recovery after ruin for insurance companies was developed and suggested for the first time in insurance mathematics.

We were able to investigate the behavior of the value function numerically and the results indicated that the value function increases irrespective of the initial wealth and time. The results on studying the behavior of the value function with respect to the volatility of the stock price were similar to those of [4] since we observed that insurance company's utility maximization can be realized for large volatility of the stock price. Also, this study observed that in case ruin occurs and a company performs refinancing the value function increases very rapidly with the increase in refinancing amount. This observation was also supported by the behavior of value function with respect to the possibility of recovery after ruin, we also observed that as the possibility of recovery after ruin increases the value function also increases.

This study recommends that all insurance companies should have well-trained staff in risk management who can study the company's portfolio and give suggestions to managers on how to avoid or minimize ruin and how to recover in case ruin occurs.

Further study can be done as an extension where one can incorporate the use of stochastic interest rates instead of the fixed one for both stocks and bonds to realize the actual fluctuation of the interest rates at any given time.

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Authors' Contributions. All the authors contribute equally to this work.

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