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Design of robust L_2-L_∞ filtering for uncertain Lur'e systems with sector and slope restricted nonlinearities

Chun Yin **and** ***Shou-ming Zhong***

School of Mathematics Science
University of Electronic Science and Technology of China, Chengdu, China, 610054
E-mail: yinchun.86416@163.com

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Abstract. This paper considers the problem of robust $L_2 - L_\infty$ filtering for uncertain Lur'e systems with sector and slope restricted nonlinearities. The filter is designed to not only guarantee stability of the estimation error system but also satisfy a prescribed $L_2 - L_\infty$ performance. The nonlinearities are expressed as convex combinations of sector and slope bounds. Based on Finsler's Lemma, the delay-dependent conditions for the existence of $L_2 - L_\infty$ filter are derived in terms of linear matrix inequalities (LMIs). Simulation results show the effectiveness of the proposed method.

Keywords: Filter design; $L_2 - L_\infty$ performance; Uncertain Lur'e system; Sector/ slope restricted nonlinearities; Finsler's Lemma; Time delay

1. Introduction

In recent years, a special category of such systems, called Lur'e systems, have the feedback nonlinear element satisfying certain sector conditions which was introduced by Lur'e in [1]. Since time-delays are sources of poor performance and instability, the problem of absolute and robust stability of the Lur'e systems with time-delay attains considerable significance [2-4]. For example, Yin et al. obtained a criterion of robust stability for a class of Lur'e systems of neutral type in [4].

On the other hand, it is well known that state estimation with measurement noise input is an important problem in engineering applications. A powerful robust control framework has been widely developed for addressing state estimation problem of dynamic systems with different performance indexes, such as $H_2 - H_\infty$ filtering approach [5-6] and $L_2 - L_\infty$ filtering approach. Note that $L_2 - L_\infty$ control design has been received considerable attention mainly because of its insensitivity to the exact knowledge of the statistics of the noise signals. Therefore, the $L_2 - L_\infty$ filtering problem has been extensively studied [7-9]. In [8], Wu studied the problem of $L_2 - L_\infty$ filter design for stochastic systems with time-varying delay.

In this paper, the problem of robust $L_2 - L_\infty$ filter for uncertain Lur'e systems with sector and slope restricted nonlinearities and time-delay is studied. The nonlinear function is written as convex combinations of sector bounds and slope bounds. An equality constraint is obtained by utilizing convex properties of the nonlinearities. The delay-dependent conditions for the $L_2 - L_\infty$ control law are derived by utilizing Finsler's Lemma. One example is provided to demonstrate the effectiveness of the proposed method.

2. Problem statements and preliminaries

Consider the following Lur'e system with sector and slope restricted nonlinearities

$$\begin{aligned} \dot{x}(t) &= (A+\delta A)x(t) + (B+\delta B)x(t-\tau) + (F+\delta F)f(\upsilon(t)) + K\omega(t), \\ y(t) &= Cx(t) + C_1 x(t-\tau) + Df(\upsilon(t)) + E\omega(t), \\ \upsilon(t) &= Hx(t), \end{aligned} \tag{1}$$

where $x(t) \in R^n$ is the state vector; $y(t) \in R^p$ is the measurement vector; $\upsilon(t) \in R^q$ is the output to be estimated; $\omega(t) \in R^r$ is the noise input which is assumed to belong to $L_2[t_0, \infty]$. A, B, F, K, C, C_1, D, E and H are constant matrices. The time delay τ is constant. $\delta A(t), \delta B(t), \delta F(t)$ are time-varying uncertainties. They are assumed to be of the following form.

$$[\delta A(t) \quad \delta B(t) \quad \delta F(t)] = WG(t)[N_1 \quad N_2 \quad N_3],. \tag{2}$$

where W, N_1, N_2 and N_3 are known real constant matrices with appropriate dimensions and $G(t)$ is an unknown real time-varying matrix satisfying $\|G(t)\| \le 1, \ \forall t > 0$. The nonlinearity $f(\cdot): R^m \to R^m$ is a memoryless vector-valued function, whose ith element $f_i(\cdot)$ is in a certain sector, such that

$$\beta_i \le \frac{f_i(\upsilon_i(t))}{\upsilon_i(t)} \le \alpha_i, i = 1, 2, \cdots m, . \tag{3}$$

where $\upsilon_i(t)$ is the ith element of $\upsilon(t)$, α_i and β_i are upper and lower bounds.

We assume that $f(\cdot): R^m \to R^m$ is restricted by the slope bounds $[\widehat{\beta}_i, \widehat{\alpha}_i]$, i.e,

$$\widehat{\beta}_i \le \frac{d(f_i(\upsilon_i(t)))}{d(\upsilon_i(t))} \mathrel{\hat{=}} f_i'(\upsilon_i(t)) \le \widehat{\alpha}_i, i = 1, 2, \cdots m. \tag{4}$$

The nonlinear function $f_i(\cdot)$ can be expressed

$$f_i(\upsilon_i(t)) = (\Lambda_i^u(\upsilon_i)\beta_i + \Lambda_i^l(\upsilon_i)\alpha_i)\upsilon_i(t), \tag{5}$$

where $\Lambda_i^l(\upsilon_i)=\dfrac{f_i(\upsilon_i(t))-\beta_i\upsilon_i(t)}{(\alpha_i-\beta_i)\upsilon_i(t)},\Lambda_i^u(\upsilon_i)=\dfrac{\alpha_i\upsilon_i(t)-f_i(\upsilon_i(t))}{(\alpha_i-\beta_i)\upsilon_i(t)}$. Since $\Lambda_i^l(\upsilon_i(t))\geq 0$, $\Lambda_i^u(\upsilon_i(t))\geq 0$, and $\Lambda_i^l(\upsilon_i(t))+\Lambda_i^u(\upsilon_i(t))=1$, the nonlinearity $f_i(\cdot)$ can be represented as $f_i(\upsilon(t))=\Lambda_i(\upsilon_i(t))\upsilon_i(t)$, where $\Lambda_i(\sigma_i(t))\in Co\{\beta_i,\alpha_i\}$.

One assumes $\dot{f}_i(\upsilon(t))=\overline{\Lambda}_i(\upsilon_i(t))\dot{\upsilon}_i(t)$, where $\overline{\Lambda}_i(\sigma_i(t))\in Co\{\widehat{\beta}_i,\widehat{\alpha}_i\}$. Define

$$\begin{aligned}&\Lambda=\mathrm{diag}\{\Lambda_1(\upsilon_1),\cdots,\Lambda_m(\upsilon_m)\},\Delta_1=\mathrm{diag}\{\beta_1,\cdots,\beta_m\},\Delta_2=\mathrm{diag}\{\alpha_1,\cdots,\alpha_m\},\\&\overline{\Lambda}=\mathrm{diag}\{\overline{\Lambda}_1(\upsilon_1),\cdots,\overline{\Lambda}_m(\upsilon_m)\},\overline{\Delta}_1=\mathrm{diag}\{\widehat{\beta}_1,\cdots,\widehat{\beta}_m\},\overline{\Delta}_2=\mathrm{diag}\{\widehat{\alpha}_1,\cdots,\widehat{\alpha}_m\}.\end{aligned}\tag{6}$$

Then, the nonlinearities $f(\upsilon(t))$ and $\dot{f}(\upsilon(t))$ can be expressed as

$$f(\upsilon(t))=\Lambda\upsilon(t),\ \dot{f}(\upsilon(t))=\overline{\Lambda}\dot{\upsilon}(t),\tag{7}$$

and $\nabla:=\mathrm{diag}\{\Lambda,\overline{\Lambda}\}$ in the set $\Delta:=\{\mathrm{diag}\{\Lambda,\overline{\Lambda}\}\mid\Lambda\in Co\{\Delta_1,\Delta_2\},\overline{\Lambda}\in Co\{\overline{\Delta}_1,\overline{\Delta}_2\}\}$.

Let us define $p(t)=G(t)\{N_1x(t)+N_2x(t-\tau)+N_3f(v(t))\}$ from (2). Then, the system (1) can be rewritten as

$$\begin{aligned}&\dot{x}(t)=Ax(t)+Bx(t-\tau)+Ff(\upsilon(t))+K\omega(t)+Wp(t),\\&y(t)=Cx(t)+C_1x(t-\tau)+Df(\upsilon(t))+E\omega(t),\\&\upsilon(t)=Hx(t),\end{aligned}\tag{8}$$

The problem considered in this paper is a filter with the following structure

$$\begin{aligned}&\dot{\hat{x}}(t)=A_F\hat{x}(t)+B_Fy(t),\\&\hat{\upsilon}(t)=H_F\hat{x}(t),\end{aligned}\tag{9}$$

where $\hat{x}(t)\in R^n$ is the filter vector; $\hat{\upsilon}(t)\in R^q$ is the estimate of $\upsilon(t)$; A_F,B_F and H_F are constant matrices. The filter initial condition is set as $\hat{x}(0)=x(0)$. Define the estimation error $e(t)=\upsilon(t)-\hat{\upsilon}(t)$. The estimation error dynamic system can be described by

$$\begin{aligned}&\dot{\xi}(t)=\overline{A}\xi(t)+\overline{B}\overline{C}\xi(t-\tau)+\overline{F}f(\upsilon(t))+\overline{K}\omega(t)+\overline{W}p(t),\\&e(t)=\overline{H}\xi(t),\end{aligned}\tag{10}$$

where $\xi(t):=[x(t)^T\quad \hat{x}(t)^T]^T$ and the matrices are given as

$$\begin{aligned}&\overline{A}=\begin{bmatrix}A&0\\B_FC&A_F\end{bmatrix},\overline{B}=\begin{bmatrix}B\\B_FC_1\end{bmatrix},\overline{F}=\begin{bmatrix}F\\B_FD\end{bmatrix},\overline{K}=\begin{bmatrix}K\\B_FE\end{bmatrix},\overline{K}=\begin{bmatrix}W\\0\end{bmatrix},\\&\overline{C}=[I\quad 0],\overline{H}=[H\quad -H_F].\end{aligned}$$

Remark 2.1. The problem under consideration in this paper is to design a filter of the form (9) which minimizes the L_2-L_∞ norm of the estimation error dynamics in the presence of $\omega(\cdot)\in L_2$ and the uncertainties satisfying (2), under the zero initial condition. That is, the problem is to design a robust filter (9) that solves the following problem $\min\sup\limits_{\omega(\cdot)\in L_2\backslash\{0\}}\dfrac{\|e(t)\|_\infty}{\|\omega(t)\|_2}$. The estimation error without disturbance should be zero for all $t\geq 0$ in order to validate the criterion.

Definition 2.1. For a given scalar $\gamma > 0$, the estimation error system (10) is said to be exponentially stable with a weighted $L_2 - L_\infty$ performance level γ, if the system (10) is exponentially stable with $\omega(t) = 0$, and under zero initial condition, that is , it holds for all non-zero $\omega(\cdot) \in L_2(t_0, \infty)$ that $\sup_{\omega(\cdot)\in L_2\setminus\{0\}} \frac{\| e(t) \|_\infty}{\| \omega(t) \|_2}$.

The following Fact and Lemmas will be useful for deriving LMI conditions for $L_2 - L_\infty$ performance analysis of the given estimation error dynamic system.

Fact 2.1. For any real vectors a, b and any matrix $Q > 0$ with appropriate dimensions, it is following that: $2a^T b \le a^T Q a + b Q^{-1} b.$

Lemma 2.1. (Schur complement). Given constant symmetric matrices $\Sigma_1, \Sigma_2\, \Sigma_3$ where $\Sigma_1 = \Sigma_1^T$, and $0 < \Sigma_2 = \Sigma_2^T$, then $\Sigma_1 + \Sigma_3^T \Sigma_2^{-1} \Sigma_3 < 0$ if and only if

$$\begin{bmatrix} \Sigma_1 & \Sigma_3^T \\ \Sigma_3 & -\Sigma_2 \end{bmatrix} < 0, or \begin{bmatrix} -\Sigma_2 & \Sigma_3 \\ \Sigma_3^T & \Sigma_1 \end{bmatrix} < 0. \tag{11}$$

Lemma 2.2. For a positive matrix $R > 0$, any matrices $G_i, (i = 1, \cdots, 7)$ and a scalar $\delta > 0$, the following inequality holds:

$$-\int_{t-\delta}^{t} \dot{\xi}(s)^T R \dot{\xi}(s) ds \le \overline{\xi}^T(t) \hat{G} \overline{\xi}(t) + \delta \overline{\xi}^T(t) \overline{G}^T R^{-1} \overline{G} \overline{\xi}(t), \tag{12}$$

where

$$\overline{G} = [G_1, G_2, G_3, G_4, G_5, G_6, G_7],$$

$$\overline{\xi}(t) = [\xi^T(t), \xi^T(t-\tau), f^T(v(t)), (\int_{t-\delta}^{t} \xi(s) ds)^T, \omega^T(t), p^T(t), \dot{f}^T(v(t))]^T,$$

$$\hat{G} = \begin{bmatrix} 0 & 0 & 0 & G_1^T & 0 & 0 & 0 \\ * & 0 & 0 & G_2^T & 0 & 0 & 0 \\ * & * & 0 & G_3^T & 0 & 0 & 0 \\ * & * & * & G_4^T + G_4 & G_5 & G_6 & G_7 \\ * & * & * & * & 0 & 0 & 0 \\ * & * & * & * & * & 0 & 0 \\ * & * & * & * & * & * & 0 \end{bmatrix}.$$

Proof. Utilizing Fact 2.1, we have

$$\begin{aligned} -\int_{t-\delta}^{t} \dot{\xi}^T(s) R \dot{\xi}(s) ds &\le 2(\int_{t-\delta}^{t} \dot{\xi}(s) ds)^T \overline{G} \overline{\xi}(t) + \int_{t-\delta}^{t} \overline{\xi}^T(t) \overline{G}^T R^{-1} \overline{G} \overline{\xi}(t) ds \\ &= \overline{\xi}^T(t) \hat{G} \overline{\xi}(t) + \delta \overline{\xi}^T(t) \overline{G}^T R^{-1} \overline{G} \overline{\xi}(t). \end{aligned} \tag{13}$$

Lemma 2.3. [24] Let a matrix F, $Q = Q^T$ and a compact subset of real matrices $\hbar$ be given. The following statements are equivalent:

(1) For each $H \in \hbar$, $\xi^T Q \xi < 0$ for all $\xi \neq 0$ such that $HF\xi \neq 0$.

(2) There exists $\Theta = \Theta^T$ such that $Q + F^T \Theta F < 0, \Psi_\hbar^T \Theta \Psi_\hbar \geq 0, \quad \forall H \in \hbar$, where $\Psi_\hbar$ is a matrix belong to a null space of H.

3. Main results

Theorem 3.1. Consider the estimation error system (10), if there exist positive-definite symmetric matrices L, P, Q, R, S, U, Θ, any matrices $M_i, (i = 1, \cdots, 7)$ with appropriate dimensions and scalars $\gamma > 0, \eta > 0$ such that the following LMIs hold:

$$\begin{bmatrix} \Sigma_1 + \Sigma_2^T \Theta \Sigma_2 & N^T U & \tau \overline{M}^T \\ UN & -U & 0 \\ \tau \overline{M} & 0 & -\tau R \end{bmatrix} < 0, \begin{bmatrix} I \\ \nabla \end{bmatrix}^T \Theta \begin{bmatrix} I \\ \nabla \end{bmatrix} \geq 0, \forall \nabla \in \Delta, \begin{bmatrix} L & \overline{H}^T \\ * & I \end{bmatrix} > 0, \qquad (14)$$

where

$$\Sigma_1 = \begin{bmatrix} \Sigma_1^{11} & \Sigma_1^{12} & \Sigma_1^{13} & \Sigma_1^{14} & \Sigma_1^{15} & \Sigma_1^{16} & \Sigma_1^{17} \\ * & \Sigma_1^{22} & \Sigma_1^{23} & \Sigma_1^{24} & \Sigma_1^{25} & \Sigma_1^{26} & 0 \\ * & * & \Sigma_1^{33} & \Sigma_1^{34} & \Sigma_1^{35} & \Sigma_1^{36} & \Sigma_1^{37} \\ * & * & * & \Sigma_1^{44} & \Sigma_1^{45} & \Sigma_1^{46} & \Sigma_1^{47} \\ * & * & * & * & \Sigma_1^{55} & \Sigma_1^{56} & 0 \\ * & * & * & * & * & \Sigma_1^{66} & 0 \\ * & * & * & * & * & * & 0 \end{bmatrix},$$

$$\Sigma_1^{11} = X\overline{A} + \overline{A}^T X + \tau \overline{A}^T R \overline{A} + L\overline{A} + (L\overline{A})^T + Q + \eta S, \Sigma_1^{12} = X\overline{BC} + \tau \overline{A}^T R \overline{BC} + L\overline{BC},$$

$$\Sigma_1^{13} = X\overline{F} + \overline{A}^T Y + \tau \overline{A}^T R \overline{F} + L\overline{F}, \Sigma_1^{14} = M_1^T, \Sigma_1^{15} = X\overline{K} + \tau \overline{A}^T R \overline{K} + L\overline{K},$$

$$\Sigma_1^{16} = X\overline{W} + \tau \overline{A}^T R \overline{W} + L\overline{W}, \Sigma_1^{17} = Y, \Sigma_1^{22} = -Q + \tau (\overline{BC})^T R \overline{BC},$$

$$\Sigma_1^{23} = \tau (\overline{BC})^T R \overline{F} + (\overline{BC})^T Y, \Sigma_1^{24} = M_2^T, \Sigma_1^{25} = \tau (\overline{BC})^T R \overline{K}, \Sigma_1^{26} = \tau (\overline{BC})^T R \overline{W},$$

$$\Sigma_1^{33} = Y^T \overline{F} + \overline{F}^T Y + \tau \overline{F}^T R \overline{F}, \Sigma_1^{34} = M_3^T, \Sigma_1^{35} = Y^T \overline{K} + \tau \overline{F}^T R \overline{K}, \Sigma_1^{36} = Y^T \overline{W} + \tau \overline{F}^T R \overline{W},$$

$$\Sigma_1^{37} = Z, \Sigma_1^{44} = M_4^T + M_4, \Sigma_1^{45} = M_5, \Sigma_1^{46} = M_6, \Sigma_1^{47} = M_7, \Sigma_1^{55} = \tau \overline{K}^T R \overline{K} - \gamma^2 I,$$

$$\Sigma_1^{56} = \tau \overline{K}^T R \overline{W}, \Sigma_1^{66} = \tau \overline{W}^T R \overline{W} - U,$$

$$\Sigma_2 = \begin{bmatrix} \overline{H} & 0 & 0 & 0 & 0 & 0 & 0 \\ \overline{HA} & \overline{HBC} & \overline{HF} & 0 & \overline{HK} & \overline{HW} & 0 \\ 0 & 0 & -I & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -I \end{bmatrix}, P = \begin{bmatrix} X & Y \\ Y^T & Z \end{bmatrix},$$

$$\overline{M} = [M_1, M_2, M_3, M_4, M_5, M_6, M_7].$$

Then, the system (10) is asymptotically stable when $\omega(t) \equiv 0$, and the norm of the system (10) in the presence of $\omega(\cdot) \in L_2 \setminus \{0\}$ is less than γ under the zero initial condition.

Proof. For simplicity, define the matrix and augmented vectors:

$$\xi_a(t) = [\xi^T(t), \xi^T(t-\tau)]^T,$$

$$\bar{\xi}(t) = [\xi^T(t), \xi^T(t-\tau), f^T(\upsilon(t)), (\int_{t-\tau}^{t} \xi(s)ds)^T, \omega^T(t), p^T(t), \dot{f}^T(\upsilon(t))]^T,$$

$$\Xi_4 = \begin{bmatrix} \tau\bar{A}^T R\bar{A} & \tau\bar{A}^T R\bar{B}\bar{C} & \tau\bar{A}^T R\bar{F} & 0 & \tau\bar{A}^T R\bar{K} & \bar{A}^T R\bar{W} & 0 \\ * & \tau(\bar{B}\bar{C})^T R\bar{B}\bar{C} & \tau(\bar{B}\bar{C})^T R\bar{F} & 0 & \tau(\bar{B}\bar{C})^T R\bar{K} & (\bar{B}\bar{C})^T R\bar{W} & 0 \\ * & * & \tau\bar{F}^T R\bar{F} & 0 & \tau\bar{F}^T R\bar{K} & \tau\bar{F}^T R\bar{W} & 0 \\ * & * & * & 0 & 0 & 0 & 0 \\ * & * & * & * & \tau\bar{K}^T R\bar{K} & \bar{K}^T R\bar{W} & 0 \\ * & * & * & * & * & \bar{W}^T R\bar{W} & 0 \\ * & * & * & * & * & * & 0 \end{bmatrix},$$

$$\Xi_5 = \begin{bmatrix} 0 & 0 & 0 & M_1^T & 0 & 0 & 0 \\ * & 0 & 0 & M_2^T & 0 & 0 & 0 \\ * & * & 0 & M_3^T & 0 & 0 & 0 \\ * & * & * & M_4^T + M_4 & M_5 & M_6 & M_7 \\ * & * & * & * & 0 & 0 & 0 \\ * & * & * & * & * & 0 & 0 \\ * & * & * & * & * & * & 0 \end{bmatrix}.$$

For positive definite matrices L, P, Q, R, let us take a Lyapunov functional as follows:

$$V(\xi(t)) = V_1(\xi(t)) + V_2(\xi(t)) + V_3(\xi(t)) + V_4(\xi(t)), \tag{15}$$

where

$$V_1(\xi(t)) = \xi^T(t)L\xi(t), \qquad V_2(\xi(t)) = \xi_a^T(t)P\xi_a(t),$$

$$V_3(\xi(t)) = \int_{t-\tau}^{t} \xi^T(s)Q\xi(s)ds, \qquad V_4(\xi(t)) = \int_{t-\tau}^{t}\int_{u}^{t} \xi^T(s)R\xi(s)dsdu.$$

The derivatives of $V_1(\xi(t)), V_2(\xi(t)), V_3(\xi(t)), V_4(\xi(t))$ are

$$\begin{aligned} &\dot{V}_1(\xi(t)) = 2\xi^T(t)L\dot{\xi}(t), \quad \dot{V}_2(\xi(t)) = 2\xi_a^T(t)P\dot{\xi}_a(t), \\ &\dot{V}_3(\xi(t)) = \xi^T(t)Q\xi(t) - \xi^T(t-\tau)Q\xi(t-\tau), \\ &\dot{V}_4(\xi(t)) \le \bar{\xi}^T(t)(\Xi_4 + \Xi_5 + \tau\bar{M}^T R^{-1}\bar{M})\bar{\xi}(t). \end{aligned} \tag{16}$$

Define a function $J(\bar{\xi}(t))$ as follows $J(\bar{\xi}(t)) = \dot{V}(\xi(t)) - \gamma^2\omega^T(t)\omega(t)$. From (16), one has

$$J(\bar{\xi}(t)) \le \bar{\xi}^T(t)(\Xi + \tau\bar{M}^T R^{-1}\bar{M})\bar{\xi}(t), \tag{17}$$

where

$$\Xi = \begin{bmatrix} \Xi_{11} & \Xi_{12} & \Xi_{13} & \Xi_{14} & \Xi_{15} & \Xi_{16} & \Xi_{17} \\ * & \Xi_{22} & \Xi_{23} & \Xi_{24} & \Xi_{25} & \Xi_{26} & 0 \\ * & * & \Xi_{33} & \Xi_{34} & \Xi_{35} & \Xi_{36} & \Xi_{37} \\ * & * & * & \Xi_{44} & \Xi_{45} & \Xi_{46} & \Xi_{47} \\ * & * & * & * & \Xi_{55} & \Xi_{56} & 0 \\ * & * & * & * & * & \Xi_{66} & 0 \\ * & * & * & * & * & * & 0 \end{bmatrix},$$

$\Xi_{11} = X\overline{A} + \overline{A}^T X + \tau\overline{A}^T R\overline{A} + L\overline{A} + (L\overline{A})^T + Q, \Xi_{12} = X\overline{B}\overline{C} + \tau\overline{A}^T R\overline{B}\overline{C} + L\overline{B}\overline{C},$

$\Xi_{13} = X\overline{F} + \overline{A}^T Y + \tau\overline{A}^T R\overline{F} + L\overline{F}, \Xi_{14} = M_1^T, \Xi_{15} = X\overline{K} + \tau\overline{A}^T R\overline{K} + L\overline{K},$

$\Xi_{16} = X\overline{W} + \tau\overline{A}^T R\overline{W} + L\overline{W}, \Xi_{17} = Y, \Xi_{22} = -Q + \tau(\overline{B}\overline{C})^T R\overline{B}\overline{C}, \Xi_{23} = \tau(\overline{B}\overline{C})^T R\overline{F} + (\overline{B}\overline{C})^T Y,$

$\Xi_{24} = M_2^T, \Xi_{25} = \tau(\overline{B}\overline{C})^T R\overline{K}, \Xi_{26} = \tau(\overline{B}\overline{C})^T R\overline{W}, \Xi_{33} = Y^T\overline{F} + \overline{F}^T Y + \tau\overline{F}^T R\overline{F}, \Xi_{34} = M_3^T,$

$\Xi_{35} = Y^T\overline{K} + \tau\overline{F}^T R\overline{K}, \Xi_{36} = Y^T\overline{W} + \tau\overline{F}^T R\overline{W}, \Xi_{37} = Z, \Xi_{44} = M_4^T + M_4, \Xi_{45} = M_5, \Xi_{46} = M_6,$

$\Xi_{47} = M_7, \Xi_{55} = \tau\overline{K}^T R\overline{K} - \gamma^2 I, \Xi_{56} = \tau\overline{K}^T R\overline{W}, \Xi_{66} = \tau\overline{W}^T R\overline{W}.$

If the matrix $S = S^T > 0$ and a constant $\eta > 0$ satisfy the following condition:

$$\overline{\Xi} = \Xi + \tau\overline{M}^T R^{-1}\overline{M} + diag\{\eta S, 0, 0, 0, 0, 0, 0\} < 0, \tag{18}$$

then, we have

$$J(\overline{\xi}(t)) < -\overline{\xi}^T(t) \begin{bmatrix} \eta S & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \overline{\xi}(t). \tag{19}$$

From (31), we can derive that

$$\dot{V}(x(t))\big|_{w(t)=0} < -\eta\lambda_{\min}(S)\|x(t)\|^2 < 0, \quad \forall x(t) \neq 0. \tag{20}$$

Therefore, the estimation error system (10) is exponentially stable for $w(t) \equiv 0$.

By integrating the function in (31) from 0 to t, we have

$$V(t) - V(0) - \int_0^t \gamma^2 \omega^T(t)\omega(t)dt \le 0. \tag{21}$$

With zero initial condition, $V(t) - \gamma^2 \int_0^t \omega^T(s)\omega(s)ds \le 0.$ one has

$$e^T(t)e(t) \le \gamma^2 \int_0^t \omega^T(t)\omega(t)dt \le \gamma^2 \int_0^\infty \omega^T(t)\omega(t)dt. \tag{22}$$

We have $\| e(t) \|_{\infty} < \gamma \| \omega(t) \|_2$ for any nonzero $\omega(\cdot) \in L_2[0,\infty)$. By Definition 2.1, the (10) is said to be exponentially stable with a $L_2 - L_\infty$ performance level $\gamma > 0$. Here, we will solve the sufficient condition (19). First, for any positive symmetries matrix T, one obtains $p^T(t)Tp(t) \le \xi^T(t)N^T TN\xi(t)$. By S-procedure [4], if there exist $\varepsilon > 0$ satisfying

$$\bar{\xi}^T(t)(\bar{\Xi} + N^T \varepsilon TN + \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -\varepsilon T & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix})\bar{\xi}(t) < 0. \tag{23}$$

Using convex properties of the nonlinear function (11), the following constraints is satisfied

$$\begin{bmatrix} \Lambda\bar{H} & 0 & -I & 0 & 0 & 0 & 0 \\ \bar{\Lambda}\bar{H}\bar{A} & \bar{\Lambda}\bar{H}\bar{B}\bar{C} & \bar{\Lambda}\bar{H}\bar{F} & 0 & \bar{\Lambda}\bar{H}\bar{K} & \bar{\Lambda}\bar{H}\bar{W} & -I \end{bmatrix}\xi(t) = 0. \tag{24}$$

It can be represented as $[\nabla \quad -I]\Sigma_2\xi(t) = 0, \forall\nabla \in \Delta$. If there exist a symmetric matrix Θ satisfying

$$\begin{aligned} &\bar{\Sigma}_1 + \tau\bar{M}^T R^{-1}\bar{M} + N^T \varepsilon TN + \Sigma_2^T\Theta\Sigma_2 < 0, \\ &\begin{bmatrix} I \\ \nabla \end{bmatrix}^T \Theta \begin{bmatrix} I \\ \nabla \end{bmatrix} \ge 0, \forall\nabla \in \Delta, \end{aligned} \tag{25}$$

where

$$\bar{\Sigma}_1 = \begin{bmatrix} \bar{\Sigma}_{11} & \bar{\Sigma}_{12} & \bar{\Sigma}_{13} & \bar{\Sigma}_{14} & \bar{\Sigma}_{15} & \bar{\Sigma}_{16} & \bar{\Sigma}_{17} \\ * & \bar{\Sigma}_{22} & \bar{\Sigma}_{23} & \bar{\Sigma}_{24} & \bar{\Sigma}_{25} & \bar{\Sigma}_{26} & 0 \\ * & * & \bar{\Sigma}_{33} & \bar{\Sigma}_{34} & \bar{\Sigma}_{35} & \bar{\Sigma}_{36} & \bar{\Sigma}_{37} \\ * & * & * & \bar{\Sigma}_{44} & \bar{\Sigma}_{45} & \bar{\Sigma}_{46} & \bar{\Sigma}_{47} \\ * & * & * & * & \bar{\Sigma}_{55} & \bar{\Sigma}_{56} & 0 \\ * & * & * & * & * & \bar{\Sigma}_{66} & 0 \\ * & * & * & * & * & * & 0 \end{bmatrix},$$

$$\bar{\Sigma}_{11} = X\bar{A} + \bar{A}^T X + \tau\bar{A}^T R\bar{A} + L\bar{A} + (L\bar{A})^T + Q, \bar{\Sigma}_{12} = X\bar{B}\bar{C} + \tau\bar{A}^T R\bar{B}\bar{C} + L\bar{B}\bar{C},$$

$$\bar{\Sigma}_{13} = X\bar{F} + \bar{A}^T Y + \tau\bar{A}^T R\bar{F} + L\bar{F}, \bar{\Sigma}_{14} = M_1^T, \bar{\Sigma}_{15} = X\bar{K} + \tau\bar{A}^T R\bar{K} + L\bar{K},$$

$$\bar{\Sigma}_{16} = X\bar{W} + \tau\bar{A}^T R\bar{W} + L\bar{W}, \bar{\Sigma}_{17} = Y, \bar{\Sigma}_{22} = -Q + \tau(\bar{B}\bar{C})^T R\bar{B}\bar{C},$$

$$\bar{\Sigma}_{23} = \tau(\bar{B}\bar{C})^T R\bar{F} + (\bar{B}\bar{C})^T Y, \bar{\Sigma}_{24} = M_2^T, \bar{\Sigma}_{25} = \tau(\bar{B}\bar{C})^T R\bar{K}, \bar{\Sigma}_{26} = \tau(\bar{B}\bar{C})^T R\bar{W},$$

$\bar{\Sigma}_{33} = Y^T \bar{F} + \bar{F}^T Y + \tau \bar{F}^T R \bar{F}, \bar{\Sigma}_{34} = M_3^T, \bar{\Sigma}_{35} = Y^T \bar{K} + \tau \bar{F}^T R \bar{K}, \bar{\Sigma}_{36} = Y^T \bar{W} + \tau \bar{F}^T R \bar{W},$
$\bar{\Sigma}_{37} = Z, \bar{\Sigma}_{44} = M_4^T + M_4, \bar{\Sigma}_{45} = M_5, \bar{\Sigma}_{46} = M_6, \bar{\Sigma}_{47} = M_7, \bar{\Sigma}_{55} = \tau \bar{K}^T R \bar{K} - \gamma^2 I,$
$\bar{\Sigma}_{56} = \tau \bar{K}^T R \bar{W} - \varepsilon T, \bar{\Sigma}_{66} = \tau \bar{W}^T R \bar{W}.$

Then the conditions (18) and (21) are satisfied by the Finsler's Lemma 2.

$$\begin{bmatrix} \bar{\Sigma}_1 + \Sigma_2^T \Theta \Sigma_2 & N^T & \tau \bar{M}^T \\ N & -(\varepsilon T)^{-1} & 0 \\ \tau \bar{M} & 0 & -\tau R \end{bmatrix} < 0, \tag{26}$$

$$\begin{bmatrix} I \\ \nabla \end{bmatrix}^T \Theta \begin{bmatrix} I \\ \nabla \end{bmatrix} \geq 0, \forall \nabla \in \Delta,$$

define $U = \varepsilon T$, and pre-multiply by $diag\{I, U, I\}$ on the left hand side of (41), then inequ ality (41) is obtained. This completes our proof.

4. Numerical example

In this section, one example is given to illustrate the effectiveness of the proposed approach in this paper.

Example 1. Consider the following system

$$\begin{aligned} \dot{x} &= Ax + Ff(\upsilon(t)) + K\omega(t), \\ y &= Cx(t) + Df(\upsilon(t)), \\ v(t) &= Hx(t), \end{aligned} \tag{27}$$

where

$$A = \begin{bmatrix} -15.893 & 1.092 & -1.872 \\ -17.792 & -8.971 & -0.853 \\ -18.346 & -22.251 & -1.869 \end{bmatrix}, F = \begin{bmatrix} -9 \\ 0 \\ 0 \end{bmatrix}, K = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix},$$

$$C = \begin{bmatrix} -3.085 & 0 & 0 \\ 6.021 & 24.213 & 6.021 \\ 0 & 0 & 1 \end{bmatrix}, D = \begin{bmatrix} -9 \\ 0 \\ 0 \end{bmatrix}, H = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix},$$

$$f = \frac{1}{2}\sum_{i=0}^{4}(m_i - m_{i+1})(|x_1 + c_i| - |x_1 - c_i|), m = [0.9/7, -3/7, 3.5/7, -2.7/7, 6/7, -2.4/7].$$

$f(\cdot)$ belongs to $[\beta_1, \alpha_1]$ and $[\hat{\beta}_1, \hat{\alpha}_1]$, with $\beta_1 = 0, \alpha_1 = 24.5, \hat{\beta}_1 = 0, \hat{\alpha}_1 = 10$. The noise disturbance is taken as $\omega(t) = \dfrac{1}{1+3t^2}, t > 0$. Regarding to (10), the estimation error dynamic system is given as follows

$$\begin{aligned} \dot{\xi}(t) &= \bar{A}\xi(t) + \bar{F}f(v(t)) + \bar{K}\omega(t), \\ e(t) &= \bar{H}\xi(t), \end{aligned} \tag{28}$$

where $\xi(t) := [x(t)^T \quad \hat{x}(t)^T]^T$ and the matrices are given as

$$A_F = \begin{bmatrix} -18.9941 & -7.3759 & 0 \\ 0 & -9.0213 & 0 \\ 0 & 0 & -0.9978 \end{bmatrix}, B_F = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, H_F = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}.$$

Choose $x(0) = [0.21, 0.45, 0.19]^T$, the filter initial condition can be set as $\hat{x}(0) = x(0)$. Figure 1-3 are the simulation results for designing the $L_2 - L_\infty$ filter of the estimation error dynamic system (45). First, without disturbance signal $\omega(t)$, the time responses of the estimation error dynamic system is given in Figure 1. It shows that the state converges to zero exponentially. Figure 2 depicts the time response of the estimation error $e(t)$. To observe the $L_2 - L_\infty$ performance with external disturbance attenuation and zero initial conditions, the time response of the estimation error $e(t)$ is shown in figure 5, which shows the $L_2 - L_\infty$ filter reduce the effect of the disturbance input $\omega(t)$ on the error estimation $e(t)$ to within a attenuation level $\gamma = 1.561$.

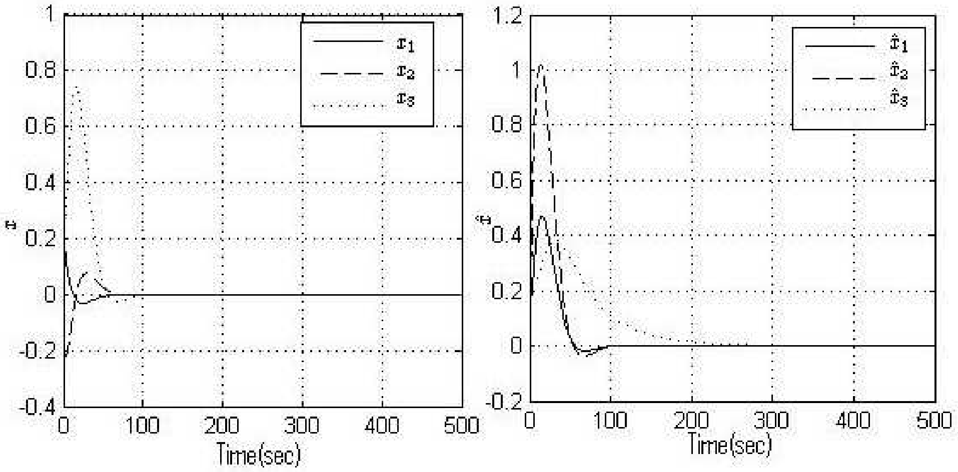

Figure 1. ***The state trajectories for the estimation error system (30) without disturbance signal.***

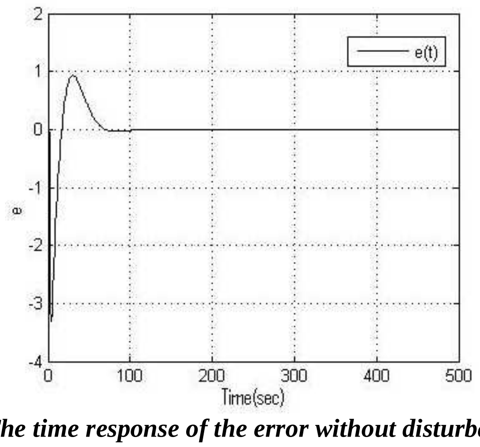

Figure 2. ***The time response of the error without disturbance signal.***

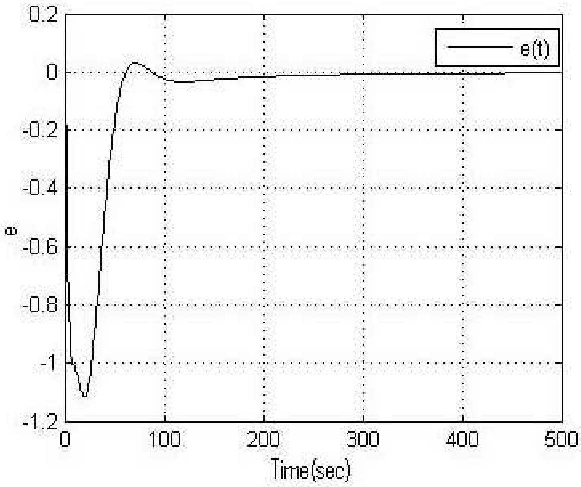

Figure 3. ***The estimation error with zero initial condition and disturbance signal.***

5. Conclusion

In this paper, the $L_2 - L_\infty$ filtering problem has been studied for uncertain Lur'e system. Based on the Finsler's lemma, the sufficient conditions for the existence of $L_2 - L_\infty$ filtering have been derived in terms of LMI respectively. The filter has been designed to not only guarantee stability of the error system but also satisfy a prescribed $L_2 - L_\infty$ performance. Finally, one numerical example has been given to illustrate the effectiveness of the proposed method.

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