Journal of Mathematics and Informatics
Vol. 11, 2017, 119-123
ISSN: 2349-0632 (P), 2349-0640 (online)
Published 11 December 2017
www.researchmathsci.org
DOI: http://dx.doi.org/10.22457/jmi.v11a15

Journal of Mathematics and Informatics

Special Dio 3-tuples for Pentatope Number

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Received 6 November 2017; accepted 9 December 2017

Abstract. We search for three distinct polynomials with integer coefficients such that the product of any two members of the set added with their sum and increased by a non-zero integer (or polynomial with integer coefficients) is a perfect square.

Keywords: Dio 3-tuples, Pentatope number, polynomials.

AMS Mathematics Subject Classification (2010): 11D25

1. Introduction

Mathematics is the language of patterns and relationships, and is used to describe anything that can be quantified. The main goal of Number theory is to discover interesting and unexpected relationships. It is devoted primarily to the study of natural numbers and integers. In [1-7], theory of numbers were discussed. Many mathematicians considered the problem of the existence of Diophantine triples & special dio 3-tuples with the property $D(n)$ for any arbitrary integer n and also for any linear polynomials n [8-10].

In this communication, we present a few special dio 3-tuples for Pentatope numbers of different ranks with their corresponding properties.

Notation

$$\mathrm{PT}_n = \text{Pentatope number of rank } n = \frac{1}{24}\, n(n+1)(n+2)(n+3)$$

2. Basic definition

A set of three distinct polynomials with integer coefficients (a_1, a_2, a_3) is said to be a special dio 3-tuple with property $D(n)$ if $a_i * a_j + (a_i + a_j) + n$ is a perfect square for all $1 \le i < j \le 3$, where n may be non-zero integer or polynomial with integer coefficients.

3. Method of analysis

Case 1: Construction of Dio 3-tuples for Pentatope number of rank n and $n-2$.

Let $a = 24\ \mathrm{PT}_n$, $b = 24\ \mathrm{PT}_{n-2}$ be Pentatope numbers of rank n and $n-2$ respectively such that $ab + (a+b) + 1$ is a perfect square say α^2.

Let c be any non-zero integer such that

$$ac+(a+c)+1=\beta^2 \tag{1}$$

$$bc+(b+c)+1=\gamma^2 \tag{2}$$

On solving equations (1) and (2), we get

$$(b+1)\beta^2-(a+1)\gamma^2=0 \tag{3}$$

Assume $\beta=x+(a+1)y$ and $\gamma=x+(b+1)y$, (4)

it reduces to $x^2=(a+1)(b+1)y^2$ (5)

The initial solution of the equation (5) is given by

$$x_0=n^4+2n^3-3n^2-4n-1,\quad y_0=1$$

Therefore, $\beta=2n^4+8n^3+8n^2+2n$

On substituting the values of a and β in equation (1), we get

$$c=4n^4+8n^3+4n^2-1=6\,\mathrm{PT}_{2n}-(4n^3+7n^2+3n+1)$$

Hence, The triple $\left(24\,\mathrm{PT}_n, 24\,\mathrm{PT}_{n-2}, 6\,\mathrm{PT}_{2n}-(4n^3+7n^2+3n+1)\right)$ is a Dio 3-tuple with property $D(1)$.

A few numerical examples of the Dio 3-tuples satisfying the above property are mentioned below.

Table 1:

n	Dio 3-tuples with property $D(1)$
1	(24, 0, 15)
2	(120, 0, 143)
3	(360, 24, 575)
4	(840, 120, 1599)
5	(1680, 360, 3599)

We present below, some of the Dio 3-tuple for Pentatope number of rank mentioned above with suitable properties.

Table 2:

a	b	c	$D(n)$
24 PT_n	24 PT_{n-}	$6\,\mathrm{PT}_{2n}-(4n^3+7n^2+3n-1)$	$D\left(2n^4+4n^3-6n^2-8n\right)$
24 PT_n	24 PT_{n-}	$6\,\mathrm{PT}_{2n}-(4n^3+7n^2+3n-3)$	$D\left(4n^4+8n^3-12n^2-16n+1\right)$
24 PT_n	24 PT_{n-}	$6\,\mathrm{PT}_{2n}-(4n^3+7n^2+3n-5)$	$D\left(6n^4+12n^3-18n^2-24n+4\right)$
24 PT_n	24 PT_{n-}	$6\,\mathrm{PT}_{2n}-(4n^3+7n^2+3n-7)$	$D\left(8n^4+16n^3-24n^2-32n+9\right)$
24 PT_n	24 PT_{n-}	$6\,\mathrm{PT}_{2n}-(4n^3+7n^2+3n-9)$	$D(10n^4+20n^3-30n^2-40n+1$

In general, it is noted that the triple

$$\left(24\ \mathrm{PT_n},\ 24\ \mathrm{PT_{n-2}},\ 6\,\mathrm{PT_{2n}} - (4n^3 + 7n^2 + 3n - (2k-1))\right)$$

is a Dio 3-tuple with the property $D\left(2kn^4 + 4kn^3 - 6kn^2 - 8kn + (k-1)^2\right)$, where $k = 0,1,2,\ldots$

Case 2: Construction of Dio 3-tuples for Pentatope number of rank n and $n-1$.

Let $a = 24\ \mathrm{PT}_n$, $b = 24\ \mathrm{PT}_{n-1}$ be Pentatope number of rank n and $n-1$ respectively such that $ab + (a+b) + 1$ is a perfect square say α^2.

Let c be any non-zero integer such that

$$ac + (a+c) + 1 = \beta^2 \tag{6}$$

$$bc + (b+c) + 1 = \gamma^2 \tag{7}$$

Solving (6), (7) and using (4), we have

$$x^2 = (a+1)(b+1)y^2 \tag{8}$$

The initial solution of the equation (8) is given by

$$x_0 = n^4 + 4n^3 + 3n^2 - 2n - 1, \quad y_0 = 1$$

Therefore, $\beta = 2n^4 + 10n^3 + 14n^2 + 4n$

On substituting the values of a and β in equation (6), we get

$$c = 4n^4 + 16n^3 + 16n^2 - 1 = 6\,\mathrm{PT_{2n}} + (4n^3 + 5n^2 - 3n - 1)$$

Hence, the triple $\left(24\ \mathrm{PT_n},\ 24\ \mathrm{PT_{n-1}},\ 6\,\mathrm{PT_{2n}} + (4n^3 + 5n^2 - 3n - 1)\right)$ is a Dio 3-tuple with property $D(1)$.

A few numerical examples of the Dio 3-tuples satisfying the above property are mentioned below.

Table 3:

n	Dio 3-tuples with property $D(1)$
1	(24, 0, 35)
2	(120, 24, 255)
3	(360, 120, 899)
4	(840, 360, 2303)
5	(1680, 840, 4899)

We present below, some of the Dio 3-tuple for Pentatope number of rank mentioned above with suitable properties.

Table 4:

a	b	c	$D(n)$
$24\ \mathrm{PT}_n$	$24\ \mathrm{PT}_{n-1}$	$6\,\mathrm{PT}_{2n}+(4n^3+5n^2-3n+1)$	$D\left(2n^4+8n^3+6n^2-4n\right)$
$24\ \mathrm{PT}_n$	$24\ \mathrm{PT}_{n-1}$	$6\,\mathrm{PT}_{2n}+(4n^3+5n^2-3n+3)$	$D\left(4n^4+16n^3+12n^2-8n+1\right)$
$24\ \mathrm{PT}_n$	$24\ \mathrm{PT}_{n-1}$	$6\,\mathrm{PT}_{2n}+(4n^3+5n^2-3n+5)$	$D\left(6n^4+24n^3+18n^2-12n+4\right)$
$24\ \mathrm{PT}_n$	$24\ \mathrm{PT}_{n-1}$	$6\,\mathrm{PT}_{2n}+(4n^3+5n^2-3n+7)$	$D\left(8n^4+32n^3+24n^2-16n+9\right)$
$24\ \mathrm{PT}_n$	$24\ \mathrm{PT}_{n-1}$	$6\,\mathrm{PT}_{2n}+(4n^3+5n^2-3n+9)$	$D\left(10n^4+40n^3+30n^2-20n+1\right.$

In general, it is noted that the triple $\left(24\ \mathrm{PT}_n, 24\ \mathrm{PT}_{n-1}, 6\,\mathrm{PT}_{2n}+(4n^3+5n^2-3n+(2k-1))\right)$ is a Dio 3-tuple with the property $D\left(2k\,n^4+8k\,n^3+6k\,n^2-4k\,n+(k-1)^2\right)$, where $k=0,1,2,\dots$

Case 3: Construction of Dio 3-tuples for Pentatope number of rank n and $n-3$.

Let $a=24\ \mathrm{PT}_n$, $b=24\ \mathrm{PT}_{n-3}$ be Pentatope number of rank n and $n-3$ respectively such that $ab+(a+b)+1$ is a perfect square say α^2.

Proceeding as in case1, we have the initial solution of the equation as

$$x_0=n^4-7n^2+1,\quad y_0=1$$

Therefore, $$\beta=2n^4+6n^3+4n^2+6n+2$$

On substituting the values of a and β, we get

$$c=4n^4+8n^2+3=6\,\mathrm{PT}_{2n}-(12n^3+3n^2+3n-3)$$

Hence, The triple $\left(24\ \mathrm{PT}_n, 24\ \mathrm{PT}_{n-3}, 6\,\mathrm{PT}_{2n}-(12n^3+3n^2+3n-3)\right)$ is a Dio 3-tuple with property $D(1)$.

A few numerical examples of the Dio 3-tuples satisfying the above property are mentioned below.

Table 5:

n	Dio 3-tuples with property $D(1)$
1	(24, 0, 15)
2	(120, 0, 99)
3	(360, 0, 399)
4	(840, 24, 1155)

5	(1680, 120, 2703)

We present below, some of the Dio 3-tuple for Pentatope number of rank mentioned above with suitable properties.

Table 6:

a	b	c	$D(n)$
$24\,\mathrm{PT}_n$	$24\,\mathrm{PT}_{n-3}$	$6\,\mathrm{PT}_{2n}-(12n^3+3n^2+3n-5)$	$D(2n^4-14n^2+4)$
$24\,\mathrm{PT}_n$	$24\,\mathrm{PT}_{n-3}$	$6\,\mathrm{PT}_{2n}-(12n^3+3n^2+3n-7)$	$D(4n^4-28n^2+9)$
$24\,\mathrm{PT}_n$	$24\,\mathrm{PT}_{n-3}$	$6\,\mathrm{PT}_{2n}-(12n^3+3n^2+3n-9)$	$D(6n^4-42n^2+16)$
$24\,\mathrm{PT}_n$	$24\,\mathrm{PT}_{n-3}$	$6\,\mathrm{PT}_{2n}-(12n^3+3n^2+3n-11)$	$D(8n^4-56n^2+25)$
$24\,\mathrm{PT}_n$	$24\,\mathrm{PT}_{n-3}$	$6\,\mathrm{PT}_{2n}-(12n^3+3n^2+3n-13)$	$D(10n^4-70n^2+36)$

In general, it is noted that the triple $\left(24\,\mathrm{PT}_n, 24\,\mathrm{PT}_{n-3}, 6\,\mathrm{PT}_{2n}-(12n^3+3n^2+3n-(2k+3))\right)$ is a Dio 3-tuple with the property $D\left(2k\,n^4-14k\,n^2+(k+1)^2\right)$, where $k=0,1,2,\ldots$

4. Conclusion

In this paper we have presented a few examples of constructing a special Dio 3-tuples for Pentatope number of different ranks with suitable properties. To conclude one may search for Dio 3-tuples for other numbers with their corresponding suitable properties.

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