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Construction of the Diophantine Triple Involving Stella Octangula Number

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Abstract. We search for three distinct polynomials with integer coefficients such that the product of any two numbers increased by a non-zero integer (or polynomials with integer coefficients) is a perfect square.

Keywords: Diophantine triples, Stella Octangula number, Perfect square.

AMS Mathematics Subject Classification (2010): 11D25

1. Introduction

Let n be an integer. A set of positive integers $\{a_1, a_2, \ldots, a_m\}$ is said to have the property $D(n)$ if $a_i a_j + n$ is a perfect square for all $1 \le i < j \le m$; such a set is called a Diophantine m-tuple of size m. The problem of construction of such set was studied by Diophantus. Many mathematicians considered the problem of the existence of Diophantine quadruples with the property $D(n)$ for any arbitrary integer n [1] and also for any linear polynomials n. Further, various authors considered the connections of the problem of Diaphanous, Davenport and Fibonacci numbers in [2-14].

In this communication, we present three sections where in each of which we find the Diophantine triples from Stella Octangula number with different ranks. A few interesting relations among the numbers in each of the above Diophantine triples are presented.

We use Stella Octangula Number of rank n = $n(2n^2 - 1)$

2. Method of analysis

2.1. Section A

Let $a = 2n^3 - 6n^2 + 5n - 1$ and $b = 2n^3 - n$ be Stella Octangula number of rank $n-1$ and n respectively such that $ab + (9n^4 - 16n^3 + 9n^2 - n)$ is a perfect square say α^2.

Let c be any non-zero integer such that

$$ac+\left(9n^4-16n^3+9n^2-n\right)=\beta^2 \qquad (1)$$

$$bc+\left(9n^4-16n^3+9n^2-n\right)=\gamma^2 \qquad (2)$$

Setting $\beta=a+\alpha$ and $\gamma=b+\alpha$, then subtracting (1) from (2), we get

$$c(b-a)=\gamma^2-\beta^2=(\gamma+\beta)(\gamma-\beta)$$

$$=(a+b+2\alpha)(b-a)$$

Thus, we get $c=a+b+2\alpha$

Similarly by choosing $\beta=a-\alpha$ and $\gamma=b-\alpha$, we obtain $c=a+b-2\alpha$

Here we have $\alpha=2n^3-3n^2+2n$ and thus two values of c are given by $c=8n^3-12n^2+8n-1$ and $c=-1$.

Thus, we observe that $\{2n^3-6n^2+5n-1,\ 2n^3-n,\ 8n^3-12n^2+8n-1\}$ and $\{2n^3-6n^2+5n-1,\ 2n^3-n,\ -1\}$ are Diophantine triples with the property $D(9n^4-16n^3+9n^2-n)$.

Some numerical examples are given below in the following table.

Table 1:

n	Diophantine Triples	$D(9n^4-16n^3+9n^2-n)$
1	(0,1,3) and (0,1,-1)	1
2	(1,14,31) and (1,14,-1)	50
3	(14,51,131) and (14,51,-1)	375

We present below, some of the Diophantine triples for Stella Octangula number of rank mentioned above with suitable properties.

Table 2:

a	b	c	$D(n)$
$2n^3-6n^2+5n-1$	$2n^3-n$	$8n^3-12n^2+6n-1$	$5n^4-10n^3+6n^2-n$
		$2n-1$	
$2n^3-6n^2+5n-1$	$2n^3-n$	$8n^3-12n^2+10n-1$	$13n^4-22n^3+14n^2-n$
		$-2n-1$	
$2n^3-6n^2+5n-1$	$2n^3-n$	$8n^3-12n^2+12n-1$	$17n^4-28n^3+21n^2-n$
		$-4n-1$	

2.2. Section B

Let $a=2n^3-12n^2+23n-14$ and $b=2n^3-n$ be Stella Octangula number of rank $n-2$ and n respectively such that $ab+\left(-4n^4+4n^3+24n^2-14n\right)$ is a perfect square say α^2.

Let c be any non-zero integer such that

$$ac+\left(-4n^4+4n^3+24n^2-14n\right)=\beta^2 \qquad (3)$$

$$bc+\left(-4n^4+4n^3+24n^2-14n\right)=\gamma^2 \qquad (4)$$

Applying the procedure as mentioned in section (A), we obtain

$$c=8n^3-24n^2+24n-14 \text{ and } c=20n-14$$

Thus, we observe that

$\{2n^3-12n^2+23n-14,\ 2n^3-n,\ 8n^3-24n^2+24n-14\}$ and $\{2n^3-12n^2+23n-14,\ 2n^3-n,\ 20n-14\}$ are Diophantine triples with the property $D(-4n^4+4n^3+24n^2-14n)$.

Some numerical examples are given below in the following table.

Table 3:

n	Diophantine Triples	$D(-4n^4+4n^3+24n^2-14n)$
1	(-1,1,-6) & (-1,1,6)	10
2	(0,14,2) & (0,14,26)	36
3	(1,51,58) & (1,51,46)	-42

We present below, some of the Diophantine triples for Stella Octangula number of rank mentioned above with suitable properties.

Table 4:

a	b	c	$D(n)$
$2n^3-12n^2+23n-14$	$2n^3-n$	$8n^3-24n^2+26n-14$	$-8n^3+27n^2-14n$
		$18n-14$	
$2n^3-12n^2+23n-14$	$2n^3-n$	$8n^3-24n^2+28n-14$	$4n^4-20n^3+32n^2-14n$
		$16n-14$	
$2n^3-12n^2+23n-14$	$2n^3-n$	$8n^3-24n^2+30n-14$	$8n^4-32n^3+39n^2-14n$
		$14n-14$	

2.3. Section C

Let $a=2n^3-12n^2+23n-14$ and $b=2n^3-6n^2+5n-1$ be Stella Octangula number of rank $n-2$ and $n-1$ respectively such that

$ab+\left(-35n^4+174n^3-202n^2+93n-14\right)$ is a perfect square say α^2.

Let c be any non-zero integer such that

$$ac+\left(-35n^4+174n^3-202n^2+93n-14\right)=\beta^2 \qquad (5)$$

$$bc+\left(-35n^4+174n^3-202n^2+93n-14\right)=\gamma^2 \qquad (6)$$

Applying the procedure as mentioned in section (A), we obtain

$$c = 8n^3 - 36n^2 + 34n - 15 \text{ and } c = 22n - 15$$

Thus, we observe that
$\{2n^3 - 12n^2 + 23n - 14,\ 2n^3 - 6n^2 + 5n - 1,\ 8n^3 - 36n^2 + 34n - 15\}$ and
$\{2n^3 - 12n^2 + 23n - 14,\ 2n^3 - 6n^2 + 5n - 1,\ 22n - 15\}$ are Diophantine triples with the property $D(-35n^4 + 174n^3 - 202n^2 + 93n - 14)$.

Some numerical examples are given below in the following table.

Table 5:

n	Diophantine Triples	$D(-35n^4 + 174n^3 - 202n^2 + 93n - 14)$
1	(-1,0,-9) and (-1,0,7)	16
2	(0,1,-27) and (0,1,29)	196
3	(1,14,-21) and (1,14,51)	310

We present below, some of the Diophantine triples for Stella Octangula number of rank mentioned above with suitable properties.

Table 6:

a	b	c	$D(n)$
$2n^3 - 12n^2 + 23n - 14$	$2n^3 - 6n^2 + 5n - 1$	$8n^3 - 36n^2 + 30n - 15$	$-43n^4 + 210n^3 - 210n^2 + 93n - 14$
		$26n - 15$	
$2n^3 - 12n^2 + 23n - 14$	$2n^3 - 6n^2 + 5n - 1$	$8n^3 - 36n^2 + 32n - 15$	$-39n^4 + 192n^3 - 207n^2 + 93n - 14$
		$24n - 15$	
$2n^3 - 12n^2 + 23n - 14$	$2n^3 - 6n^2 + 5n - 1$	$8n^3 - 36n^2 + 36n - 15$	$-31n^4 + 156n^3 - 195n^2 + 93n - 14$
		$20n - 15$	

3. Conclusion

In this paper, we have presented a few examples of constructing a Diophantine triples for Stella Octangula number of different rank with suitable properties. To conclude one may search for Diophantine triples for other numbers with their corresponding suitable properties.

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