Journal of Mathematics and Informatics
Vol. 1, 2013-14, 66-75
ISSN: 2349-0632 (P), 2349-0640 (online)
Published on 22 May 2014
www.researchmathsci.org

Journal of
Mathematics and Informatics

Periodic solutions for a class of nonautomous second order systems

Er Ye and Yabin Shao

College of Mathematics and Computer Science
Northwest University for Nationalities, Lanzhou – 730030, Lanzhou, China
E-mail: yb-shao@163.com

Received 14 April 2014; accepted 20 may 2014

Abstract. In this paper, some multiplicity theorem is obtained for periodic solutions of nonautomous second order systems with partially periodic potential by the minimax methods.

Keywords: Nonautomous second order systems; periodic solutions; minimax methods

AMS Mathematics Subject Classification (2010): 34A34, 34A12

1. Introduction and main results

Consider the second-order systems

$$\begin{cases} \ddot{u}(t)-\nabla F(t,u(t))=0 & \text{a.e.}\, t\in[0,T], \\ u(0)-u(T)=\dot{u}(0)-\dot{u}(T)=e(t) \end{cases} \tag{1}$$

where $T>0$, and $F:[0,T]\times R^N\to R$ satisfies the following assumption:

$(H)\,F(t,x)$ is measurable in t for each $x\in R^N$ and continuously differentiable in x for $a.e.t\in[0,T]$ and there exist $a\in C(R^+,R^+)$, $b\in L^1(0,T;R^+)$ such that

$$|F(t,x)|\le a(|x|)b(t), |\nabla F(t,x)|\le a(|x|)b(t)$$

for all $x\in\times R^N$ and $a.e.t\in[0,T]$.

The existence of periodic solutions for problem (1) has been studied extensively, a lot of existence and multiplicity results have been obtained, we refer the readers to [1-17] and the reference therein.

Suppose that $F(t,x)$ is T_i – periodic in x_i, $1\le i\le r$, that is

$$F\left(t,x+\sum_{i=1}^{r}k_iT_ie_i\right)=F(t,x), \tag{2}$$

for all $a.e.t\in[0.T]$, $x\in\times R^N$ and all integers k_i, $1\le i\le r$, where $\{e_i\}(1\le i\le r)$ is the canonical basis of R^N.

With periodic potentials, that is, (2) holding with r=N, the existence and multiplicity theorems are obtained for the nonautonomous second-order system (1) in [1] and [2] respectively. When the nonlinearity is bounded, In [3-4], Chang and Liu have studied the nonautonomous second order system (1) with partially periodic (that is,(2)holding with $1\le r\le N$)a- nd partially uniformly coercive potentials ($F(t,x)\to+\infty$ for every $(x,$ $x_2\cdots,x_r)\in R^r$ as $(x_{r+1},x_{r+2}\cdots x_N)$ tends to infinity in R^{N-r}).

In [5], Wu has obtained the following result.

Theorem A. Suppose that F satisfies conditions (H) and (2). Assume that there exist $g\in L^1(0,T;R^+)$ such that

$$|\nabla F(t,x)| \le g(t) \qquad \text{and} \quad \int_0^T F(t,x)dt \to +\infty,$$

for every $(x_1, x_2 \cdots, x_r) \in R^r$ as $(x_{r+1}, x_{r+2} \cdots, x_N)$ tends to infinity in R^{N-r} .Then problem (1) has at least $r+1$ geometrically distinct solutions in H_T^1.

In 2003, Tang [6] generalized Theorem A and obtained the following results.

Theorem B. Assume that there exist $f, g \in L^1(0,T;R^+)$ and $0 \le \alpha < 1$ such that

$$|\nabla F(t,x)| \le f(t)|x|^{\alpha} + g(t) \tag{3}$$

for all $x \in R^N$ and $a.e. t \in [0,T]$. Suppose that F satisfies conditions (H) and (2). and

$$|x|^{-2\alpha}\int_0^T F(t,x)dt \to +\infty, \tag{4}$$

for every $(x_1, x_2 \ldots, x_r) \in R^r$ as $(x_{r+1}, x_{r+2} \cdots, x_N)$ tends to infinity in R^{N+r} .Then problem (1) has at least $r+1$ geometrically distinct solutions in H_T^1.

In 2011, Zhang and Tang [7] investigated the existence of periodic solutions for problem (1) and obtained the following results.

Theorem C. Assume that there exist f, $g \in L^1(0,T;R^+)$ and $0 \le \alpha < 1$ such that

$$|\nabla F(t,x)| \le f(t)|x|^{\alpha} + \mathrm{g}(t),$$

for all $x \in R^N$ and $a.e.t \in [0,T]$. Suppose that F satisfies conditions (H) and (2).and

$$\liminf_{|x|\to\infty} |x|^{-2\alpha}\int_0^T F(t,x)dt > \frac{T}{8}\left(\int_0^T f(t)dt\right)^2,$$

for every $(x_1, x_2 \cdots, x_r) \in R^r$ as $(x_{r+1}, x_{r+2} \cdots, x_N)$ tends to infinity in R^{N-r}.Then problem (1) has at least $r+1$ geometrically distinct solutions in H_T^1 .

In this paper, we consider the following second-order systems:

$$\begin{cases} \ddot{u}(t) + A\dot{u}(t) + \nabla F(t, u(t)) = 0, & a.e. t \in [0,T], \\ u(0) - u(T) = \dot{u}(0) - \dot{u}(T) = 0 \end{cases} \tag{5}$$

where $T > 0$, A is antisymmetry constant matric with $\|A\| < \frac{2\pi}{T}$.

The Hilbert space H_T^1 is defined by $H_T^1 = \{u : [0,T] \mapsto R^N \mid$ u is a absolutely continuous $u(0) = u(T)$ and $\dot{u} \in L^2(0,T;R^N)\}$ and H_T^1 is endowed with the norm

$$\|u\| = \left(\int_0^T |\dot{u}|^2 dt + \int_0^T |u|^2 dt\right)^{\frac{1}{2}},$$

By sobolev embeded theorems, there exist C>0, for all $u \in H_T^1$, such that

$$\|u\|_{\infty} \le C\|u\| \cdot \tag{6}$$

where $\|u\|_{\infty} = \max_{t\in[0,T]} |u(t)|$ .Motivated by [1-7], The following main results are obtained by the minimax methods.

Theorem 1.1. Suppose that F satisfies conditions (H), and there exist $f, g \in L^1(0,T;R^+)$ such that

$$|\nabla F(t,x)| \le f(t)|x| + g(t), \tag{7}$$

for all $x \in R^N$ and $a.e.t \in [0,T]$, where

$$M_1=\int_0^T f(t)dt,\delta=\min\left\{\frac{1}{2},\frac{2\pi^2}{T^2}\right\}\left(1-\frac{T}{2\pi}\|A\|\right).$$

$$2M_1C^2<\delta,$$

Assume that (2) holds and

$$|x|^{-2}\int_0^T F(t,x)dt\to+\infty, \tag{8}$$

as x tends to infinity in $\in 0\times R^{N-r}$.Then problem (5) has at least $r+1$geometrically distinct solutions in H_T^1.

Remark 1.1. System (5) generalizes system (1) obviously, for if A= 0, then system (5) becomes system (1). We consider the system (5) with partially periodic potentials and linear nonlinearity.

Theorem 1.1 is a new result, which completes the theorem B and theorem C with α=1 in (3). Theorem 1.1 generalized Theorem A , which is the special case of Theorem 1.1 corresponding to f(t)≡0.

Example 1.1. There are functions F satisfying our Theorem 1.1 and not satisfying the results in [1]-[7]. For example, let

$$F(t,x)=r+1+\sin x_1+\sin x_2+\cdots+\sin x_r+\sum_{j=r+1}^{N}|x_j|^2,$$

where $x=\{x_1,x_2,\cdots,x_N\}\in R^N$.

Theorem 1.2. Suppose that F satisfies conditions (H), (2),(7) and

$$|x|^2\int_0^T F(t,x)dt\to-\infty, \tag{9}$$

as x tends to infinity in $\in 0\times R^{N-r}$.Then problem (5) has at least $r+1$ geometrically distinct solutions in H_T^1.

Example 1.2. There are functions F satisfying our Theorem 1.2 and not satisfying the results in [1]-[7]. For example, let

$$F(t,x)=-(r+1+\sin x_1+\sin x_2+\cdots+\sin x_r+\sum_{j=r+1}^{N}|x_j|^2),$$

where $x=\{x_1,x_2,\cdots,x_N\}\in R^N$.

2. Preliminaries

For $u\in H_T^1$,let $\bar{u}=\frac{1}{T}\int_0^T u(t)dt$,and $\tilde{u}(t)=u(t)-\bar{u}$.Then we have Sobolev's inequality

$$\|\tilde{u}\|^2\le\frac{T}{12}\int_0^T|\dot{u}(t)|^2dt, \tag{10}$$

and Wirtinger's inequality

$$\int_0^T|\tilde{u}(t)|^2dt\le\frac{T^2}{4\pi^2}\int_0^T|\dot{u}(t)|^2dt \tag{11}$$

for all $u\in H_T^1$ (see Proposition 1.3 in [8]).

Put $\hat{u}(t)=P\bar{u}+Q\bar{u}+\tilde{u}(t)$, where

$$P\bar{u}=\sum_{i=r+1}^{N}(\bar{u},e_i)e_i,\ Q\bar{u}=\sum_{i=1}^{r}\left[(\bar{u},e_i)-l_iT_i\right]e_i$$

and $l_i(1\le i\le r)$ is the unique integer such that $0\le(\bar{u},e_i)-l_iT_i<T_i$

Define the functional φ on H_T^1 by

$$\varphi(u)=\frac{1}{2}\int_0^T|\dot{u}(t)|^2dt+\frac{1}{2}\int_0^T(Au(t),\dot{u}(t))dt-\int_0^T F(t,u(t))dt$$

Then φ is continuously differentiable by Lemma 1 in [9] and the solutions of systems (5) correspond to the critical points of φ. Moreover, one has

$$\langle\varphi(u),v\rangle=\int_0^T(u(t),v(t))dt+\frac{1}{2}\int_0^T(Au(t),v(t))dt-\int_0^T(\nabla F(t,u(t)),v(t))dt$$

Let

$$G=\left\{\sum_{i=1}^{r}k_iT_ie_i\mid k_i\in Z,1\le i\le r\right\}$$

be a discrete subgroup of H_T^1 and let $\pi: H_T^1\to H_T^1/G$ be the canonical surjection. It is obvious that $H_T^1/G=X\times V$,
where $X=Y\oplus Z$, $Y=\tilde{H}_T^1=\{u\in H_T^1\mid\bar{u}=0\}$ $Z=span\{e_{r+1},\cdots,e_N\}$,and $V=span\{e_1,\cdots,e_r\}/G$ is isomorphic to the torus T^r.Define $\psi;X\times Y\to R$ by

$$\psi(\pi(u))=\varphi(u).$$

It follows from (2) that ψ is well-defined. Moreover, ψ is continuously differentiable.

3. Proof of theorem

Now we begin to prove our main result, for the sake of convenience, c denote some constant.

Proof of Theorem 1.1. the proof relies on the generalized saddle point theorem due to Liu [4]. Assume that $\pi((u_n))\to 0$ is a (PS) sequence for ψ ,that is, $\psi(\pi(u_n))$ is bounded and $\psi'(\pi(u_n))\to 0$.Then $\varphi(u_n)$ is bounded and $\phi'(u_n)\to 0$.

Let $\{u_n\}\in H_T^1$ be such that

$$|\varphi(u_n)|\le c,\varphi'(u_n)\to 0,(n\to\infty)\qquad(12)$$

First,we shall prove that $\{u_n\}$ is bounded in H_T^1.By contradiction, we assume $\|u_n\|\to\infty$ as $n\to\infty$.

For $\{u_n\}\in H_T^1$,let $\bar{u}=\frac{1}{T}\int_0^T u(t)dt$,and $\tilde{u}(t)=u(t)-\bar{u}$.By (12), for $\forall\phi\in H_T^1$,we have

$$\int_0^T(\dot{u}_n,\dot{\phi})dt+\int_0^T(Au_n,\dot{\phi})dt-\int_0^T(\nabla F(t,u_n),\phi)dt=0(\|\phi\|).$$

Taking $\phi=\tilde{u}_n$,we have

$$\int_0^T|\dot{u}_n|^2dt+\int_0^T(Au_n,\dot{u}_n)dt-\int_0^T(\nabla F(t,u_n),\tilde{u}_n)dt\le\|\tilde{u}_n\|.\qquad(13)$$

By the Wirtinger inequality (11) and $\|A\|\le\frac{2\pi}{T}$, we have

$$\int_0^T|\dot{u}_n|^2dt+\int_0^T(Au_n,\dot{u}_n)dt$$

$$\geq (1-\frac{T}{2\pi}\|A\|)\int_0^T |\dot{u}|^2 dt$$

$$\geq \frac{1}{2}(1-\frac{T}{2\pi}\|A\|)\int_0^T |\dot{u}_n|^2 dt + \frac{1}{2}\frac{4\pi^2}{T^2}(1-\frac{T}{2\pi}\|A\|)\int_0^T |\tilde{u}_n|^2 dt$$

$$\geq \min\left\{\frac{1}{2},\frac{2\pi^2}{T^2}\right\}(1-\frac{T}{2\pi}\|A\|)\|\tilde{u}_n\|^2 = \delta\|\tilde{u}_n\|^2 \tag{14}$$

where $\delta = \min\left\{\frac{1}{2},\frac{2\pi^2}{T^2}\right\}(1-\frac{T}{2\pi}\|A\|)$.

By (7), (11) and the *Hölder* inequality, one has

$$\left|\int_0^T (\nabla F(t,u_n),\tilde{u}_n)\right| dt$$

$$\leq \int_0^T f(t)|u_n||\tilde{u}_n| dt + \int_0^T g(t)|\tilde{u}_n| dt$$

$$\leq \int_0^T f(t)dt\|u\|_\infty\|\tilde{u}_n\|_\infty + \int_0^T g(t)dt\|\tilde{u}_n\|_\infty$$

$$\leq C^2\int_0^T f(t)dt\|u_n\|\|\tilde{u}_n\| + C\int_0^T g(t)dt\|\tilde{u}_n\|$$

$$= M_1C^2\|u_n\|\|\tilde{u}_n\| + C\int_0^T g(t)dt\|\tilde{u}_n\| \tag{15}$$

where $M_1 = \int_0^T f(t)dt$.

By (13),(14) and (15), we have

$$\|u_n\| \geq \int_0^T |\dot{u}_n|^2 dt + \int_0^T (Au_n,\dot{u}_n)dt - \int_0^T (\nabla F(t,u_n),\tilde{u}_n)dt$$

$$\geq \delta\|\tilde{u}_n\|^2 - M_1C^2\|u_n\|\|\tilde{u}_n\| - C\int_o^T g(t)dt\|\tilde{u}_n\|,$$

Therefore

$$\|\tilde{u}_n\| \leq \frac{M_1C^2}{\delta}\|u_n\| + C \tag{16}$$

It follows from (15) and (16) that

$$\left|\int_0^T (\nabla F(t,u_n),\tilde{u}_n)dt\right|$$

$$\leq M_1C^2\|u_n\|\|\tilde{u}_n\| + C\int_0^T g(t)dt\|\tilde{u}_n\|$$

$$\leq \frac{M_1^2C^4}{\delta}\|u_n\|^2 + C\|u_n\| + C, \tag{17}$$

By (16), $2M_1C^2 < \delta$ and the boundedness of $\|Q\bar{u}_n\|$, we have

$$\frac{\|\tilde{u}_n\|}{\|u_n\|} \to m \leq \frac{M_1C^2}{\delta} < \frac{1}{2}, \quad (n\to\infty)$$

$$\frac{\|P\bar{u}_n\|}{\|u_n\|} \geq 1 - \frac{\|\tilde{u}_n\|}{\|u_n\|} - \frac{\|Q\bar{u}_n\|}{\|u_n\|} \to 1 - m \geq \frac{1}{2} - \frac{M_1 C^2}{\delta} \quad , \ (n \to \infty) \tag{18}$$

By (18), for $\forall \varepsilon > 0$, such that $\frac{1}{2} - \frac{M_1 C^2}{\delta} - \varepsilon > 0$, we have

$$\|u_n\| \leq \frac{1}{\frac{1}{2} - \frac{M_1 C^2}{\delta} - \varepsilon} \|P\bar{u}_n\| \cdot \tag{19}$$

It follows from (13),(16), (17) and (19) that

$$\int_0^T |\dot{u}_n|^2 dt + \int_0^T (Au_n, \dot{u}_n) dt \leq \int_0^T (\nabla F(t, u_n), \tilde{u}_n) dt + \|\tilde{u}_n\| \leq \frac{M_1^2 C^4}{\delta} \|u_n\|^2 + c\|u_n\| + c$$

$$\leq \frac{M_1^2 C^4}{\delta} \frac{1}{\frac{1}{2} - \frac{M_1 C^2}{\delta} - \varepsilon} \|P\bar{u}_n\|^2 + c\|P\bar{u}_n\| + c \tag{20}$$

By (16) and (19), we have

$$\|\tilde{u}_n\| \leq \frac{M_1 C^2}{\delta} \|u_n\| + c \leq \frac{M_1 C^2}{\delta} \frac{1}{\frac{1}{2} - \frac{M_1 C^2}{\delta} - \varepsilon} \|P\bar{u}_n\| \cdot \tag{21}$$

By (6),(7),(21), the *Hölder* inequality and the boundedness of $\|Q\bar{u}_n\|$, we have

$$\left| \int_0^T F(t, \hat{u}_n) dt - \int_0^T F(t, P\bar{u}_n) dt \right| \leq \int_0^T \int_0^1 |\nabla F(t, P\bar{u}_n + s(Q\bar{u}_n + \tilde{u}_n))|$$

$$\leq \int_0^T f(t)(|P\bar{u}_n| + |Q\bar{u}_n + \tilde{u}_n|)|Q\bar{u}_n + \tilde{u}_n| dt + \int_0^T g(t)|Q\bar{u}_n + \tilde{u}_n| dt$$

$$\leq \int_0^T f(t) dt (\|P\bar{u}_n\|_\infty + \|Q\bar{u}_n + \tilde{u}_n\|_\infty) \|Q\bar{u}_n + \tilde{u}_n\|_\infty + \int_0^T g(t) dt \|Q\bar{u}_n + \tilde{u}_n\|_\infty$$

$$\leq C^2 \int_0^T f(t) dt (\|P\bar{u}_n\| + \|Q\bar{u}_n + \tilde{u}_n\|) \|Q\bar{u}_n + \tilde{u}_n\|$$

$$C\int_0^T g(t) dt \|Q\bar{u}_n + \tilde{u}_n\| = M_1 C^2 \|P\bar{u}_n\| \|Q\bar{u}_n + \tilde{u}_n\| + M_1 C^2 \|Q\bar{u}_n + \tilde{u}_n\|^2 + C\int_0^T g(t) dt \|Q\bar{u}_n + \tilde{u}_n\|$$

$$\leq \frac{M_1^2 C^4}{\delta} \frac{1}{\frac{1}{2} - \frac{M_1 C^2}{\delta} - \varepsilon} \|P\bar{u}_n\|^2 + c\|P\bar{u}_n\| + c + \frac{M_1^2 C^4}{\delta^2} \frac{1}{(\frac{1}{2} - \frac{M_1 C^2}{\delta} - \varepsilon)^2} M_1 C^2 \|P\bar{u}_n\|^2$$

$$= \frac{M_1^2 C^4}{\delta} \left[\frac{1}{\frac{1}{2} - \frac{M_1 C^2}{\delta} - \varepsilon} - \frac{\frac{M_1 C^2}{\delta}}{(\frac{1}{2} - \frac{M_1 C^2}{\delta} - \varepsilon)^2} \right] \|P\bar{u}_n\|^2 + c\|P\bar{u}_n\| + c$$

$$= \frac{M_1^2 C^4}{\delta} \frac{\frac{1}{2} - \varepsilon}{(\frac{1}{2} - \frac{M_1 C^2}{\delta} - \varepsilon)^2} \|P\bar{u}_n\|^2 + c\|P\bar{u}_n\| + c \tag{22}$$

By (20) and (22), we have

$$\varphi(u_n) = \varphi(\hat{u}_n) \le \frac{M_1^2 C^4}{\delta} \frac{1}{(\frac{1}{2} - \frac{M_1 C^2}{\delta} - \varepsilon)^2} \|P\bar{u}_n\|^2 + c\|P\bar{u}_n\| + c$$

$$- \int_0^T \left[F(t, \hat{u}_n) - F(t, P\bar{u}_n)\right] dt - \int_0^T F(t, P\bar{u}_n) dt$$

$$\le \frac{M_1^2 C^4}{\delta} \frac{1}{(\frac{1}{2} - \frac{M_1 C^2}{\delta} - \varepsilon)^2} \|P\bar{u}_n\|^2 t + \frac{M_1^2 C^4}{\delta} \frac{\frac{1}{2} - \varepsilon}{(\frac{1}{2} - \frac{M_1 C^2}{\delta} - \varepsilon)^2} \|P\bar{u}_n\|^2$$

$$+ c\|P\bar{u}_n\| + c - \int_0^T F(t, P\bar{u}_n) dt$$

$$= \frac{M_1^2 C^4}{\delta} \left[\frac{1 - \varepsilon}{(\frac{1}{2} - \frac{M_1 C^2}{\delta} - \varepsilon)^2} - \frac{1}{\|P\bar{u}_n\|^2} \int_0^T F(t, P\bar{u}_n) dt \right]$$

$$\|P\bar{u}_n\|^2 + c\|P\bar{u}_n\| + c \tag{23}$$

Since $\|u_n\| \to \infty$ as $n \to \infty$, by (19), we have $\|P\bar{u}_n\| \to \infty$ as $n \to \infty$.

Let ε small enough, by (8), we get that $\phi(u_n) \to -\infty$ as $n \to \infty$,this contradicts (12) and hence $\{u_n\}$ is bounded in H_T^1 .Arguing then as in Proposition 4.1 in [9], we conclude that the (PS) condition is satisfied.

Now we check the link condition that

(a) $\inf\{\psi(\pi(u)) \mid \pi(u) \in Y \times X\} > -\infty$.

(b) $\psi(\pi(x)) \to -\infty$ uniformly for $\pi(Qx)$ as $|Px| \to \infty$,where $x \in R^N$.

For $\pi(u) \in Y \times V$, $u = \tilde{u} + Q\bar{u}$.

In a similar way to (22), we can get that

$$\left| \int_0^T F(t, \tilde{u}(t) + Q\bar{u}) dt - \int_0^T F(t, 0) dt \right|$$

$$\le M_1 C^2 \|\tilde{u}\|^2 + M_2 |T|^{\frac{1}{2}} C \|\tilde{u}\| + c, \tag{24}$$

It follows from (24), $\|A\| < \frac{2\pi}{T}$ and the boundedness of $\|Q\bar{u}_n\|$ that

$$\psi(\pi(u)) = \psi(\pi(\tilde{u} + Q\bar{u})) = \varphi(\tilde{u} + Q\bar{u}) = \frac{1}{2}\int_0^T |\dot{u}|^2 dt + \frac{1}{2}\int_0^T (Au(t), \dot{u}(t)) dt$$

$$- \left[\int_0^T F(t, \tilde{u}(t) + Q\bar{u}) dt - \int_0^T F(t, 0) dt \right] - \int_0^T F(t, 0) dt$$

$$\geq \frac{1}{2}\min\left\{\frac{1}{2},\frac{2\pi^2}{T^2}\right\}(1-\frac{T}{2\pi}\|A\|)\|\tilde{u}\|^2 - M_1C^2\|\tilde{u}\|^2 - M_2|T|^{\frac{1}{2}}C\|\tilde{u}\|+c-\int_0^T F(t,0)dt$$

By $2M_1C^2<\delta$, we have $\psi(\pi(x))\to+\infty$, as $\|u\|\to\infty$, for all $\pi(u)\in Y\times V$,which implies (a).

It follows from (7) and the boundedness of $|Q\bar{u}_n|$ that

$$\psi(\pi(x))=\varphi(x)=\varphi(\hat{x})=-\int_0^T F(t,Px+Qx)dt$$

$$=-\int_0^T\int_0^1(\nabla F(t,Px+sQx),Qx)dsdt-\int_0^T F(t,Px)dt$$

$$\leq -\int_0^T F(t,Px)dt+\int_0^T M_1|Px+Qx||Qx|dt+\int_0^T M_2|Qx|dt$$

$$\leq -\int_0^T F(t,Px)dt+c|Px|+c = -|Px|^2\left[\frac{1}{|Px|^2}\int_0^T F(t,Px)dt+\frac{c}{|Px|}+\frac{c}{|Px|^2}\right],$$

for all $x\in R^N$.Hence we have, by (8),(b) is satisfied. It follows from the generalized saddle point theorem (Theorem 1.7 in [4]) that has at least $r+1$ critical points .Hence φ has at least $r+1$ geometrically distinct critical points. Therefore, problem (5) has at least $r+1$ geometrically distinct solutions in H_T^1 .

Proof of Theorem 1.2 the proof relies on Theorem 4.12 in [8]. In a similar way to (22), we can get that

$$\left|\int_0^T F(t,\hat{u})dt-\int_0^T F(t,P\bar{u})dt\right|$$

$$\leq M_1C^2\|P\bar{u}\|\|\tilde{u}\|+c\|P\bar{u}\|+M_1C^2\|\tilde{u}\|^2+c\|\tilde{u}\|+c \tag{25}$$

It follows from (14) and (25) that

$$\varphi(u)=\varphi(\hat{u})$$

$$\geq\frac{\delta}{2}\|\tilde{u}\|^2-\left[\int_0^T F(t,\hat{u})dt-\int_0^T F(t,P\bar{u})dt\right]-\int_0^T F(t,P\bar{u})dt$$

$$\geq(\frac{\delta}{2}-M_1C^2)\|\tilde{u}\|^2-M_1C^2\|P\bar{u}\|\|\tilde{u}\| \quad +c\|P\bar{u}\|+c\|\tilde{u}\|+c-\int_0^T F(t,P\bar{u})$$

$$=(\frac{\delta}{2}-M_1C^2)\left[\|\tilde{u}\|-\frac{M_1C^2}{2(\frac{\delta}{2}-M_1C^2)}\|P\bar{u}\|\right]^2 \quad +c\|P\bar{u}\|+c\|\tilde{u}\|+c$$

$$-\left[\frac{M_1^2C^4}{4(\frac{\delta}{2}-M_1C^2)}+\frac{1}{\|P\bar{u}\|^2}\int_0^T F(t,P\bar{u})dt\right]\|P\bar{u}\|^2$$

$$=(\frac{\delta}{2}-M_1C^2)\left[\|\tilde{u}\|-\frac{M_1C^2}{2(\frac{\delta}{2}-M_1C^2)}\|P\bar{u}\|\right]^2+c\|P\bar{u}\|+c\|\tilde{u}\|+c$$

$$-\left[\frac{M_1^2C^4}{4(\frac{\delta}{2}-M_1C^2)}+\frac{1}{\|P\bar{u}\|^2}\int_0^T F(t,P\bar{u})dt\right]\|P\bar{u}\|^2 \tag{26}$$

By $2M_1C^2<\delta$ and (9), we get that φ is bounded from below. Moreover, the functional φ satisfies the $(PS)_c$ condition; that is, for every sequence (u_n) in H_T^1 such that $\varphi(u_n)\to 0$ is bounded and $\varphi(u_n)\to 0$,the sequence $\pi(u_n)$ has a convergent subsequence (see Definition 4.2 in [8]). In fact, the boundedness of $\varphi(u_n)$, (9) and (26) imply that $(\tilde{u}_n)$ and (Pu_n) are bounded. Hence $(\hat{u}_n)$ bounded. As in the proof of Proposition 4.1 in [8], $(\hat{u}_n)$ has a convergent subsequence, so we have $\pi(u_n)=\pi(\hat{u}_n)$.

Now the Theorem in this case follows from Theorem 4.12 in [8].

Acknowledgments
This work is supported by the National Natural Scientific Fund of China(11161041).

REFERENCES

1. M.Willem, Oscillations forces de systemes hamiltoniens, *Public Semin.Analyse nonlineaire Univ.* Besancon, 1981.
2. P.H.Rabinowitz, On a class of functionals invariant under a Zn action, *Trans. Amer.Math. Soc,* 310(1) (1988) 303-311.
3. K.C. Chang, on the periodic nonlinearity and the multiplicity of solutions, *Nonlinear Analysis TMA*, 13(5) (1989) 527-537.
4. J.Q.Liu, A generalized saddle point theorem, *J. Differential Equations*, 82 (1989) 372-385.
5. X.P.Wu, Periodic solutions for nonautonomous second-order systems with bounded nonlinearity, *J. Math. Anal, Appl.*, 230(1) (1999) 135-141.
6. C.L.Tang, A note on periodic solutions of nonautonomous second order systems, *Porc. Amer. Math. Soc.,* 132 (5) (2003) 1295-1393.
7. X.Y.Zhang and X.H.Tang, Periodic solutions for an ordinary p-Laplacian system, *Taiwanese Journal of Mathematic,* 3 (2011) 1369-1396.
8. J.Mawhin and M.Willem, *Critical point theory and Hamiltonian systems,* Springer-Verlag, New York 1989.
9. F.J.Meng and J.Yang, Periodic Solutions for a Class of Non-autonomous Second Order Systems, *International Journal of Nonlinear Science*, 10(3) (2010) 342-348.
10. Z.Y.Wang and J.H.Zhang, Periodic solutions of nonautomous second orderr systems with p-Laplacian, *Electronic J. Differential Equations*, 17 (2009) 1-12.

11. J.X.Feng and Z.Q.Han, Periodic solutions to differential systems with unbounded or periodic nonlinearities, *J. Math. Anal. Appl*, 323 (2006) 1264-1278.
12. Y.W.Ye and C.L.Tang, Periodic solutions for some nonautonomous second order Hamiltonian systems, *J. Math. Anal .Appl.*, 344(1) (2008) 462-471.
13. Q.Meng and X.H.Tang, Solutions of a second order Hamiltonian systems, *Nonlinear Anal*, 9(2010) 1053-1067.
14. Y.Liu and H.b.Chen, Existence and multiplicity of periodic solutions generated by impulses, *Abstract and Applied Analysis,* 2 (2011) 1-15.
15. X. H. Tang and X.Y.Zhang, Periodic solutions for second-order Hamiltonian systems with a p-Laplacian, *Annales Universitatis Mariae*, 1 (2010) 93-113.
16. P.Zhang and C.Tang, Infinitely many periodic solutions for nonautonomous sublinear second order Hamiltonian systems, *Abstract and Applied Analysis,* 12 (2010) 1-10.
17. Y.H.Su and Z.S.Feng, A nonautonomous Hamiltonia *n* system on time scales, *Nonlinear Anal*, 10 (2012) 4126-4136.