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Time Minimizing Multi-Index Bulk Transportation Problem

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Abstract. In industries, usually, two or more than two types of commodities are manufactured to maximize the profit. Sometimes, commodities are supplied to multiple warehouses via different modes of transportation. To deal such type of problems in transportation, transportation problems are extended into Multi-Index Transportation Problems(MITP). In this paper, Multi-Index Bulk Transportation problem(MIBTP) which is anextension of Bulk Transportation Problem(BTP) having two modes of transportation with time minimizing objective is considered.In the present paper, VAM is extended to study the time minimizing MIBTP already studied by Latha [23] and a comparative study is done on the existing method and proposed amethod.

Keywords: Transportation, Bulk Transportation, Multi-Index Bulk Transportation

AMS Mathematics Subject Classification (2010): 90C08

1. Introduction

Transportation model is a special class of the linear programming problem that deals with the distribution ofnumber of units of a commodityfrom multiple warehouses to multiple destinations satisfying supply and demand constraints with cost minimizing objective. Transportation model plays an important role in supply-chain management and logistics for reducing the cost of transportation in industries. The classical transportation problem was presented first by Hitchcock [1]. Later on, Koopmans [2] and Dantzig [3] further developed the theory of transportation problem. Several other authors, like, Balas [4], Bhatia et al. [5], Hammer [6], Szwarc [7], Hakim [8], Ahmed et al. [9] and Pramila and Uthra [10] using different approaches for solving the transportation problems.

Bulk transportation problem (BTP) is a special class of transportation problem in which entire requirement of a destination should be met from only one source; however, a source can supply to any number of destinations depending on the availability ofnumber of units of the product. Maio and Roveda [11] were the first who presented the theory of BTP with cost minimizing objective. The authors solved the problem by an iterative procedure. Later on, Srinivasan and Thompson[12] presented an algorithm based on branch and bound method for solving the earlier problem. Several other authors like Bhatia [13], Foulds and Gibbons [14] and Verma and Puri [15] studied the different BTPusing different approaches.

To maximize the profit, generally, two or more than two types of products are manufactured in factories and transported to various destinations to meet their requirements. Sometimes, products are supplied to various destinations through different modes of transportation.To optimize the profit in all such cases, transportation problems are extended intoMITP. The problem is extended into MITP by considering commodities or modes of transportation as additional indices.MITP was first introduced by Schell [16] and Galler and Dwyer [17] in literature. Several other authorslike Haley [18], Junginer [19], Korsnikov [20], Pandian and Anuradha [21], Patel and Tripathy [22] studied MITP using different approaches. Latha [23] studied MIBTP with time minimizing objective using Lexi-Search algorithm.

In the present paper, the MIBTP solved by Latha [23] is considered and an algorithm is proposed for minimizing thetime of MIBTP. In Section 2, the formulation of the problem is discussed and in Section 3, steps of the proposed algorithm are presented. In Section 4, a numerical example is worked out to illustrate the proposed algorithm. Finally, in section 5, theconclusion is presented.

2. Formulation of the problem

Let there are‘m’ sources producing a certain product and ‘n’ destinations having some requirements. Let the products are supplied to multiple destinations through two modes of transportation to meet their requirements.Let ‘T’ denotes the total time of MIBTP. The mathematical formulation of time minimizing MIBTP is as follows:

Minimize

$$T = \max\{ t_{ijk}: x_{ijk} = 1, i = 1,2, \dots, m; j = 1,2, \dots., n; k = 1,2\} \tag{1}$$

subject to the constraints

$$\sum_{j=1}^{n} \sum_{k=1}^{2} b_j x_{ijk} \leq a_i \tag{2}$$

$$\sum_{i=1}^{m} \sum_{k=1}^{2} x_{ijk} = 1 \tag{3}$$

$$x_{ijk} = 1 \ or \ 0 \tag{4}$$

$$x_{i_1 j_1 k_1} = x_{i_2 j_2 k_2} = 1 \text{ for } i_1 = i_2, j_1 \neq j_2 \ and \ k_1 \neq k_2 \tag{5}$$

where $a_i, b_j \ and \ t_{ijk}$ are non-negative real numbers.

Here ‘a_i’ and‘b_j’ denote the amount of product available at the ith source and amount of product required at the jth destination respectively. Here ‘t_{ijk}’denotes the bulk time of transportation from ith source to jth destination availing the facility ‘k’ and ‘x_{ijk}’ is the decision variable assuming the value 1 or 0 depending upon whether the requirement of the jth destination is met or not met from the source ‘i’ availing the facility ‘k’.

3. Proposed solution procedure

The proposed solution procedure comprises two main steps. In step 1, an algorithm is separately proposed for solving the bulk transportation problems for each mode of transportation. Step 2 provides theminimumtime of MIBTP.

Step 1.

(i) Delete the cells (i, j) from the table for which number of units of the product available at the i^{th} source is less than the requirement of j^{th} destination.

(ii) Select the two lowest bulk transportation time for each row and column and find their difference. This difference indicates the penalty. This penalty represents an additionaltime which has to be paid if the cell having lowestbulk transportation time remainsunallocated.

(iii) Select the largest penalty among all the penalties so obtained and allocate 1 to the lowesttimecell (i, j) corresponding to the largest penalty.This means that requirement of j^{th} destination will be met fromi^{th} source. In case of tie among the largest penalties, select thecell (i, j) having lowest time. Again, if there is tie, then select the cell $(i, j)$for which maximum allocation can be done.

(iv) Decrease the number of units of the product available at the i^{th}source by the demand ofj^{th} destination whose demand has been met.

(v) Remove the rows from the table having zero availability or which can't satisfy the requirement of any destination. Also, remove the destinations whose demand has been met.

Repeat steps (i) to (v) until the requirement of all destinations are satisfied.

Step 2.Select minimum of the corresponding times associated with the decision variables through facility k = 1 and k = 2 for each destination. Then, maximum of all such times gives the minimum the time of MIBTP.

4. Numerical problem

The problem studied by Latha [23] is considered here in which there are 3 sources, 5 destinations,and 2 facilities. The availabilities and requirements of the sources and destinations are 25, 30, 35 and 10, 12, 15, 8, 10 respectively.In BTP P_1 and P_2, bulk times of transportation are given through facility k = 1 and k = 2 respectively. The proposed method is applied to the problem to determine the minimum time of transportation. The tableau representation of the numerical problem is given is given below.

P_1 (Representation of times of BTP through 1^{st} facility)

	D_1	D_2	D_3	D_4	D_5	$a_i \downarrow$
S_1	15	13	7	9	4	25
S_2	1	7	12	9	12	30
S_3	22	20	6	11	13	35
$b_j \rightarrow$	10	12	15	8	10	

P_2 (Representation of times of BTP through 2nd facility)

	D_1	D_2	D_3	D_4	D_5	$a_i \downarrow$
S_1	9	1	3	8	12	25
S_2	7	10	12	5	4	30
S_3	3	18	4	2	14	35
$b_j \rightarrow$	10	12	15	8	10	

Applying step 1 for solving the problem P_1. The penalties for sources$S_1, S_2$and S_3 are 3, 6 and 5 respectively and the penalties for destinations D_1, D_2, D_3,$D_4$and $D_5$are 14, 6, 1, 0 and 8 respectively. Among these penalties, the largest penalty is 14 corresponding to the destination D_1.Lowest time in the column associated with destination D_1 is 1 in the cell $(2,1)$, so making allocation atthe cell $(2,1)$ $i.e. x_{211} = 1$. This fulfills the demand of destination D_1 . Remove the destination D_1 from the table and update the table to obtain Table 4.1

Table 4.1: Reduced Table after 1st allocation

	D_2	D_3	D_4	D_5	$a_i \downarrow$
S_1	13	7	9	4	25
S_2	7	12	9	12	20
S_3	20	6	11	13	35
$b_j \rightarrow$	12	15	8	10	

In Table 4.1, the penalties for sources $S_1, S_2$and $S_3$are 3, 2 and 5 respectively and for destinations D_2,D_3, D_4 and D_5 are 6, 1, 0 and 8 respectively. Among these penalties, the largest penalty is8 associated with destination D_5. Lowesttime in the column associated with destination D_5 is 4 in the cell $(1,5)$, so select the cell $(1,5) i.e. x_{151} = 1$. This fulfills the demand of destination D_5.Remove the destination$D_5$from the table and update the table to obtain table 4.2

Table 4.2: Reduced Table after 2nd allocation

	D_2	D_3	D_4	$a_i \downarrow$
S_1	13	7	9	15
S_2	7	12	9	20
S_3	20	6	11	35
$b_j \rightarrow$	12	15	8	

In Table 4.2, the penalties for sources S_1, S_2 and S_3 are 2, 2 and 5 respectively and for destinations D_2, D_3 and D_4 are 6, 1 and 0 respectively. Among these penalties, the largestpenalty is 6 associated with destination D_2. Lowesttime in the column associated with destination D_2 is 7 in the cell$(2,2)$, so making allocation at the cell $(2,2)$ $i.e. x_{221} = 1$. This fulfills the demand of destination D_2. Remove the destination

D_2 from the table. Further,since the remaining availability of the source S_2 is less than the requirement of destination D_3, deleting the entry of the cell (2, 3), we have the reduced Table 4.3.

Table 4.3: Reduced Table after 3rd allocation

	D_3	D_4	$a_i \downarrow$
S_1	7	9	15
S_2	-	9	8
S_3	6	11	35
$b_j \rightarrow$	15	8	

In Table 4.3, the penalties for sources S_1, S_2 and S_3 are 2, 9 and 5 respectively and for destinations D_3 and D_4 are 1 and 0 respectively. Among these penalties, the highest penalty is 9 associated with source S_2. Lowest time in the row associated with source S_2 is 9 in the cell (2,4), so making an allocation at the cell $(2,4)$ $i.e. x_{241} = 1$. This fulfills the demand of destination D_4 and this also exhausts the availability of source S_2. Remove the destination D_4 and source the$S_2$from the table, we have the reduced Table 4.4.

Table 4.4: Reduced Table after 4th allocation

	D_3	$a_i \downarrow$
S_1	7	15
S_3	6	35
$b_j \rightarrow$	15	

In Table 4.4, the penalties for sources S_1 and S_3 are 7 and 6 respectively and for destination D_3 is 1. Among these penalties, the largest penalty is 7 associated with source S_1. Lowest time in the rowassociated with sourceS_1 is 7 in the cell (1,3). So, making allocation in the cell (1,3), $i.e. x_{131} = 1$. This satisfies the demand of destination D_3.

Thus, all destinations met their demands. The decision variables which assume value 1 for the BTP P_1 are $x_{211}, x_{221}, x_{131}, x_{241}$ and x_{151}. Similarly, on solving the second BTP P_2 satisfying all the constraints, we have the decision variables which assume value 1 are $x_{312}, x_{122}, x_{332}, x_{242}$ *and* x_{252}. The following table gives the allocations through facility k=1 & k=2 and the final solution of the considered problem.

Table 4.5: Optimal solution of the MIBTP

Decision variables through facility k=1 and associated time	Decision variables through facility k=2 and associated time	Minimum of the corresponding timesassociated with decision variables through facility k=1 & k=2 and associated decision variables
$x_{211}, t_{211} = 1$ $x_{221}, t_{221} = 7$ $x_{131}, t_{131} = 7$	$x_{312}, t_{312} = 3$ $x_{122}, t_{122} = 1$ $x_{332}, t_{332} = 4$	$t_{211} = 1, x_{211}$ $t_{122} = 1, x_{122}$ $t_{332} = 4, x_{332}$

$x_{241}, t_{241} = 9$ $x_{151}, t_{151} = 4$	$x_{242}, t_{242} = 5$ $x_{252}, t_{252} = 4$	$t_{242} = 5, x_{242}$ $t_{151} = 4, x_{151}$

Thus, the solution of the considered MIBTP is $X = \{x_{211}, x_{122}, x_{332}, x_{242}, x_{151}\}$ and the minimum time of MIBTP is T = Max {1, 1, 4, 5, 4} = 5.

Table 4.6: Comparative study

Decision variables and Optimal time of MIBTP by the method proposed by Latha [23]	Decision variables and Optimal time of MIBTP by the proposed method
$x_{211}, x_{122}, x_{332}, x_{242}, x_{151}$ and T = 5	$x_{211}, x_{122}, x_{332}, x_{242}, x_{151}$and T = 5

Thus, we observe that the proposed method provides the same optimal solution as provided by Latha [23]. However, the proposed method is much simpler as compared toLexi-Search algorithm based upon pattern recognition technique proposed by Latha [23].

5. Conclusion

In the present paper, MIBTP already solved by Latha [23] is considered and is solved by the proposed method which gives the same optimal solution as by Latha [23]. Latha [23] used Lexi- Search algorithm based on pattern recognition technique to solve the problem. But, the method in this paper is quite simple as compared to theLexi-Search algorithm as it involves a less number of calculations.Also, the proposed method is very easy to apply on the problem and an alternative method for solving the time minimizing MIBTP.

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