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Domination Dharwad Indices of Graphs

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Abstract. In this paper, we introduce the domination Dharwad index, modified domination Dharwad index and their corresponding exponentials of a graph. Furthermore, we compute these indices for some standard graphs like French windmill graphs and friendship graphs.

Keywords: domination Dharwad index, modified domination Dharwad index, graph.

AMS Mathematics Subject Classification (2010): 05*C*10, 05*C*69

1. Introduction

In this paper, *G* denotes a finite, simple, connected graph, *V*(*G*) and *E*(*G*) denote the vertex set and edge set of G. The degree $d_G(u)$ of a vertex u is the number of vertices adjacent to *u*. For undefined terms and notations, we refer the books [1, 2]. Graph indices have their applications in various disciplines of Science and Technology.

The domination degree $d_d(u)$ [3] of a vertex *u* in a graph *G* is defined as the number of minimal dominating sets of *G* which contains *u*.
Ref. [3] was soon followed by a series of publications [4, 5, 6, 7, 8, 9, 10, 11].

Recently, the so-called Dharwad index was put forward, defined as [12]

$$D(G)=\sum_{uv\in E(G)}\sqrt{d_G(u)^3+d_G(v)^3}.$$

Inspired by work on Dharwad index, we introduce the domination Dharwad index of a graph *G* as follows:

The domination Dharwad index of a graph *G* is defined as

$$DD(G)=\sum_{uv\in E(G)}\sqrt{d_d(u)^3+d_d(v)^3}.$$

where $d_d(u)$ is the domination degree of a vertex *u* in *G*.

Considering the domination Dharwad index, we introduce the domination Dharwad exponential of a graph *G* and defined it as

$$DD(G,x)=\sum_{uv\in E(G)}x^{\sqrt{d_d(u)^3+d_d(v)^3}}.$$

We define the modified domination Dharwad index of a graph *G* as

$$^{m}DD(G)=\sum_{uv\in E(G)}\frac{1}{\sqrt{d_d(u)^3+d_d(v)^3}}.$$

Considering the modified domination Dharwad index, we introduce the modified domination Dharwad exponential of a graph G and defined it as

$$^{m}DD(G,x)=\sum_{uv\in E(G)}x^{\frac{1}{\sqrt{d_d(u)^3+d_d(v)^3}}}.$$

Recently, some topological indices were studied in [13, 14, 15, 16].

In this paper, we determine the domination Dharwad index and modified domination Dharwad index of some standard graphs, French windmill graphs and friendship graphs.

2. Domination Dharwad index

2.1. Results for some standard graphs

Proposition 1. If K_n is a complete graph with n vertices, then

$$DD(K_n)=\frac{n(n-1)}{\sqrt{2}}.$$

Proof: If K_n is a complete graph, then $d_d(u)$ =1. From definition, we have

$$DD(K_n)=\sum_{uv\in E(K_n)}\sqrt{d_d(u)^3+d_d(v)^3}=\frac{n(n-1)}{2}\sqrt{1^3+1^3}=\frac{n(n-1)}{\sqrt{2}}.$$

Proposition 2. If S_{n+1} is a star graph with $d_d(u)$ =1, then $DD(S_{n+1})=\sqrt{2}n.$

Proposition 3. If $S_{p+1,q+1}$ is a double star graph with $d_d(u)$ =2, then

$$DD(S_{p+1,q+1})=4(p+q+1).$$

Proposition 4. Let $K_{m,n}$ be a complete bipartite graph with $2\le m\le n$. Then

$$DD(K_{m,n})=mn\sqrt{(m+1)^3+(n+1)^3}.$$

Proof: Let $G=K_{m,n}$, $m, n\geq 2$ with $d_d(u)=m+1$

$= n+1$, for all $u\in V(K_{m,n})$.

From definition, we have

$$DD(K_{m,n})=\sum_{uv\in E(K_{m,n})}\sqrt{d_d(u)^3+d_d(v)^3}=mn\sqrt{(m+1)^3+(n+1)^3}.$$

In proposition 5, by using definition, we obtain the domination Dharwad exponential of K_n, S_{n+1}, $S_{p+1,q+1}$ and $K_{m,n}$.

Proposition 5. The domination Dharwad exponential of K_n, S_{n+1}, $S_{p+1,q+1}$ and $K_{m,n}$ are given by

(i) $DD(K_n, x) = \sum_{uv \in E(G)} x^{\sqrt{d_d(u)^3 + d_d(v)^3}} = \frac{n(n-1)}{2} x^{\sqrt{1^3+1^3}} = \frac{n(n-1)}{2} x^{\sqrt{2}}$.

(ii) $DD(S_{n+1}, x) = nx^{\sqrt{2}}$.

(iii) $DD(S_{p+1,q+1}, x) = (p+q+1)x^4$.

(iv) $DD(K_{m,n}, x) = mnx^{\sqrt{(m+1)^3 + (n+1)^3}}$.

2.2. Results for French Windmill graphs

The French windmill graph F_n^m is the graph obtained by taking $m \geq 3$ copies of K_n, $n \geq 3$ with a vertex in common. The graph F_n^m is presented in Figure 1. The French windmill graph F_3^m is called a friendship graph.

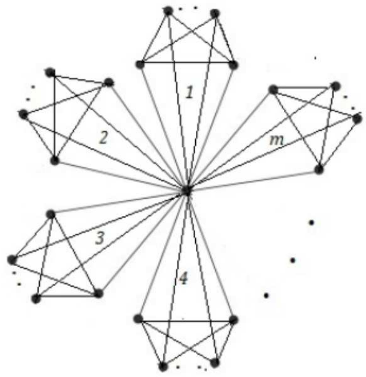

Figure 1: French windmill graph F_n^m

Let F be a French windmill graph F_n^m. Then

$d_d(u)$ =1, if u is in center

$= (n-1)^{m-1}$, otherwise.

Theorem 1. Let F be a French windmill graph F_n^m. Then

$$DD(F) = m(n-1)\sqrt{1 + (n-1)^{(m-1)3}} + [(mn(n-1)/2) - m(n-1)]\sqrt{2(n-1)^{(m-1)3}}.$$

Proof: In F, there are two sets of edges. Let E_1 be the set of all edges which are incident with the center vertex and E_2 be the set of all edges of the complete graph. Then

$$DD(F) = \sum_{uv \in E_1(F)} \sqrt{d_d(u)^3 + d_d(v)^3} + \sum_{uv \in E_2(F)} \sqrt{d_d(u)^3 + d_d(v)^3}$$

$$= m(n-1)\sqrt{1^3 + (n-1)^{(m-1)3}}$$

$$+ [(mn(n-1)/2) - m(n-1)]\sqrt{(n-1)^{(m-1)3} + (n-1)^{(m-1)3}}$$

$$= m(n-1)\sqrt{1+(n-1)^{(m-1)3}} + [(mn(n-1)/2) - m(n-1)]\sqrt{2(n-1)^{(m-1)3}}.$$

Corollary 1.1. Let F_3^m be a friendship graph. Then

$$DD\left(F_3^m\right) = 2m\sqrt{1+2^{(m-1)3}} + m\sqrt{2^{3m-2}}.$$

Theorem 2. The domination Dharwad exponential of F_n^m, F_3^m are given by

(i) $DD\left(F_n^m, x\right) = m(n-1)x^{\sqrt{1+(n-1)^{(m-1)3}}} + [(mn(n-1)/2) - m(n-1)]x^{\sqrt{2(n-1)^{(m-1)3}}}$.

(ii) $DD\left(F_3^m, x\right) = 2mx^{\sqrt{1+2^{(m-1)3}}} + mx^{\sqrt{2^{3m-2}}}$.

3. Modified domination Dharwad index

3.1. Results for Some Standard graphs

Proposition 6. If K_n is a complete graph with n vertices, then

$$^{m}DD(K_n) = \frac{n(n-2)}{2\sqrt{2}}.$$

Proof: If K_n is a complete graph, then $d_d(u)$ =1. From definition, we have

$$^{m}DD(K_n) = \sum_{uv\in E(K_n)} \frac{1}{\sqrt{d_d(u)^3 + d_d(v)^3}} = \frac{n(n-1)}{2}\frac{1}{\sqrt{1^3+1^3}} = \frac{n(n-1)}{2\sqrt{2}}.$$

Proposition 7. If S_{n+1} is a star graph with $d_d(u)$ =1, then $^{m}DD(S_{n+1}) = \frac{n}{\sqrt{2}}$.

Proposition 8. If $S_{p+1,q+1}$ is a double star graph with $d_d(u)$ =2, then

$$^{m}DD(S_{p+1,q+1}) = \frac{p+q+1}{4}.$$

Proposition 9. Let $K_{m,n}$ be a complete bipartite graph with $2 \le m \le n$. Then

$$^{m}DD(K_{m,n}) = \frac{mn}{\sqrt{(m+1)^3 + (n+1)^3}}.$$

In proposition 10, by using definition, we obtain the modified domination Dharwad exponential of K_n, S_{n+1}, $S_{p+1,q+1}$ and $K_{m,n}$.

Proposition 10. The domination Dharwad exponential of K_n, S_{n+1}, $S_{p+1,q+1}$ and $K_{m,n}$ are given by

(i) $^{m}DD(K_n, x) = \sum_{uv\in E(G)} x^{\frac{1}{\sqrt{d_d(u)^3+d_d(v)^3}}} = \frac{n(n-1)}{2}x^{\frac{1}{\sqrt{1^3+1^3}}} = \frac{n(n-1)}{2}x^{\frac{1}{\sqrt{2}}}$.

(ii) $^{m}DD(S_{n+1},x)=nx^{\frac{1}{\sqrt{2}}}$. (iii) $^{m}DD(S_{p+1,q+1},x)=(p+q+1)x^{\frac{1}{4}}$.

(iv) $^{m}DD(K_{m,n},x)=mnx^{\frac{1}{\sqrt{(m+1)^3+(n+1)^3}}}$.

3.2. Results for French Windmill graphs

Theorem 3. Let F be a French windmill graph F_n^m. Then

$$^{m}DD(F)=\frac{m(n-1)}{\sqrt{1+(n-1)^{(m-1)3}}}+\frac{[(mn(n-1)/2)-m(n-1)]}{\sqrt{2(n-1)^{(m-1)3}}}.$$

Proof: In F, there are two sets of edges. Let E_1 be the set of all edges which are incident with the center vertex and E_2 be the set of all edges of the complete graph. Then

$$\begin{aligned} ^{m}DD(F) &= \sum_{uv\in E_1(F)}\frac{1}{\sqrt{d_d(u)^3+d_d(v)^3}}+\sum_{uv\in E_2(F)}\frac{1}{\sqrt{d_d(u)^3+d_d(v)^3}} \\ &= \frac{m(n-1)}{\sqrt{1^3+(n-1)^{(m-1)3}}}+\frac{[(mn(n-1)/2)-m(n-1)]}{\sqrt{(n-1)^{(m-1)3}+(n-1)^{(m-1)3}}} \\ &= \frac{m(n-1)}{\sqrt{1+(n-1)^{(m-1)3}}}+\frac{[(mn(n-1)/2)-m(n-1)]}{\sqrt{2(n-1)^{(m-1)3}}}. \end{aligned}$$

Corollary 3.1. Let F_3^m be a friendship graph. Then

$$^{m}DD(F_3^m)=\frac{2m}{\sqrt{1+2^{(m-1)3}}}+\frac{m}{\sqrt{2^{3m-2}}}.$$

Theorem 4. The modified domination Dharwad exponential of F_n^m, F_3^m are

(i) $^{m}DD(F_n^m,x)=m(n-1)x^{\frac{1}{\sqrt{1+(n-1)^{(m-1)3}}}}+[(mn(n-1)/2)-m(n-1)]x^{\frac{1}{\sqrt{2(n-1)^{(m-1)3}}}}$.

(ii) $^{m}DD(F_3^m,x)=2mx^{\frac{1}{\sqrt{1+2^{(m-1)3}}}}+mx^{\frac{1}{\sqrt{2^{3m-2}}}}$.

4. Conclusion

In this paper, the domination Dharwad index and modified domination Dharwad index and their corresponding exponentials for some standard graphs, French windmill graphs, friendship graphs are computed.

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Conflicts of Interest. This is a single authored paper. There is no conflicts of interest.

Authors' contributions. This is the authors' sole contribution.

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