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Connections between Cylinder, Frustum of a Cone with Truncated Octahedral Number and Other Special Numbers

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Abstract. In this paper, we find the relation among the geometrical figures cylinder and frustum of the cone with truncated octahedral number and some special numbers such as gnomonic number and polygonal numbers.

Keywords: Frustum of the cone, Lateral surface area of the cylinder, Truncated Octahedral number

AMS Mathematics Subject Classification (2010): 11D09

1. Introduction

The systematic study of number theory was initiated around 300 B.C. Over a thousand years later around 972 A.D. Arab mathematicians formulated the congruent number problem to decide whether or not a given positive integer n is area of right triangle, all three of whose sides are rationals. Another 1000 years later in 1976 Diffie and Hellman introduced the first ever public-key cryptosystem which enabled to communicate secretly over public communication channels. By then there has been a rapid growth of inventions and the research continues.

For various ideas and problems one may refer [1-5].Various relations between Pythagorean triangles and special numbers are studied in [6-16].Relations between Rectangles and special numbers are studied in [17-18].

Recently in [19] some observations regarding Truncated Octahedral number and in [20] Correlation of cylinder and frustum of the cone with special numbers are recorded.

In this paper, we find the relation among the geometrical figures cylinder and frustum of the cone with truncated octahedral number and some special numbers such as gnomonic number and polygonal numbers.

2. Method of analysis

Let r, h be the base radius and height of the cylinder respectively. Suppose a frustum of the cone with the same height 'h' of the cylinder and the circular end at the bottom of this frustum coincides with the base of the cylinder is attached on either of its base.

This means that the base radius of the cylinder is equal to the radius of the circular end at the bottom of the frustum. Let R be the radius of the circular end at the top of the frustum.

Assume that three times of the difference between volume of the frustum of the cone and lateral surface area of the cylinder is equal to

$\pi[TO_h + T_{76,h} - 2T_{6,h} - Gno_h + 5]$

The mathematical statement of our assumption is

$$\pi h\ [R^2 + r^2 + Rr - 6r] = \pi h\ [16h^2 - 12] \quad (1)$$

This reduces to

$$R^2 + r^2 + Rr - 6r + 12 = 16h^2 \quad (2)$$

Introduce the linear transformation

$$\mathrm{R = u + v\,;\ r = u - v} \quad , \quad u \neq 0, v \neq 0 \quad (3)$$

Therefore (2) reduces to

$$3x^2 + y^2 = z^2 \quad (4)$$

where $x = u - 1 \quad ; \quad y = v + 3 \quad ; \quad z = 4h$ (5)

We present below different methods of solving (4) and thus obtain different choices of non-zero integral values of the radii of the circular ends at the top and bottom of the frustum and the height of the cylinder cum frustum

Pattern 1:

Most cited solution to (4) is

$$x = 2ab; y = 3a^2 - b^2; z = 3a^2 + b^2 \quad (6)$$

In view of (5) and (6)

$$\left.\begin{aligned} u &= 2ab + 1 \\ v &= 3a^2 - b^2 - 3 \\ h &= \frac{3a^2 + b^2}{4} \end{aligned}\right\} \quad (7)$$

Since our aim is to find integer solution to height and radius, take

$\mathrm{a = 2A; b = 2B.}$

Therefore we have,

$$u = 8AB + 1 \quad (8)$$

$$v = 12A^2 - 4B^2 - 3 \quad (9)$$

$$h = 3A^2 + B^2 \quad (10)$$

Using (8) and (9) in (3), the non-zero integral values of the radii of the circular ends at the top and bottom of the frustum and the height of the cylinder cum frustum satisfying (1) are given by

$$R = 12A^2 - 4B^2 + 8AB - 2$$

$$r = -12A^2 + 4B^2 + 8AB + 4$$

$h = 3A^2 + B^2$

Some numerical examples satisfying our assumption are tabulated below

Table1

s.no	a	b	A	B	R	r	h	L.H.S of (1)	R.H.S of (1)
1	1	2	2	4	46	84	28	12544	12544
2	2	2	4	4	254	4	64	65536	65536
3	2	3	4	6	238	148	84	112896	112896
4	3	4	6	8	558	212	172	473344	473344
5	4	5	8	10	1006	276	292	1364224	1364224

Pattern 2:

Let $z = a^2 + 3b^2$ (11)

Therefore (4) becomes $(a^2 + 3b^2)^2 = y^2 + 3x^2$

Using factorization define

$$[(a + i\sqrt{3}b)(a - i\sqrt{3}b)]^2 = (y + i\sqrt{3}x)(y - i\sqrt{3}x)$$

Equating real and imaginary we get

$$x = 2ab; y = a^2 - 3b^2; \quad (12)$$

In view of (5) and (12)

$$\left.\begin{aligned} u &= 2ab + 1 \\ v &= a^2 - 3b^2 - 3 \\ h &= \frac{a^2 + 3b^2}{4} \end{aligned}\right\} \quad (13)$$

Since our aim is to find integer solution to height and radius, take

a = 2A;b = 2B.

Therefore we have,

$$u = 8AB + 1 \quad (14)$$

$$v = 4A^2 - 12B^2 - 3 \quad (15)$$

$$h = A^2 + 3B^2 \quad (16)$$

Using (14) and (15) in (3), the non-zero integral values of the radii of the circular ends at the top and bottom of the frustum and the height of the cylinder cum frustum satisfying (1) are given by

$$R = 4A^2 - 12B^2 + 8AB - 2$$

$$r = -4A^2 + 12B^2 + 8AB + 4$$

$$h = A^2 + 3B^2$$

Some numerical examples satisfying our assumption are tabulated below.

Table2

s.no	a	b	A	B	R	r	h	L.H.S of (1)	R.H.S of (1)
1	3	2	6	4	142	244	84	112896	112896
2	6	4	12	8	574	964	336	1806336	1806336
3	6	5	12	10	334	1588	444	3154176	3154176
4	12	11	24	22	718	7732	2028	65804544	65804544
5	13	9	26	18	2558	4932	1648	43454464	43454464

Pattern 3:

Write (4) as

$$z^2 = 1.\,(y^2 + 3x^2) \tag{17}$$

Taking z as in (11), write 1= $\dfrac{(1+i\sqrt{3})(1-i\sqrt{3})}{2^2}$

Using factorization define (17) as

$$[(a + i\sqrt{3}b)(a - i\sqrt{3}b)]^2 = \frac{(1+i\sqrt{3})(1-i\sqrt{3})}{2^2}[(y+i\sqrt{3}x)(y-i\sqrt{3}x)]$$

Equating real and imaginary parts we have,

$$\begin{aligned} x &= \frac{1}{2}[a^2 - 3b^2 + 2ab] \\ y &= \frac{1}{2}[a^2 - 3b^2 - 6ab] \end{aligned} \tag{18}$$

In view of (5) and (18)

$$\left.\begin{aligned} u &= \frac{1}{2}[a^2 - 3b^2 + 2ab + 2] \\ v &= \frac{1}{2}[a^2 - 3b^2 - 6ab - 6] \\ h &= \frac{a^2 + 3b^2}{4} \end{aligned}\right\} \tag{19}$$

Since our aim is to find integer solution to height and radius, take a = 2A;b = 2B

Therefore we have,

$$\left.\begin{aligned} u &= 2A^2 - 6B^2 + 4AB + 1 \\ v &= 2A^2 - 6B^2 - 12AB - 3 \end{aligned}\right\} \tag{20}$$

$$h = A^2 + 3B^2 \tag{21}$$

Using (20) in (3) with (21), the non-zero integral values of the radii of the circular ends at the top and bottom of the frustum and the height of the cylinder cum frustum satisfying (1) are given by

$$R = 4A^2 - 12B^2 - 8AB - 2$$

$$r = -16AB + 4$$

$$h = A^2 + 3B^2$$

Some numerical examples satisfying our assumption are tabulated below.

Table 3									
s.no	a	b	A	B	R	R	H	L.H.S of (1)	R.H.S of (1)
1	6	1	12	2	334	388	156	389376	389376
2	7	1	14	2	510	452	208	692224	692224
3	8	2	16	4	318	1028	304	1478656	1478656
4	9	2	18	4	526	1156	372	2214144	2214144
5	10	2	20	4	766	1284	448	3211264	3211264

Pattern 4:

Write (4) as in (17)

Taking z as in (11) and write $1 = \frac{(1+i4\sqrt{3})(1-i4\sqrt{3})}{7^2}$

Using factorization define (17) as

$$[(a + i\sqrt{3}b)(a - i\sqrt{3}b)]^2 = \frac{(1+i4\sqrt{3})(1-i4\sqrt{3})}{7^2}[(y+i\sqrt{3}x)(y-i\sqrt{3}x)]$$

Equating real and imaginary parts we have

$$\left.\begin{aligned} x &= \frac{1}{7}[4a^2 - 12b^2 + 2ab] \\ y &= \frac{1}{7}[a^2 - 3b^2 - 24ab] \end{aligned}\right\} \tag{22}$$

In view of (5) and (22)

$$\left.\begin{aligned} u &= \frac{1}{7}[4a^2 - 12b^2 + 2ab + 7] \\ v &= \frac{1}{7}[a^2 - 3b^2 - 24ab - 21] \\ h &= \frac{1}{4}[a^2 + 3b^2] \end{aligned}\right\} \tag{23}$$

Since our aim is to find integer solution to height and radius, take

$a = 14A; b = 14B.$

Therefore we have,

$$\left.\begin{aligned} u &= 112A^2 - 336B^2 + 56AB + 1 \\ v &= 28A^2 - 84B^2 - 672AB - 3 \end{aligned}\right\} \quad (24)$$

$$h = 49A^2 + 147B^2 \quad (25)$$

Using (24) in (3) with (25), the non-zero integral values of the radii of the circular ends at the top and bottom of the frustum and the height of the cylinder cum frustum satisfying (1) are given by

$$R = 140A^2 - 420B^2 - 616AB - 2$$

$$r = 84A^2 - 252B^2 + 728AB + 4$$

$$h = 49A^2 + 147B^2$$

Some numerical examples satisfying our assumption are tabulated below.

Table4

a	b	A	B	R	r	h	L.H.S of (1)	R.H.S of (1)
35	5	490	70	10427198	43904004	12485200	2494083504640000	2494083504640000
12	2	168	28	724414	5597764	1498224	35914802466816	35914802466816
39	4	546	56	21584302	46510804	15068676	3633039942287620	3633039942287620
49	5	686	70	34245118	73253828	23779504	9047436967776260	9047436967776260
7	1	98	14	417086	1756164	499408	3990533607424	3990533607424

3. Conclusion

In this paper, we expose connections between cylinder, frustum of a cone with truncated octahedral number and other special numbers and presented four patterns of solutions. In this approach, one may also search for various patterns of solutions and compare some other Geometrical figures with figurate numbers.

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