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Hexagonal Numbers and Pythagorean Triangles

P.Jayakumar [1] **and** ***G.Shankarakalidoss*** [2]

[1]Department of Mathematics, A.V.V.M. Sri Pushpam College (Autonomous)
Poondi, Thanjavur-613 503
Email: pjkumar58@gmail.com
[2]Department of Mathematics, Kings College of Engineering
Punalkulam, Pudukkottai (Dt) -613 303
Email: shankarakalidoss@yahoo.com

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Abstract. The oblong numbers were first studied by Pythagorean. These numbers are studied in terms of special Pythagorean Triangles. The perimeters of such triangles are obtained as a double of hexagonal numbers. Existence of Pythagorean triangles with two consecutive sides and their perimeters as a double of hexagonal numbers is also investigated.

Key words: Hexagonal numbers, Pythagorean Triangles.

AMS Mathematics Subject Classification (2010): 11D09

1. Introduction

In 2005, Gopalan and Devibala [2] studied Special Pythagorean triangle. In 2008, Gopalan and Janaki [3] investigated Pythagorean triangles with perimeter as a pentagonal number. In 2010, Gopalan and Vijayalakshmi [1] observed Special Pythagorean triangles generated through the integral solutions of the equation $y^2 = (k^2 + 1)\, x^2 + 1$. After that Mita [4] investigated about oblong numbers and Pythagorean triangles. He found that perimeter of the Pythagorean triangles are as oblong numbers.

Inspired by all the aforementioned results, this paper aim to study the perimeter of the Pythagorean triangles are as a double of hexagonal numbers.

2. Method of analysis

The primitive solutions of the Pythagorean Equation,

$X^2 + Y^2 = Z^2$, is given by [5] (1)

$X = m^2 - n^2,\ Y = 2\,m\,n,\ Z = m^2 + n^2$ (2)

for some integers m, n of opposite parity such that $m > n > 0$ and $(m, n) = 1$

3. Perimeter is a double of hexagonal number

Definition 3.1. A natural number 'h' is called a double of hexagonal numbers if it can be written in the form $2u\,(2\,u - 1),\ u \in N$.

If the perimeter of the Pythagorean triangles (X, Y, Z) are as a double of hexagonal numbers 'h', then

$X + Y + Z = 2u(2u - 1) = h$ (3)

From the equations (2) & (3)

$2m^2 + 2mn = 2u(2u - 1), u \varepsilon N.$

$m(m + n) = u(2u - 1)$ (4)

4. Hypotenuse and one leg are consecutive

In such cases, $m = n + 1$ (5)

This gives equation (4) as

$(n + 1)(2n + 1) = u(2u - 1)$

Take $u = n + 1$ (6)

Equations (2), (5) and (6) give solution of equations (1) in correspondence with equations (3) and (4) i.e.,

$X = 2n + 1;$

$Y = 2n(n + 1);$

$Z = 2n(n + 1) + 1;$

First ten such special Pythagorean triangles (X, Y, Z) are given in the Table 4.1 below:

S. No.	n	u	h	X	Y	Z	X + Y + Z = 2u (2u-1)
1	1	2	12	3	4	5	12=2.2.3
2	2	3	30	5	12	13	30=2.3.5
3	3	4	56	7	24	25	56=2.4.7
4	4	5	90	9	40	41	90=2.5.9
5	5	6	132	11	60	61	132=2.6.11
6	6	7	182	13	84	85	182=2.7.13
7	7	8	240	15	112	113	240=2.8.15
8	8	9	306	17	144	145	306=2.9.17
9	9	10	380	19	180	181	380=2.10.19
10	10	11	462	21	220	221	462=2.11.21

Table 4.1: Special Pythagorean Triangles and Verification of $X + Y + Z = 2u(2u-1)$.

5. Remarkable observations

1. $(2X - Y + Z)^2 = X^2 + 2(X + Y + Z) + 2(X + Z)$
2. $(3X + Z - Y)^2 = 3[3(Y + Z) + 2X] + Z – Y$

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