Journal of Mathematics and Informatics
Vol. 11, 2017, 143-146
ISSN: 2349-0632 (P), 2349-0640 (online)
Published 11 December 2017
www.researchmathsci.org
DOI: http://dx.doi.org/10.22457/jmi.v11a18

Journal of
Mathematics and Informatics

Stability Analysis by Lyapunov Function in Neural Network

***S.Mehar Banu*[1] and *S.Ramadevi*[2]**

[1]Department of Mathematics, Government Arts College (Autonomous)
Salem, Tamilnadu, India. E-mail: meharizh@gmail.com
[2]Department of Mathematics
Vivekanandha College of Arts and Sciences for Women (Autonomous)
Elayampalayam, Thiruchengode-637205, Namakkal, Tamilnadu, India.
e-mail: ramadevimurugan@gmail.com

Received 3 November 2017; accepted 9 December 2017

Abstract. This paper is connected with the stability analysis of Discrete-Time recurrent neural networks (RNNs) with time delays as random variable drawn from some probability distribution. By introducing the variation probability of the time delay, a common delayed Discrete-Time RNN system is transformed into one with stochastic parameters. Improved condition for the mean square stability of these systems are obtained by employing new Lyapunov function and more techniques are used to achieves delay dependence. The merit of the proposed condition lies in its reduced conservation which is made possible by considering not only the range of the time delays but also the variation probability distribution.

Keywords : Stability, Lyapunov function, dynamical system

AMS Mathematics Subject Classification (2010): 34D08

1. Introduction

In the theory of ODEs Lyapunov functions are scalar function that may be used to prove the stability of an equilibrium of an ODE, named after the Russian Mathematician Aleksandermikhaulovich Lyapunov. Who published his book the general problem of stability of motion in 1892. Lyapunov was the first person to consider the modifications which are necessary in non linear system to the linear theory of stability based on linearizing near a point of equilibrium.

2. Preliminaries

2.1. Definition of Lyapunov function

A Lyapunov function for an autonomous dynamical system

$$\begin{cases} G: \ \mathrm{R}^n \to \mathrm{R}^n \\ \quad \dot{y} = g(y) \end{cases}$$

With equilibrium point at y=0 is a scalar function $V:R^n \to R$ that is continuous, has continuous derivatives, is locally positive definite and for which $-\nabla v.g$ is also locally positive definite. The condition that–∇v.g is locally positive definite is sometimes stated as∇ v.g is locally negative definite.

Locally asymptotically stable equilibrium
If V is a lyapunov function, then the equilibrium is lyapunov stable. The converse is also true and was proved by J.L Massena.

Stable equilibrium
If the lyapunov-candidate –function V is locally positive definite and the time derivative of the lyapunov-candidate-function is locally negative semidefinite.$\dot{V}$ (x)<0 $\forall$ x$\in$B/{0}, for some neighborhood B of origin then the equilibrium is proven to be locally asymptotically stable.

Globally asymptotically stable equilibrium
If the lyapunov–candidate function V is globally positive definite ,radially unbounded and the time derivative of the lyapunov-candidate-function is globally negative definite $\dot{V}$ (x)<0 $\forall$ x$\in R^n\setminus\{0\}$ then the equilibrium is prove to be globally asymptotically stable.The lyapunov –candidate –function V(x) is radially unbounded if $\|x\|\longrightarrow \infty \Rightarrow v(x)\longrightarrow \infty$

Example: Consider the following differential equation with solution x on R : $\dot{x}$=-x.

Consider that x^2 is always positive around the origin it is a natural candidate to be a lyapunov function to help us study x. So let v(x) =x^2 on R. then $\dot{V}$ (x) = v'(x)=f(x) =2x.(-x)=-2.

This correctly shows that the above differential equation, x is asymptotically stable about the origin. Note that using the same lyapunov candidate one can show that the equilibrium is also globally asymptotically stable.

These are important to stability theory of dynamical system (1) and control theory. A similar concept appear in the theory of general state space markov chains, usually under the name foster-lyapunov functions. For certain classes of ODEs the existence of lyapunov function is a necessary and sufficient condition for stability. whereas there is no general technique for constructing lyapunov function for ODEs, in many specific cases, the construction of lyapunov function is known. For instance quadratic functions suffice for systems with one state ; the solution of a particular linear matrix inequality provider lyapunovfunctions.for linear system and conservation laws can often be used to construct lyapunov function for physical system.

Lyapunov stability
This is about the non linear systems along with asymptotic stability. For stability of linear system, we can consider the exponential stability.

Various types of stability may be discussed for the solution of difference (or) differential equation describing dynamical system (2). The most important type is that concerning the solution has the stability near to a equilibrium point. This can be get the results by the theory of lyapunov. In simple terms if the solution that start out near an equilibrium point x_e stay near x_e forever, then we can say thatx_e is lyapunov stable.
More strongly, if $x_e$is lyapunov stable and all solutions start near byx_e converge to x_e then x_e is asymptotically stable. The motion of exponential stability gives a minimal rate

of decay. i.e.,an estimate of how quickly the solution converge. The idea of lyapunov stability can e extended to infinite dimentional manifolds, it is known as structural stability.

The “second method of lyapunov was found to be applicable to the stability of aerospace guidance system.The lyapunov exponent which received wide interest in connection with chaos theory (3).

3. Main results

3.1. Lyapunov –Krasovskii theory

As in the study of systems without delay, an effective method for determining the stability of a time-delay system is Lyapunov method for a system without delay, this requires the construction of a lyapunov function V(t,x(t)),which is some sense is a potential measure quantifying the deviation of the state x(t) from the trivial solution O. Since for a delay-free system x(t) is needed to specify the systems future evolution beyond t, and since in a time –delay system the “state” at time t required for the same purpose is the value of x(t) in the interval [t-r, t].i.e.,x_t it is netural to expect that for a time delay system,the corresponding lyapunov function be a functional V(t, x_t) depending on x_t which also should measure the deviation of x_t from the trivial solution O. Such a functional is known as a lyapunov-krasovskii functional.

3.2. Main result

Definition for continuous- time system:

Consider an autonomous nonlinear dynamical system$\dot{x}$ = g(x(t)), x(0)=x_0, when x(t)$\in$ D$\leq R^n$ denotes the system state vector, D an open set containing the origin and g:D$\rightarrow R^n$ continuous on D. suppose g has an equilibrium at x_e so that f(x_e)=0 then

(1) The type of equilibrium is said to be Lyapunov stable, if for every $\in$>0, there exists a δ>0 such that, if$\|x(0)-x_e\| < \delta$ then for every t $\geq$0 we have $\|x(0)-x_e\|<\in$.

(2) The equilibrium of the above system is said to be asymptotically stable, if it is lyapunov stable and there exists δ>0 such that if $\|x(0)-x_e\|<\delta$ then$\lim_{t\to\infty}\left\|x(t)-x_e\right\| = 0$.

(3) The equilibrium of the above system is said to be exponentially stable if it is asymptotically stable and there exist α >0, β>0, δ >0 such that if$\|x(0)-x_e\|<\delta$ then $\|x(t)-x_e\| \leq \alpha\|x(0)-x_e\|e^{-\beta t}$; for all t$\geq$0.

Following are the meaning of the above.

(1) Lyaounov stability of an equilibrium means that solutions starting ”close enough” to the equilibrium remains “close enough” forever.

(2) Asymptotic stability means that solution that start close enough not only remains close enough but also eventually converge to the equilibrium.

(3) Exponential stability means that the solution not only converge, but in fact converge faster than or at least as fast as a particular known rate$\alpha\|x(0)-x_e\|e^{-\beta t}$.

Definition for discrete-time system

The definition for discrete – time system (4) is almost identical to that for continuous-time systems.

Let (x,d) be a matrix space and f:x$\rightarrow$x a continuous function. A point x in X is said to be lyapunov stable,if$\forall \in$>0 $\exists\delta$> 0$\forall$ $y\epsilon\, x[d(x,y) < \delta \Rightarrow$ we say that x is asymptotically stable if it belongs to the interior of its stable set i.e., if $\exists\, \delta > 0[d(x, y) < \delta \Rightarrow \lim_{n\to\infty} d(f^n\ (x)]$

4. Conclusion

This is about the non linear system along with asymptotic stability. For stability of linear system we can consider exponential stability for that we are in need of equilibrium, stochastic delay with control theory in time delay differential equation. We can analysis the control theory of neural network with the help of lyapunov function.

REFERENCES

1. N.P.Bhatia, et al., Stability theory of dynamical systems, Springer, (2002).
2. G.Teschi, ODEs and Dynamical systems Providence: American Mathematical Society, (2012).
3. S.Wiggins, Introduction to Applied Nonlinear Dynamical Systems and Chaos (2nd edition) New York city: Springer Verlag, (2003).