Journal of Mathematics and Informatics
Vol. 1, 2013-14, 13-24
ISSN: 2349-0632 (P), 2349-0640 (online)
Published on 1 October 2013
www.researchmathsci.org

Journal of
Mathematics and Informatics

A stock model for uncertain European finance markets

Xichang Yu

Liaocheng Vocational and Technical College, Liaocheng, 252000, China
Email: chinalcyxc@sina.com

Received 3 August 2013; accepted 1 September 2013

Abstract. This paper presents a type of stock models with jumps for uncertain markets by using uncertainty theory. A stock model with jumps for uncertain European markets is formulated by the tool of uncertain differential equation based on two classes of uncertain processes, canonical process and renewal process. Some option pricing formulas on the proposed uncertain stock model are investigated and some numerical calculations are illustrated. Finally, extended European option models are studied.

***Keywords*:** Stock model, uncertain process, renewal process, finance, option price

1. Introduction

The Black-Scholes model, which was proposed by Black and Scholes [2] has been widely used in the analysis of derivatives pricing and portfolio management (see Black and Karasinski [1], Hull and White [8], Karatzas and Shreve [6, 7]). It has become an indispensable tool in today's daily financial market practice. As a further research, Liu [10] proposed a stock model based on a fuzzy process. Then Qin and Li [19] derived the corresponding European option pricing formulas. Soon after, Gao and Gao [5] presented a mean-reverting stock model for fuzzy markets.

As we known, most human decisions are made in the state of uncertainty. In order to research the behavior of uncertain phenomena in human systems, uncertainty theory was founded by Liu [9] in 2007 and refined by Liu [13] in 2010 based on normality, monotonicity, self-duality, countable subadditivity, and product measure axioms, and then it became a branch of axiomatic mathematics for modeling human uncertainty. Nowadays, uncertainty theory has become a branch of axiomatic mathematics for modeling human uncertainty. Some basic theoretical work of uncertainty theory have been studied, such as You [21] proved some convergence theorems of uncertain sequences. Gao [4] showed some properties of continuous uncertain measure. Liu and Ha [15] calculated expected value of function of uncertain variables. To determine an uncertainty distribution of an uncertain variable, Liu [13]designed a questionnaire survey for collecting expert's experimental data, and introduced the empirical uncertainty distribution.

In order to dealing with the evolution of uncertain phenomena with time, the concept of uncertain process was presented by Liu [10] in 2008. A basic uncertain process called renewal process was introduced by Liu [10] and the canonical process was designed by Liu [11] in 2009. The most important and useful uncertain process is canonical process which is the counterpart of Brown motion. Different from Brownian motion, the canonical process is a Lipschitz continuous uncertain process with stationary and independent increments and each increment is a normal uncertain variable. Based on this process, uncertain calculus was initialized by Liu [11] in 2009 to deal with differentiation and integration of functions of uncertain processes. Following that, uncertain differential equation, a type of differential equation driven by canonical process, was defined by Liu [10]. Soon afterwards, Chen and Liu [3] proved an existence and uniqueness theorem of solution for uncertain differential equation under Lipschitz condition and linear growth condition. In many cases mathematical finance models are expressed in terms of stochastic differential equation [16, 20]. For uncertain financial market, the uncertain differential equation also plays an indispensible role. Following the argument that stock price follows geometric canonical process, Liu [9] proposed a basic stock model for uncertain financial market. After that, Xu and Peng [17] studied barrier options pricing in uncertain financial market, Peng and Yao [18] proposed another stock model to describe the stock price in long-run, Yu[22,23] studied expected payoff of trading strategies with jumps for uncertain financial markets and a stock model with jumps for uncertain markets.

In above models, the stock price is assumed to be continuous. However, in many cases the stock price is not continuous because of economic crisis, war, stock daily limit or stock lower limit. In order to incorporate these uncertainties into uncertain stock models, we develop a stock model with jumps for uncertain financial market. This paper presents an alternative assumption that stock price follows geometric canonical process and renewal process. Based on this assumption, the European option price models are formulated.

The rest of this paper is organized as follows. The next section is intended to introduce some useful concepts of uncertain process as needed. The uncertain differential equation with jumps is presented in Section 3. A stock model with jumps for uncertain financial markets is proposed in Section 4. European option price on the proposed uncertain stock model with jumps are investigated in Section 5. Furthermore uncertain currency model with jumps is studied in Section 6. Finally, a brief conclusion is made in Section 7.

2. Preliminaries

Uncertainty theory was founded by Liu [9] to provide a mathematical model for dealing with uncertain phenomena in human system. A set function M is called an uncertain measure if it satisfies the normality, monotonicity, self-duality, countable subadditivity, and product measure axioms. An uncertain variable is a measurable function from an uncertainty space (Γ, L, M) to the set of real numbers. The uncertainty distribution $\Phi:\Re\to[0,1]$ of an uncertain variable ξ is defined by $\Phi(x) = M\{\xi \le x\}$.

Definition 1. *(Liu [12]) Let ξ be an uncertain variable. Then the expected value of ξ is defined by*

$$E[\xi]=\int_0^{+\infty} M\{\xi\geq r\}dr-\int_{-\infty}^0 M\{\xi<r\}dr \qquad (1)$$

provided that at least one of the two integrals is finite.

The uncertain variables $\xi_1,\xi_2,\cdots,\xi_m$ are said to be independent if

$$M\left\{\bigcap_{i=1}^m\{\xi_i\in B_i\}\right\}=\min_{1\leq i\leq m} M\{\xi_i\in B_i\}$$

for any Borel sets $B_1,B_2,\cdots,B_m$ of real numbers. An uncertain process is essentially a sequence of uncertain variables indexed by time or space. An important uncertain process, the canonical process will be introduced as follows.

Definition 2. *(Liu [10]) An uncertain process C_t is said to be a canonical Liu process if*

(i) $C_0=0$ and almost all sample paths are Lipschitz continuous,

(ii) C_t has stationary and independent increments,

(iii) every increment $C_{s+t}-C_s$ is a normal uncertain variable with uncertainty distribution

$$\Phi(x)=\left(1+\exp\left(-\frac{\pi x}{\sqrt{3}}\right)\right)^{-1},\ x\in R.$$

Let C_t be a canonical process. Then for any real numbers e and σ, the uncertain process $X_t=et+\sigma C_t$ is called an arithmetic canonical process, where e is called the drift and σ is called the diffusion. The uncertain process $X_t=\exp(et+\sigma C_t)$ is an geometric canonical process.

Definition 3. *(Liu [10]) Let $\xi_1,\xi_2,...\xi_n$ be iid positive uncertain variables. Define $S_0=0$ and $S_n=\xi_1+\xi_2+...+\xi_n$ for $n\geq 1$. Then the uncertain process*

$$N_t=\max_{n\geq 0}\{n\,|\,S_n\leq t\}$$

is called a renewal process.

If $\xi_1,\xi_2,\cdots,\xi_n$ denote the interarrival times successive events. Then S_n can be regarded as the waiting time until the occurrence of the n th event. And N_t is the number of renewals in (0,t]. Note that N_t is a right-continuous and increasing uncertain process. Without loss of generality, we assume renewal time is a fixed number $a\ (a>0)$. Then we have $N_t=\lfloor t/a\rfloor$.

Definition 4. *(Liu [11]) Let X_t be an uncertain process and let C_t be a canonical process. For any partition of closed interval $[a,b]$ with $a=t_1<t_2<\cdots<t_k=b$, the mesh is written as*

$$\Delta=\max_{1\leq i\leq k}|t_{i+1}-t_i|.$$

Then the uncertain integral of X_t with respect to C_t is

$$\int_a^b X_t \mathrm{d}C_t = \lim_{\Delta\to 0}\sum_{i=1}^{k} X_{t_i}\cdot(C_{t_{i+1}} - C_{t_i})$$

provided that the limit exists almost surely and is an uncertain variable.

3. Uncertain Differential Equation with Jumps

In this section, we will introduce uncertain differential equation with jumps.

Definition 5. *Let C_t be a canonical Liu process, N_t a renewal process and Z_t an uncertain process. If there exist uncertain processes μ_s, σ_s and γ_s such that*

$$Z_t = Z_0 + \int_0^t \mu_s \mathrm{d}s + \int_0^t \sigma_s \mathrm{d}C_s + \int_0^t \gamma_s \mathrm{d}N_s$$

for any $t \ge 0$, then Z_t is said to have an uncertain differential

$$\mathrm{d}Z_t = \mu_t \mathrm{d}t + \sigma_t \mathrm{d}C_t + \gamma_t \mathrm{d}N_t.$$

For this case, Z_t is called a differentiable uncertain process with drift μ_t, diffusion σ_t and jump γ_t.

Example 1. Let C_t be a canonical process. Then for any partition $0 = t_1 < t_2 < \cdots < t_{k+1} = s$, we have

$$\int_0^s \mathrm{d}C_t = \lim_{\Delta\to 0}\sum_{i=1}^{k}(C_{t_{i+1}} - C_{t_i}) \equiv C_s - C_0 = C_s.$$

That is,

$$\int_0^s \mathrm{d}C_t = C_s. \tag{2}$$

Thus the uncertain process C_t is differentiable and has an uncertain differential $\mathrm{d}C_t$.

Example 2. Let N_t be an uncertain renewal process. we have

$$\int_0^s \mathrm{d}N_t = \lim_{\Delta\to 0}\sum_{i=1}^{k}(N_{t_{i+1}} - N_{t_i}) \equiv N_s - N_0 = N_s.$$

That is,

$$\int_0^s \mathrm{d}N_t = N_s. \tag{3}$$

Thus the uncertain process N_t is differentiable and has an uncertain differential $\mathrm{d}N_t$.

Example 3. Let $Z_t = \mu t + \gamma N_t$ be an uncertain process. Since

$$Z_t = \int_0^t \mu \mathrm{d}s + \int_0^t \gamma \mathrm{d}N_s,$$

we obtain that the uncertain process Z_t is differentiable and has an uncertain differential

$$\mathrm{d}Z_t = \mu \mathrm{d}t + \gamma \mathrm{d}N_t.$$

Example 4. Let $Z_t = \mu t + \sigma C_t + \gamma N_t$ be an uncertain process. Since

$$Z_t = \int_0^t \mu \mathrm{d}s + \int_0^t \sigma \mathrm{d}C_s + \int_0^t \gamma \mathrm{d}N_s,$$

we obtain that the uncertain process Z_t is differentiable and has an uncertain differential

$$\mathrm{d}Z_t = \mu \mathrm{d}t + \sigma \mathrm{d}C_t + \gamma \mathrm{d}N_t.$$

Theorem 1. *(Fundamental Theorem) Let C_t be a canonical Liu process, N_t a renewal process, and $h(t, C_t, N_t)$ a continuously differentiable function. Then the uncertain process $Z_t = h(t, C_t, N_t)$ has an uncertain differential*

$$\mathrm{d}Z_t = \frac{\partial h}{\partial t}(t, C_t, N_t)\mathrm{d}t + \frac{\partial h}{\partial C_t}(t, C_t, N_t)\mathrm{d}C_t + h(t, C_t, N_t) - h(t, C_t, N_{t-}).$$

Proof: Since the function h is continuous differentiable, by Taylor series expansion, we have

$$\begin{aligned}
\Delta Z_t &= h(t, C_t, N_t) - h(t - \Delta t, C_{t-\Delta t}, N_{t-\Delta t}) \\
&= h(t, C_t, N_{t-\Delta t}) - h(t - \Delta t, C_t, N_{t-\Delta t}) + h(t, C_t, N_t) - h(t, C_t, N_{t-\Delta t}) \\
&= \frac{\partial h}{\partial t}(t, C_t, N_{t-\Delta t})\Delta t + \frac{\partial h}{\partial C_t}(t, C_t, N_{t-\Delta t})\Delta C_t \\
&\quad + h(t, C_t, N_t) - h(t, C_t, N_{t-\Delta t}) + o(\Delta t) + o(\Delta C_t).
\end{aligned}$$

Letting $\Delta t \to 0$, we have

$$\mathrm{d}h(t, C_t, N_t) = \frac{\partial h}{\partial t}(t, C_t, N_t)\mathrm{d}t + \frac{\partial h}{\partial C_t}(t, C_t, N_t)\mathrm{d}C_t + h(t, C_t, N_t) - h(t, C_t, N_{t-}).$$

Example 5. Consider the uncertain process $\mu t + \gamma N_t$. For this case, we have $h(t, n) = \mu t + \gamma n.$ It is clear that

$$\frac{\partial h}{\partial t}(t, n) = \mu, \quad h(t, N_t) - h(t, N_{t-}) = \gamma \mathrm{d}N_t.$$

It follows from the fundamental theorem that

$$\mathrm{d}(\mu t + \gamma N_t) = \mu \mathrm{d}t + \gamma \mathrm{d}N_t.$$

Definition 6. *Suppose C_t is a canonical process, N_t is a renewal process, and f, g and h are some given functions. Then*

$$\mathrm{d}X_t = f(t, X_t)\mathrm{d}t + g(t, X_t)\mathrm{d}C_t + h(t, X_t)\mathrm{d}N_t \tag{4}$$

is called an uncertain differential equation with jumps. A solution is an uncertain process X_t that satisfies (4) identically in t.

Remark: The uncertain differential equation (4) is equivalent to the uncertain integral equation

$$X_t = X_0 + \int_0^t f(s, X_s)\mathrm{d}s + \int_0^t g(s, X_s)\mathrm{d}C_s + \int_0^t h(s, X_s)\mathrm{d}N_s.$$

Example 6. Let α,β and γ be real numbers. Consider an uncertain differential equation with jumps

$$\mathrm{d}X_t = \alpha\mathrm{d}t + \beta\mathrm{d}C_t + \gamma\mathrm{d}N_t.$$

Integrate on both sides, and we have

$$X_t - X_0 = \int_0^t \alpha\mathrm{d}s + \int_0^t \beta\mathrm{d}C_s + \int_0^t \gamma\mathrm{d}N_s.$$

Thus the uncertain differential equation has a solution

$$X_t = X_0 + \alpha t + \beta C_t + \gamma N_t.$$

Example 7. Let α,β and γ be real numbers. Consider an uncertain differential equation with jumps

$$\mathrm{d}X_t = \alpha X_t\mathrm{d}t + \beta X_t\mathrm{d}C_t + \gamma X_t\mathrm{d}N_t.$$

It is easy to verify that the uncertain differential equation has a solution

$$X_t = X_0(1+\gamma)^{N_t}\exp(\alpha t + \beta C_t).$$

4. An Uncertain Stock Model with Jumps

Here, our goal is to introduce a general stock model with jumps for uncertain financial markets. Let X_t be the bond price, and Y_t the stock price. Assume that stock follows a canonical process denoted by C_t and a renewal process denoted by N_t. Then we express the price dynamics as follows

$$\begin{cases} dX_t = rX_t dt \\ dY_t = \alpha Y_t dt + \beta Y_t dC_t + \gamma Y_t dN_t \end{cases} \tag{5}$$

where r is the riskless interest rate, α is the stock drift, β is the stock diffusion and γ is the stock renewal coefficient.

5. European Option

European options are financial instruments which gives its holder the right without being under obligation to trade an underlying asset at any time $t \le s$ for strike price K. The pricing of European option is one of the most important problems in financial markets. In many cases the stock price is not continuous because of economic crisis, war, stock daily limit or stock lower limit. In order to incorporate those into European stock model, we should develop an uncertain calculus with jump process. This paper presents an alternative assumption that stock price follows geometric canonical process and renewal process. Based on this assumption, the European option price models are formulated in this section.

5.1 European Call Option Pricing with Jumps

A European call option gives the holder the right, but not the obligation, to buy a stock at a specified time for a specified price. Considering a general stock model with jumps for uncertain financial markets, we assume that a European call option has strike price K and expiration time s. If Y_s is the final price of the underlying stock, then the payoff from buying a European call option is given by

$$(Y_t - K)^+ = \begin{cases} Y_t - K, & if \quad Y_t > K, t \le s \\ 0, & otherwise. \end{cases}$$

Considering the time value of money resulted from the bond, the present value of this payoff is $exp(-rt)(Y_t - K)^+$.

Definition 7. *Let X_t and Y_t be the bond price and the stock price, respectively. Let C_t and N_t be a canonical Liu process and a renewal process, respectively. Suppose that X_t and Y_t satisfy the price dynamics described by the general stock model with jumps. Then the European call option pricing formula is given by*

$$f_c(Y_0, K, \alpha, \beta, \gamma) = E[exp(-rt)(Y_t - K)^+]$$

where K is the strike price at exercise time s.

Let us consider the financial market described by the stock (5), The European call option price is

Theorem 2. *(European Call Option Pricing Formula) Assume a European call option for the stock model (5) has a strike price K and an expiration time t. Then the European call option price is*

$$f_c = exp(-rs)Y_0 \int_{K/Y_0}^{+\infty} \left(1 + exp\frac{\pi\left(\ln y - \alpha s - \lfloor s/a \rfloor \ln(1+\gamma)\right)}{\sqrt{3}\beta s}\right)^{-1} dy. \qquad (6)$$

Proof: It is clear that the stock price Y_t follows a geometric canonical process and a renewal process, i.e., $Y_t = Y_0(1+\gamma)^{N_t} exp(\alpha t + \beta C_t)$. Then the European call option price is

$$\begin{aligned} & f_c(Y_0, K, \alpha, \beta, \gamma) = E[exp(-rt)(Y_t - K)^+] \\ & = exp(-rs)E[Y_0(1+\gamma)^{N_t} exp(\alpha t + \beta C_t) - K]^+ \\ & = exp(-rs)\int_0^{+\infty} M\{Y_0(1+\gamma)^{N_t} exp(\alpha t + \beta C_t) - K) \ge x\}dx \\ & = exp(-rs)Y_0 \int_{K/Y_0}^{+\infty} M\{(1+\gamma)^{N_t} exp(\alpha t + \beta C_t) \ge y\}dy \\ & = exp(-rs)Y_0 \int_{K/Y_0}^{+\infty} M\left\{\alpha s + \beta C_s + \lfloor s/a \rfloor \ln(1+\gamma) \ge \ln y\right\}dy \\ & = exp(-rs)Y_0 \int_{K/Y_0}^{+\infty} M\left\{\beta C_s \ge \ln y - \alpha s - \lfloor s/a \rfloor \ln(1+\gamma)\right\}dy \\ & = exp(-rs)Y_0 \int_{K/Y_0}^{+\infty} \left(1 + exp\frac{\pi\left(\ln y - \alpha s - \lfloor s/a \rfloor \ln(1+\gamma)\right)}{\sqrt{3}\beta s}\right)^{-1} dy. \end{aligned}$$

This yields the desired result and completes the proof.

Theorem 3. *European call option formula* $f_c(Y_0, K, \alpha, \beta, \gamma) = E[\exp(-rs)(Y_t - K)^+]$ *has the following properties:*

(i) f_c is an increasing and convex function of Y_0;

(ii) f_c is a decreasing and convex function of K;

(iii) f_c is an increasing function of α;

(iv) f_c is an increasing function of β;

(v) f_c is a decreasing function of r;

(vi) f_c is an increasing of γ if $\gamma > 0$.

Proof: (i) This property means that if the other variables remain unchanged, then the option price is an increasing and convex function of the stock's initial price. To prove it, first note that for any positive constant b, the $\exp(-rt)(Y_0 b - K)^+$ is an increasing and convex function of Y_0. Consequently, the quantity

$$\exp(-rt)Y_0(1+\gamma)^{N_t}(\exp(\alpha t + \beta C_t) - K)^+$$

is increasing and convex in Y_0. Since the uncertainty distribution of $(1+\gamma)^{N_t}\exp(\alpha t + \beta C_t)^+$ does not depend on Y_0, the desired result is verified.

(ii) This follows from the fact that $\exp(-rt)Y_0((1+\gamma)^{N_t}\exp(\alpha t + \beta C_t) - K)^+$ is decreasing and convex in K. It means that European call option price is a decreasing and convex function of the stock's strike price when the other variables remain unchanged.

(iii) It is obvious that $f_c(Y_0, K, \alpha, \beta, \gamma) = E[\exp(-rt)(Y_t - K)^+]$ is a increasing function of α. This means that European call option price will increase with the stock drift.

(iv) This follows from the fact that $f_c(Y_0, K, \alpha, \beta, \gamma) = E[(\exp(-rt)Y_t - K)^+]$ is an increasing functions of β immediately. This property means that European call option price will increase with the stock diffusion.

(v) Since $f_c(Y_0, K, \alpha, \beta, \gamma) = E[\exp(-rt)(Y_t - K)^+]$ is a decreasing function of r and the expected value is independent of r, the result is verified. This means European call option price will decrease with the riskless interest rate.

(vi) This follows from the fact that $f_c(Y_0, K, \alpha, \beta, \gamma) = E[\exp(-rt)(Y_t - K)^+]$ is an increasing functions of γ if $\gamma > 0$ immediately. This property means that European call option price will increase with the stock renewal coefficient if $\gamma > 0$.

5.2. European Put Option Pricing Formula with Jumps

A European put option gives the holder the right, but not the obligation, to sell a stock at a specified time for a specified price. We assume that a European put option has strike price K and expiration time s. If Y_s is the final price of the underlying stock, then the payoff from buying a European put option is given by

$$(K-Y_t)^+ = \begin{cases} K-Y_t, & if \;\; Y_t < K, t \le s \\ 0, & otherwise. \end{cases}$$

Considering the time value of money resulted the bond, The present value of this payoff is $exp(-rt)(K-Y_t)^+$. The definition is given as follows:

Definition 8. *European put option pricing formula with jumps is given by*

$$f_p(Y_0, K, \alpha, \beta, \gamma) = E[exp(-rt)(K-Y_t)^+]$$

where K is the strike price at exercise time s.

Let us consider the financial market described by the stock (5). The European put option price is given by:

Theorem 4. *European put option formula with jumps for* (5) *is*

$$f_p = exp(-rs)Y_0 \int_0^{K/Y_0} \left(1 + exp\frac{\pi\left(\alpha s + \lfloor s/a \rfloor \ln(1+\gamma) - \ln y\right)}{\sqrt{3}\beta s}\right)^{-1} dy. \quad (7)$$

Proof: By the definition of expected value of uncertain variable, we have

$$\begin{aligned} f_p &= exp(-rs)E[Y_0(K - \exp(\alpha s + \beta C_s)(1+\gamma)^{N_t}] \\ &= exp(-rs)\int_0^{+\infty} M\left\{Y_0(K - exp(\alpha s + \beta C_s)(1+\gamma)^{N_t} \ge x\right\} dx \\ &= exp(-rs)Y_0 \int_{K/Y_0}^{+\infty} M\left\{exp\left(\alpha s + \beta C_s\right)(1+\gamma)^{N_t} \le y\right\} dy \\ &= exp(-rs)Y_0 \int_0^{K/Y_0} M\left\{\alpha s + \beta C_s + + \lfloor s/a \rfloor \ln(1+\gamma) \le \ln y\right\} dy \\ &= exp(-rs)Y_0 \int_0^{K/Y_0} M\left\{\beta C_s \le \ln y - \alpha s - \lfloor s/a \rfloor \ln(1+\gamma)\right\} dy \\ &= exp(-rs)Y_0 \int_0^{K/Y_0} \left(1 + exp\frac{\pi\left(\alpha s + \lfloor s/a \rfloor \ln(1+\gamma) - \ln y\right)}{\sqrt{3}\beta s}\right)^{-1} dy. \end{aligned}$$

This yields the desired result and completes the proof.

Theorem 5. *European put option formula* $f_p(Y_0, K, \alpha, \beta, \gamma) = E[exp(-rt)(K-Y_t)^+]$ *has the following properties:*

(i) f_p is a decreasing and convex function of Y_0;

(ii) f_p is an increasing and convex function of K;

(iii) f_p is a decreasing function of α;

(iv) f_p is an increasing function of β;

(v) f_p is a decreasing function of r;

(vi) f_p is a decreasing function of γ if $\gamma > 0$.

Proof: It is clear that the European put option price is a decreasing function of interest rate r. That is, the European put option price is an increasing function of strike price K; the European put option is decreasing function of the log-drift α; and the European put option price is an increasing function of the log-diffusion β, In addition, the European put option price is a decreasing function of the log-renewal coefficient $\gamma > 0$.

6. Extensions of Models with Jumps

In stock models with jumps, there are three important constants: stock drift stock α, diffusion β, stock renewal coefficient γ, they play an important role in European option price which is a major concern to investors. We know that change process of stock price is also a dynamic process, this three constants will also change along with the time change in European option transactions. We assume that three items of stock model are dynamic function with respect to time t, the stock models with jumps naturally became a general stock models.

6.1. A General Stock Model with Jumps

In order to adapt the above models to be more consistent with the general cases, we introduce an extension of the model with time-dependent parameters. The most general model with jumps satisfies the following uncertain differential equation

$$\begin{cases} dX_t = r_t X_t dt \\ dY_t = u(t, Y_t)dt + v(t, Y_t)dC_t + m(t, Y_t)dN_t \end{cases} \tag{8}$$

where $r_t, u(t, Y_t), v(t, Y_t), m(t, Y_t)$ are deterministic functions of time t.

6.2. Uncertain Currency Model with Jumps

We assumed that the exchange rate follows two classes of uncertain process: the canonical process and the renewal process, the uncertain currency model with jumps satisfies the following uncertain differential equation

$$\begin{cases} dX_t = uX_t dt \quad (DomesticCurrency) \\ dY_t = vY_t dt \quad (ForeignCurrency) \\ dZ_t = eZ_t dt + \beta Z_t dC_t + \gamma Z_t dN_t \quad (ExchangeRate) \end{cases} \tag{9}$$

where X_t represents the domestic currency with domestic interest rate u, Y_t represents the foreign currency with foreign interest rate v, and Z_t represents the exchange rate that is domestic currency price of one unit of foreign currency at time t, β is the stock diffusion and γ is the stock renewal coefficient.

7. Conclusion

The uncertain differential equations with jumps were studied in this paper. Based on this type of differential equations, a stock model with jumps for uncertain European Financial markets follows two classes of uncertain process: the canonical process and the renewal process were proposed. The European option pricing formulas with jumps were calculated and extended European option models were studied.

Acknowledgments
This work was supported by National Natural Science Foundation of China No.60874038 and No.70671050, Science Foundation Project of Liaocheng Vocational and Technical College No.2009LZY31, China.

REFERENCES

1. F.Black and P.Karasinski, Bond and option pricing when short-term rates are lognormal, Financial Analgsts Journal, 47(4) (1991) 52-59.
2. F.Black and M.Scholes, The pricing of options and corporate liabilities, Journal of Political Economy, 81 (1973) 637-654.
3. X.Chen and B.Liu, Existence and uniqueness theorem for uncertain differential equations, Fuzzy Optimization and Decision Making, 9(1) (2010) 69-81.
4. X.Gao, Some properties of continuous uncertain measure, International Journal of Uncertainty, Fuzziness and Knowledge-Based Systems, 17(3) (2009) 419-426.
5. J.Gao and X.Gao, Credibilistic option pricing: A new model, Journal of Uncertain Systems, 2(4) (2008) 243-247.
6. L.Karatzas and S.Shreve, Brownian Motion and Stochastic Calculus, Springer-Verlag, New York, 1991.
7. L.Karatzas and S.Shreve, Methods of Mathematical Finance, Springer-Verlag, New York, 1998.
8. J.Hull and A.White, Pricing interest rate derivative Securities, Review of Financial Studies, 3 (1990) 573-592.
9. B.Liu, Uncertainty Theory, 2nd ed, Springer-Verlag, Berlin, 2007.
10. B.Liu, Fuzzy process, hybrid process and uncertain process, Journal of Uncertain Systems, 2(1) (2008) 3-16.
11. B.Liu, Some research problems in uncertainty theory, Journal of Uncertain Systems, 3(1) (2009) 3-10.
12. B.Liu, Theory and Practice of Uncertain Programming, 2nd ed., Springer-Verlag, Berlin, 2009.
13. B.Liu, Uncertainty Theory: A Branch of Mathematics for Modeling Human Uncertainty, Springer-Verlag, Berlin, 2010
14. B.Liu, Extreme value theorems of uncertrain process with application to insurance risk models, http;//orsc.edu.cn/online/100722,pdf.
15. Y.Liu and M.Ha, Expected value of function of uncertain variables, Journal of Uncertain Systems, 4(3) (2010) 181-186.
16. B.Øksendal, Stochastic Differential Equations: An Introduction with Applications, 5th edition, Springer, 1998.
17. J.Xu and J.Peng, Barrier options pricing in uncertain financial market, Proceedings of the Eighth International Conference on Information and Management Sciences, Kunming, China, pp.821-826, July 20-28, 2009.
18. J.Peng and K.Yao, A new option pricing model for stocks in uncertainty markets, International Journal of Operations Research, 8(2) (2011) 18-26.

19. Z.Qin and X.Li, Option pricing formula for fuzzy financial market, Journal of Uncertain Systems, 2(1) (2008) 17-21.
20. A.Shiryaew, Essentials of Stochastic Finance, World Scientific, Singapore, 1999.
21. C.You, Some convergence theorems of uncertain sequences, Mathematical and Computer Modelling, 49(3-4) (2009) 482-487.
22. X.Yu, A stock model with jumps for uncertain markets, International Journal of Uncertainty, Fuzziness and Knowledge-Based Systems, 20(3) (2012) 421-432.
23. X.Yu, Expected payoff of trading strategies with jumps for uncertain financial markets, Information, 14(10) (2011) 3175-3186.