Journal of Mathematics and Informatics
Vol. 10, 2017, 83-88
ISSN: 2349-0632 (P), 2349-0640 (online)
Published 11 December 2017
www.researchmathsci.org
DOI: http://dx.doi.org/10.22457/jmi.v10a11

Journal of Mathematics and Informatics

Evaluation of Some Centered Polygonal Numbers by Using the Division Algorithm

V. Pandichelvi[1] and P. Sivakamasundari[2]

[1]Department of Mathematics, Urumu Dhanalakshmi College
Trichy, Tamilnadu, India
e-mail: mvpmahesh2017@gmail.com
[2]Department of Mathematics, BDUCC, Lalgudi, Trichy, India.
e-mail: kaneeska.s@gmail.com

Received 2 November 2017; accepted 7 December 2017

Abstract. In this communication, we find centered square number, centered octagonal number, centered dodecagonal number, centered hexadecagonal number and tetradecagonal number by using division algorithm.

Keywords: Division algorithm, centered polygonal numbers

AMS Mathematics Subject Classification (2010): 11D99

1. Introduction

In [1] a function $A: N \to N$ is defined by $A(n) = m$ where m is the smallest natural number such that $n \text{ divides } m + \sum_{k=1}^{n} k^2$. In [2] a function $A(n)$ such that $A(n) = m$ where m is the smallest natural number such that $n \text{ divides } m + \sum_{k=1}^{n} k \text{ and } m + \sum_{k=1}^{n} k^3$ are given. In this communication, we obtain a function $A(n)$ such that $A(n) = m$ where m is some centered polygonal numbers, such that $2PP_n$ divides $m + (a \text{ cubical polynomial})$

Notations:

$PP_n = \frac{1}{2}\left(n^3 + n\right)$ be a pentagonal pyramidal number of rank n

$CS_n = 2n^2 - 2n + 1$ be a centered square number of rank n

$CO_n = 4n^2 - 4n + 1$ be a centered octagonal number of rank n

$CD_n = 6n^2 - 6n + 1$ be a centered dodecagonal number of rank n

$CT_n = 7n^2 - 7n + 1$ be a centered tetradecagonal number of rank n

$CH_n = 8n^2 - 8n + 1$ be a centered Hexadecagonal number of rank n

2. Method of analysis

SECTION A: Evaluation of centered square number

Let $A: N \to N$ be defined by $A(n) = m$ where m is the smallest natural number such that $2PP_n$ divides $m + (n^3 - 2n^2 + 3n - 1)$. If $2PP_n$ divides $(n^3 - 2n^2 + 3n - 1)$, then $A(n) = 2n^2 - 2n + 1$, otherwise $A(n) = (2n^2 - 2n + 1) - r$ where r is the least non negative remainder when $(n^3 - 2n^2 + 3n - 1)$ is divided by $2PP_n$. Hence A is defined for all n. By division algorithm such remainder is given by $(n^3 - 2n^2 + 3n - 1) - 2qPP_n$ where q is the quotient when $(n^3 - 2n^2 + 3n - 1)$ is divided by $2PP_n$ and is given by the greatest integer function of $\frac{(n^3 - 2n^2 + 3n - 1)}{2PP_n}$,

That is $q = \left[\frac{n^3 - 2n^2 + 3n - 1}{2PP_n}\right]$

Hence,

$$A(n) = n + n^3 - \left\{(n^3 - 2n^2 + 3n - 1) - \left[\frac{(n^3 - 2n^2 + 3n - 1)}{n + n^3}\right] \times n + n^3\right\}$$

$$A(n) = n + n^3 - n^3 + 2n^2 - 3n + 1 + \left[\left(\frac{n + n^3}{n + n^3}\right) - \frac{(2n^2 - 2n + 1)}{n + n^3}\right] \times n + n^3$$

$$A(n) = 2n^2 - 2n + 1 + \left[1 - \frac{(2n^2 - 2n + 1)}{n + n^3}\right] \times n + n^3$$

$$A(n) = 2n^2 - 2n + 1 + 0 \quad \left(\because 0 < \frac{2n^2 - 2n + 1}{n + n^3} < 1\right)$$

$A(n) = 2n^2 - 2n + 1 =$ centered square number.

SECTION B: Evaluation of centered octagonal number

Let $A: N \to N$ be defined by $A(n) = m$ where m is the smallest natural number such that $2PP_n$ divides $m + (2n^3 - 4n^2 + 6n - 1)$. If $2PP_n$ divides $(2n^3 - 4n^2 + 6n - 1)$, then $A(n) = 4n^2 - 4n + 1$, otherwise $A(n) = (4n^2 - 4n + 1) - r$ where r is the least non negative remainder when $\left(2n^3-4n^2+6n-1\right)$ is divided by $2PP_n$. Hence A is defined for all n. By division algorithm such remainder is given by $\left(2n^3-4n^2+6n-1\right)-2qPP_n$ where q is the quotient when $\left(2n^3-4n^2+6n-1\right)$ is divided by $2PP_n$ and is given by the greatest integer function of $\dfrac{\left(2n^3-4n^2+6n-1\right)}{2PP_n}$,

That is $q=\left[\dfrac{2n^3-4n^2+6n-1}{2PP_n}\right]$.

Hence ,

$$A(n)=n+n^3-\left\{\left(2n^3-4n^2+6n-1\right)-\left[\frac{\left(2n^3-4n^2+6n-1\right)}{n+n^3}\right]\times n+n^3\right\}$$

$$A(n)=n+n^3-2n^3+4n^2-6n+1+\left[\left(\frac{n+n^3}{n+n^3}\right)+\frac{n^3-4n^2+5n-1}{n+n^3}\right]\times n+n^3$$

$$A(n)=-n^3+4n^2-5n+1+\left[1+\frac{n^3-4n^2+5n-1}{n+n^3}\right]\times n+n^3$$

$$A(n)=-n^3+4n^2-5n+1+n+n^3\left(\because 0<\frac{n^3-4n^2+5n-1}{n+n^3}<1\right)$$

$A(n)=4n^2-4n+1=$ centered octagonal number.

SECTION C : Evaluation of centered dodecagonal number

Let $A: N\to N$ be defined by $A(n)=m$ where m is the least natural number such that $2PP_n$ divides $m+\left(3n^3-6n^2+9n-1\right)$. If $2PP_n$ divides $\left(3n^3-6n^2+9n-1\right)$, then $A(n)=6n^2-6n+1$, otherwise $A(n)=\left(6n^2-6n+1\right)-r$ where r is the least non negative remainder when $\left(3n^3-6n^2+9n-1\right)$ is divided by $2PP_n$. Hence A is defined for all n. By division algorithm such residue is given by $\left(3n^3-6n^2+9n-1\right)-2qPP_n$ where q is the quotient when $\left(3n^3-6n^2+9n-1\right)$ is divided by $2PP_n$ and is given by the greatest integer function of $\dfrac{\left(3n^3-6n^2+9n-1\right)}{2PP_n}$,

That is $q=\left[\dfrac{3n^3-6n^2+9n-1}{2PP_n}\right]$.

Hence ,

$$A(n)=n+n^3-\left\{\left(3n^3-6n^2+9n-1\right)-\left[\frac{\left(3n^3-6n^2+9n-1\right)}{n+n^3}\right]\times n+n^3\right\}$$

$$A(n)=n+n^3-3n^3+6n^2-9n+1+\left[\left(\frac{2\left(n+n^3\right)}{n+n^3}\right)+\frac{n^3-6n^2+7n-1}{n+n^3}\right]\times n+n^3$$

$$A(n)=-2n^3+6n^2-8n+1+\left[2+\frac{n^3-6n^2+7n-1}{n+n^3}\right]\times n+n^3$$

$$A(n)=-2n^3+6n^2-8n+1+2n+2n^3\left(\because 0<\frac{n^3-6n^2+7n-1}{n+n^3}<1\right)$$

$A(n)=6n^2-6n+1=$ centered dodecagonal number.

Section D: Evaluation of tetradecagonal number

Let $A: N\rightarrow N$ be defined by $A(n)=m$ where m is the smallest natural number such that $2PP_n$ divides $m+\left(2n^3-7n^2+9n-1\right)$. If $2PP_n$ divides $\left(2n^3-7n^2+9n-1\right)$, then $A(n)=7n^2-7n+1$, otherwise $A(n)=\left(7n^2-7n+1\right)-r$ where r is the least non negative remainder when $\left(2n^3-7n^2+9n-1\right)$ is divided by $2PP_n$. Hence A is defined for all n. By division algorithm such remainder is given by $\left(2n^3-7n^2+9n-1\right)-2qPP_n$ where q is the quotient when $\left(2n^3-7n^2+9n-1\right)$ is divided by $2PP_n$ and is given by the greatest integer function of $\frac{\left(2n^3-7n^2+9n-1\right)}{2PP_n}$,

That is $q=\left[\frac{2n^3-7n^2+9n-1}{2PP_n}\right]$.

Hence ,

$$A(n)=n+n^3-\left\{\left(2n^3-7n^2+9n-1\right)-\left[\frac{\left(2n^3-7n^2+9n-1\right)}{n+n^3}\right]\times n+n^3\right\}$$

$$A(n)=n+n^3-2n^3+7n^2-9n+1+\left[\left(\frac{n+n^3}{n+n^3}\right)+\frac{\left(n^3-7n^2+8n-1\right)}{n+n^3}\right]\times n+n^3$$

$$A(n)=-n^3+7n^2-8n+1+\left[1+\frac{\left(n^3-7n^2+8n-1\right)}{n+n^3}\right]\times n+n^3$$

$$A(n) = -n^3 + 7n^2 - 8n + 1 + n + n^3 \left(\because 0 < \frac{\left(n^3 - 7n^2 + 8n - 1\right)}{n + n^3} < 1 \right)$$

$A(n) = 7n^2 - 7n + 1 =$ centered tetradecagonal number.

SECTION E: Evaluation of centered hexadecagonal number

Let $A : N \to N$ be defined by $A(n) = m$ where m is the lowest natural number such that $2PP_n$ divides $m + \left(3n^3 - 8n^2 + 11n - 1\right)$. If $2PP_n$ divides $\left(3n^3 - 8n^2 + 11n - 1\right)$, then $A(n) = 8n^2 - 8n + 1$, otherwise $A(n) = \left(8n^2 - 8n + 1\right) - r$ where r is the least non negative remainder when $\left(3n^3 - 8n^2 + 11n - 1\right)$ is divided by $2PP_n$. Hence A is defined for all n. By division algorithm such remainder is given by $\left(3n^3 - 8n^2 + 11n - 1\right) - 2qPP_n$ where q is the quotient when $\left(3n^3 - 8n^2 + 11n - 1\right)$ is divided by $2PP_n$ and is given by the greatest integer function of $\frac{\left(3n^3 - 8n^2 + 11n - 1\right)}{2PP_n}$,

That is $q = \left[\frac{3n^3 - 8n^2 + 11n - 1}{2PP_n} \right]$.

Hence ,

$$A(n) = n + n^3 - \left\{ \left(3n^3 - 8n^2 + 11n - 1\right) - \left[\frac{\left(3n^3 - 8n^2 + 11n - 1\right)}{n + n^3} \right] \times n + n^3 \right\}$$

$$A(n) = n + n^3 - 3n^3 + 8n^2 - 11n + 1 + \left[\left(\frac{2\left(n + n^3\right)}{n + n^3} \right) + \frac{n^3 - 8n^2 + 9n - 1}{n + n^3} \right] \times n + n^3$$

$$A(n) = -2n^3 + 8n^2 - 10n + 1 + \left[2 + \frac{n^3 - 8n^2 + 9n - 1}{n + n^3} \right] \times n + n^3$$

$$A(n) = -2n^3 + 8n^2 - 10n + 1 + 2n + 2n^3 \left(\because 0 < \frac{n^3 - 8n^2 + 9n - 1}{n + n^3} < 1 \right)$$

$A(n) = 8n^2 - 8n + 1 =$ centered hexadecagonal number.

3. Conclusion

In this communication, we find the centered polygonal numbers through the divisibility algorithm. In this manner, one can find the some special numbers through the divisibility algorithm.

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