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Weakly Semi-Compatible Maps and Fixed Points in Non-Archimedean Menger PM-Space

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Abstract. This paper introduces the notion of weakly semi-compatible self-maps in a Menger PM-space and establishes a fixed point theorem for six self-maps. Our result generalizes and extends the result of Cho et al. [2] as well as Sharma et al. [14].

Keywords: Non-Archimedean Menger probabilistic metric space, Common fixed points, Compatible maps, Weakly semi-compatible maps

AMS Mathematics Subject Classification (2010): 47H10, 54H25

1. Introduction

There have been a number of generalizations of metric space. One such generalization is Menger space initiated by Menger [10]. It is a probabilistic generalization in which we assign to any two points x and y, a distribution function $F_{x,y}$. Schweizer and Sklar [12] studied this concept and gave some fundamental results on this space.

The notion of compatible mapping in a Menger space has been introduced by Mishra [11]. Using the concept of compatible mappings of type (A), Jain et al. [4, 5] proved some interesting fixed point theorems in Menger space. Afterwards, Jain et al. [6] proved the fixed point theorem using the concept of weak compatible maps in Menger space.

The notion of non-Archimedean Menger space has been established by Istrătescu and Crivat [9]. The existence of fixed point of mappings on non-Archimedean Menger space has been given by Istrătescu [8]. This has been the extension of the results of Sehgal and Bharucha-Reid [13] on a Menger space. Cho. et al. [2] proved a common fixed point theorem for compatible mappings in non-Archimedean Menger PM-space. In the sequel, Singh et al. [16] established the fixed point theorem for six self maps and an example using the concept of semi-compatible self maps in a non-Archimedean Menger PM-space.

In this paper, we generalize and extend the result of Cho et al. [2] and Sharma et. al. [14] by introducing the notion of semi-compatible self maps. Also, we cited an example in support of this.

2. Preliminaries

For terminologies, notations and properties of Menger PM-space, refer to [1,8,15].

Definition 2.1. [2] Let X be a non-empty set and $\mathcal{D}$ be the set of all left-continuous distribution functions. An ordered pair (X, $\mathcal{F}$) is called a non-Archimedean probabilistic metric space (briefly, a N.A. PM-space) if $\mathcal{F}$ is a mapping from X×X into $\mathcal{D}$ satisfying the following conditions (the distribution function $\mathcal{F}(x,y)$ is denoted by $F_{x,y}$ for all $x,y \in X$) :

(PM-1) $F_{u,v}(x) = 1$, for all $x > 0$, if and only if $u = v$;

(PM-2) $F_{u,v} = F_{v,u}$;

(PM-3) $F_{u,v}(0) = 0$;

(PM-4) If $F_{u,v}(x) = 1$ and $F_{v,w}(y) = 1$ then $F_{u,w}(\max\{x, y\}) = 1$,

for all $u, v, w \in X$ and $x, y > 0$.

Definition 2.2. [2] A t-norm is a function $\Delta : [0,1] \times [0,1] \to [0,1]$ which is associative, commutative, non-decreasing in each coordinate and $\Delta(a,1) = a$ for every $a \in [0,1]$.

Definition 2.3. [2] A *N.A. Menger PM-space* is an ordered triple (X, $\mathcal{F}$, Δ), where (X, $\mathcal{F}$) is a non-Archimedean PM-space and Δ is a t-norm satisfying the following condition:

(PM-5) $F_{u,w}(\max\{x,y\}) \geq \Delta(F_{u,v}(x), F_{v,w}(y))$, for all $u, v, w \in X$ and $x, y \geq 0$.

Definition 2.4. [2] A PM-space (X, $\mathcal{F}$) is said to be of type $(C)_g$ if there exists a $g \in \Omega$ such that

$$g(F_{x,y}(t)) \leq g(F_{x,z}(t)) + g(F_{z,y}(t))$$

for all $x, y, z \in X$ and $t \geq 0$, where $\Omega = \{g \mid g : [0,1] \to [0, \infty)$ is continuous, strictly decreasing, $g(1) = 0$ and $g(0) < \infty\}$.

Definition 2.5. [2] A N.A. Menger PM-space (X, $\mathcal{F}$, Δ) is said to be of type $(D)_g$ if there exists a $g \in \Omega$ such that

$$g(\Delta(s,t) \leq g(s) + g(t)$$

for all $s, t \in [0,1]$.

Remark 2.1. [2]

(1) If a N.A. Menger PM-space (X, $\mathcal{F}$, Δ) is of type $(D)_g$ then (X, $\mathcal{F}$, Δ) is of type $(C)_g$.

(2) If a N.A. Menger PM-space (X, $\mathcal{F}$, Δ) is of type $(D)_g$, then it is metrizable, where the metric d on X is defined by

$$d(x,y) = \int_0^1 g\left(F_{x,y}(t)\right)d(t) \text{ for all } x, y \in X.$$

(*)

Throughout this paper, suppose $(X, \mathcal{F}, \Delta)$ be a complete N.A. Menger PM-space of type $(D)_g$ with a continuous strictly increasing t-norm Δ.

Let $\phi : [0,+\infty) \to [0, \infty)$ be a function satisfied the condition (Φ) :

(Φ) ϕ is upper-semicontinuous from the right and $\phi(t) < t$ for all $t > 0$.

Lemma 2.1. [2] If a function $\phi : [0,+\infty) \to [0,+\infty)$ satisfies the condition (Φ), then we have

(1) For all $t \geq 0$, $\lim_{n\to\infty} \phi^n(t) = 0$, where $\phi^n(t)$ is n-th iteration of $\phi(t)$.

(2) If $\{t_n\}$ is a non-decreasing sequence of real numbers and $t_{n+1} \leq \phi(t_n)$, n = 1, 2, … then $\lim_{n\to\infty} t_n = 0$. In particular, if $t \leq \phi(t)$ for all $t \geq 0$, then $t = 0$.

Definition 2.6. [2] Let $A, S : X \to X$ be mappings. A and S are said to be compatible if $\lim_{n\to\infty} g(F_{ASx_n,SAx_n}(t)) = 0$ for all $t > 0$, whenever $\{x_n\}$ is a sequence in X such that $\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Sx_n = z$ for some z in X.

Definition 2.7. [16] Let $A, S : X \to X$ be mappings. A and S are said to be semi-compatible if $\lim_{n\to\infty} g(F_{ASx_n,Sz}(t)) = 0$ for all $t > 0$, whenever $\{x_n\}$ is a sequence in X such that $\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Sx_n = z$ for some z in X.

Definition 2.8. Let $A, S : X \to X$ be mappings. A and S are said to be weakly semi-compatible if $\lim_{n\to\infty} g(F_{ASx_n,Sz}(t)) = 0$ or $\lim_{n\to\infty} g(F_{SAx_n,Az}(t)) = 0$ for all $t > 0$, whenever $\{x_n\}$ is a sequence in X such that $\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Sx_n = z$ for some z in X.

Clearly, semi-compatible maps are weakly semi-compatible maps but converse is not true.

Definition 2.9. [15] Self maps A and S of a N.A. Menger PM-space $(X, \mathcal{F}, \Delta)$ are said to be weakly compatible (or coincidentally commuting) if they commute at their coincidence points, i.e. if $Ap = Sp$ for some $p \in X$ then $ASp = SAp$.

Remark 2.2. [15] Compatible maps are weakly compatible but converse is not true.

Definition 2.10. [14] Self maps A and S of a N.A. Menger PM-space $(X, \mathcal{F}, \Delta)$ are said to be occasionally weakly compatible (owc) if and only if there is a point x in X which is coincidence point of A and S at which A and S commute.

Remark 2.3. [16] The concept of semi-compatibility is more general than that of compatibility.

Lemma 2.2. [2] Let $A, B, S, T : X \to X$ be mappings satisfying the condition (1) and (2) as follows :

(1) $A(X) \subset T(X)$ and $B(X) \subset S(X)$.

(2) $g(F_{Ax,By}(t)) \leq \phi(\max\{g(F_{Sx,Ty}(t)), g(F_{Sx,Ax}(t)), g(F_{Ty,By}(T)),$
$\frac{1}{2}(g(F_{Sx,By}(T)) + g(F_{Ty,Ax}(t)))\})$

for all $t > 0$, where a function $\phi : [0,+\infty) \rightarrow [0,+\infty)$ satisfies the condition (Φ). Then the sequence $\{y_n\}$ in X, defined by $Ax_{2n} = Tx_{2n+1} = y_{2n}$ and $Bx_{2n+1} = Sx_{2n+2} = y_{2n+1}$ for n = 0, 1, 2, ..., such that

$\lim_{n\to\infty} g(F_{y_n,y_{n+1}}(t)) = 0$ for all $t > 0$ is a Cauchy sequence in X.

3. Main result

Theorem 3.1. Let A, B, S, T, L, M : $X \rightarrow X$ be mappings satisfying the condition

(3.1.1) $L(X) \subset ST(X)$, $M(X) \subset AB(X)$;

(3.1.2) AB = BA, ST = TS, LB = BL, MT = TM;

(3.1.3) either AB or L is continuous;

(3.1.4) (L, AB) is weakly semi-compatible and (M, ST) is occasionally weakly compatible;

(3.1.5) $g(F_{Lx,My}(t)) \leq \phi(\max\{g(F_{ABx,STy}(t)), g(F_{ABx, Lx}(t)), g(F_{STy, My}(t)),$
$\frac{1}{2}(g(F_{ABx, My}(t)) + g(F_{STy, Lx}(t)))\})$

for all $t > 0$, where a function $\phi : [0,+\infty) \rightarrow [0,+\infty)$ satisfies the condition (Φ).

Then A, B, S, T, L and M have a unique common fixed point in X.

Proof: Let $x_0 \in X$. From condition (3.1.1) $\exists$ $x_1, x_2 \in X$ such that

$$Lx_0 = STx_1 = y_0 \quad \text{and} \quad Mx_1 = ABx_2 = y_1.$$

Inductively, we can construct sequences $\{x_n\}$ and $\{y_n\}$ in X such that

(3.1.6) $Lx_{2n} = STx_{2n+1} = y_{2n}$ and $Mx_{2n+1} = ABx_{2n+2} = y_{2n+1}$ for n = 0, 1, 2, ...

Step 1. We prove that $\lim_{n\to\infty} g(F_{y_n,y_{n+1}}(t)) = 0$ for all $t > 0$.

From (3.1.5) and (3.1.6), we have

$$\begin{aligned} g(F_{y_{2n},y_{2n+1}}(t)) &= g(F_{Lx_{2n},Mx_{2n+1}}(t)) \\ &\leq \phi(\max\{g(F_{ABx_{2n},STx_{2n+1}}(t)), g(F_{ABx_{2n}, Lx_{2n}}(t)), g(F_{STx_{2n+1}, Mx_{2n+1}}(t)), \\ &\qquad \tfrac{1}{2}(g(F_{ABx_{2n}, Mx_{2n+1}}(t)) + g(F_{STx_{2n+1}, Lx_{2n}}(t)))\}) \\ &= \phi(\max\{g(F_{y_{2n-1},y_{2n}}(t)), g(F_{y_{2n-1}, y_{2n}}(t)), g(F_{y_{2n}, y_{2n+1}}(t)), \\ &\qquad \tfrac{1}{2}(g(F_{y_{2n-1}, y_{2n+1}}(t)) + g(1))\}) \\ &\leq \phi(\max\{g(F_{y_{2n-1},y_{2n}}(t)), g(F_{y_{2n}, y_{2n+1}}(t)), \\ &\qquad \tfrac{1}{2}(g(F_{y_{2n-1}, y_{2n}}(t)) + g(F_{y_{2n}, y_{2n+1}}(t))\}). \end{aligned}$$

If $g(F_{y_{2n-1},y_{2n}}(t)) \leq g(F_{y_{2n},y_{2n+1}}(t))$ for all $t > 0$, then by (3.1.5)

$$g(F_{y_{2n},y_{2n+1}}(t)) \leq \phi(g(F_{y_{2n},y_{2n+1}}(t))),$$

on applying Lemma 2.1, we have $g(F_{y_{2n},y_{2n+1}}(t)) = 0$ for all $t > 0$.

Similarly, we have $g(F_{y_{2n+1},y_{2n+2}}(t)) = 0$ for all $t > 0$.

Thus, we have

$$\lim_{n\to\infty} g(F_{y_n, y_{n+1}}(t)) = 0 \text{ for all } t > 0.$$

On the other hand, if $g(F_{y_{2n-1}, y_{2n}}(t)) \geq g(F_{y_{2n}, y_{2n+1}}(t))$, then by (3.1.5), we have

$$g(F_{y_{2n}, y_{2n+1}}(t)) \leq \phi(g(F_{y_{2n-1}, y_{2n}}(t))) \text{ for all } t > 0.$$

Similarly, $g(F_{y_{2n+1}, y_{2n+2}}(t)) \leq \phi(g(F_{y_{2n}, y_{2n+1}}(t)))$ for all $t > 0$.

Thus, we have $g(F_{y_n, y_{n+1}}(t)) \leq \phi(g(F_{y_{n-1}, y_n}(t)))$ for all $t > 0$ & $n = 1, 2, 3, \ldots$.

Therefore, by Lemma 2.1,

$\lim_{n\to\infty} g(F_{y_n, y_{n+1}}(t)) = 0$ for all $t > 0$, which implies that $\{y_n\}$ is a Cauchy sequence in X by Lemma 2.2.

Since $(X, \mathcal{F}, \Delta)$ is complete, the sequence $\{y_n\}$ converges to a point $z \in X$. Also its subsequences converges as follows :

(3.1.7) $\{Mx_{2n+1}\} \to z$ and $\{STx_{2n+1}\} \to z$,

(3.1.8) $\{Lx_{2n}\} \to z$ and $\{ABx_{2n}\} \to z$.

Case I. AB is continuous.

As AB is continuous, $(AB)^2x_{2n} \to ABz$ and $(AB)Lx_{2n} \to ABz$.

As (L,AB) is weakly semi-compatible, so $L(AB)x_{2n} \to ABz$.

Step 2. Putting $x = ABx_{2n}$ and $y = x_{2n+1}$ for $t > 0$ in (3.1.5), we get

$$\begin{aligned} g(F_{LABx_{2n}, Mx_{2n+1}}(t)) \leq \phi(\max\{&g(F_{ABABx_{2n}, STx_{2n+1}}(t)), g(F_{ABABx_{2n}, LABx_{2n}}(t)), \\ &g(F_{STx_{2n+1}, Mx_{2n+1}}(t)), \\ &\tfrac{1}{2}(g(F_{ABABx_{2n}, Mx_{2n+1}}(t)) + g(F_{STx_{2n+1}, LABx_{2n}}(t)))\}). \end{aligned}$$

Letting $n \to \infty$, we get

$$\begin{aligned} g(F_{ABz,z}(t)) &\leq \phi(\max\{g(F_{ABz,z}(t)), g(F_{ABz, ABz}(t)), g(F_{z,z}(t)), \\ &\qquad \tfrac{1}{2}(g(F_{ABz,z}(t)) + g(F_{z, ABz}(t)))\}) \\ &= \phi(g(F_{ABz,z}(t))) \end{aligned}$$

which implies that $g(F_{ABz,z}(t)) = 0$ by Lemma 2.1 and so we have $ABz = z$.

Step 3. Putting $x = z$ and $y = x_{2n+1}$ for $t > 0$ in (3.1.5), we get

$$\begin{aligned} g(F_{Lz, Mx_{2n+1}}(t)) \leq \phi(\max\{&g(F_{ABz, STx_{2n+1}}(t)), g(F_{ABz, Lz}(t)), g(F_{STx_{2n+1}, Mx_{2n+1}}(t)), \\ &\tfrac{1}{2}(g(F_{ABz, Mx_{2n+1}}(t)) + g(F_{STx_{2n+1}, Lz}(t)))\}). \end{aligned}$$

Letting $n \to \infty$, we get

$$\begin{aligned} g(F_{Lz,z}(t)) &\leq \phi(\max\{g(F_{z,z}(t)), g(F_{z, Lz}(t)), g(F_{z,z}(t)), \\ &\qquad \tfrac{1}{2}(g(F_{z,z}(t)) + g(F_{z, Lz}(t)))\}) \\ &= \phi(g(F_{Lz,z}(t))) \end{aligned}$$

which implies that $g(F_{Lz,z}(t)) = 0$ by Lemma 2.1 and so we have $Lz = z$.

Therefore, $ABz = Lz = z$.

Step 4. Putting $x = Bz$ and $y = x_{2n+1}$ for $t > 0$ in (3.1.5), we get

$$g(F_{LBz,Mx_{2n+1}}(t)) \le \phi(\max\{g(F_{ABBz,STx_{2n+1}}(t)), g(F_{ABBz,\,LBz}(t)), g(F_{STx_{2n+1},Mx_{2n+1}}(t)), \tfrac{1}{2}(g(F_{ABBz,\,Mx_{2n+1}}(t)) + g(F_{STx_{2n+1},\,LBz}(t)))\})$$

As BL = LB, AB = BA, so we have

$$L(Bz) = B(Lz) = Bz \text{ and } AB(Bz) = B(ABz) = Bz.$$

Letting $n \to \infty$, we get

$$\begin{aligned} g(F_{Bz,z}(t)) &\le \phi(\max\{g(F_{Bz,z}(t)), g(F_{Bz,\,Bz}(t)), g(F_{z,z}(t)), \tfrac{1}{2}(g(F_{Bz,\,z}(t)) + g(F_{z,\,Bz}(t)))\}) \\ &= \phi(g(F_{Bz,z}(t))) \end{aligned}$$

which implies that $g(F_{Bz,z}(t)) = 0$ by Lemma 2.1 and so we have Bz = z.

Also, ABz = z and so Az = z.

(3.1.9) Therefore, Az = Bz = Lz = z.

Step 5. As $L(X) \subset ST(X)$, there exists $v \in X$ such that z = Lz = STv.

Putting $x = x_{2n}$ and $y = v$ for $t > 0$ in (3.1.5), we get

$$g(F_{Lx_{2n},Mv}(t)) \le \phi(\max\{g(F_{ABx_{2n},STv}(t)), g(F_{ABx_{2n},\,Lx_{2n}}(t)), g(F_{STv,\,Mv}(t)), \tfrac{1}{2}(g(F_{ABx_{2n},\,Mv}(t)) + g(F_{STv,\,Lx_{2n}}(t)))\}).$$

Letting $n \to \infty$ and using equation (3.1.8), we get

$$\begin{aligned} g(F_{z,Mv}(t)) &\le \phi(\max\{g(F_{z,z}(t)), g(F_{z,\,z}(t)), g(F_{z,\,Mv}(t)), \tfrac{1}{2}(g(F_{z,\,Mv}(t)) + g(F_{z,\,z}(t)))\}) \\ &= \phi(g(F_{z,Mv}(t))) \end{aligned}$$

which implies that $g(F_{z,Mv}(t)) = 0$ by Lemma 2.1 and so we have z = Mv.

Hence, STv = z = Mv, which shows that v is a coincidence point of ST and M.

As (M, ST) is occasionally weakly compatible, we have

STMv = MSTv.

Thus, STz = Mz.

Step 6. Putting $p = x_{2n}$, $q = z$ for $t > 0$ in (3.1.5), we get

$$g(F_{Lx_{2n},Mz}(t)) \le \phi(\max\{g(F_{ABx_{2n},STz}(t)), g(F_{ABx_{2n},\,Lx_{2n}}(t)), g(F_{STz,\,Mz}(t)), \tfrac{1}{2}(g(F_{ABx_{2n},\,Mz}(t)) + g(F_{STz,\,Lx_{2n}}(t)))\}).$$

Letting $n \to \infty$ and using equation (3.1.8) and Step 5, we get

$$\begin{aligned} g(F_{z,Mz}(t)) &\le \phi(\max\{g(F_{z,Mz}(t)), g(F_{z,\,z}(t)), g(F_{Mz,\,Mz}(t)), \tfrac{1}{2}(g(F_{z,\,Mz}(t)) + g(F_{Mz,\,z}(t)))\}) \\ &= \phi(g(F_{z,Mz}(t))) \end{aligned}$$

which implies that $g(F_{z,Mz}(t)) = 0$ by Lemma 2.1 and so we have z = Mz.

Step 7. Putting $x = x_{2n}$ and $y = Tz$ for $t > 0$ in (3.1.5), we get

$$g(F_{Lx_{2n},MTz}(t)) \le \phi(\max\{g(F_{ABx_{2n},STTz}(t)), g(F_{ABx_{2n},\,Lx_{2n}}(t)), g(F_{STTz,\,MTz}(t)), \tfrac{1}{2}(g(F_{ABx_{2n},\,MTz}(t)) + g(F_{STTz,\,Lx_{2n}}(t)))\})$$

As MT = TM and ST = TS we have

$$MTz = TMz = Tz \quad\text{and}\quad ST(Tz) = T(STz) = Tz.$$

Letting $n \to \infty$, we get

$$g(F_{z,Tz}(t)) \leq \phi(\max\{g(F_{z,Tz}(t)), g(F_{z,z}(t)), g(F_{Tz,Tz}(t)), \tfrac{1}{2}(g(F_{z,Tz}(t)) + g(F_{Tz,z}(t)))\})$$
$$= \phi(g(F_{z,Tz}(t))),$$

which implies that $g(F_{z,Tz}(t)) = 0$ by Lemma 2.1 and so we have $z = Tz$.

Now $STz = Tz = z$ implies $Sz = z$.

(3.1.10) Hence $Sz = Tz = Mz = z$.

Combining (3.1.9) and (3.1.10), we get

$$Az = Bz = Lz = Mz = Tz = Sz = z.$$

Hence, the six self maps have a common fixed point in this case.

Case II. L is continuous.

As L is continuous, $L^2x_{2n} \to Lz$ and $L(AB)x_{2n} \to Lz$.

Step 8. Putting $x = Lx_{2n}$ and $y = x_{2n+1}$ for $t > 0$ in (3.1.5), we get

$$g(F_{LLx_{2n},Mx_{2n+1}}(t)) \leq \phi(\max\{g(F_{ABLx_{2n},STx_{2n+1}}(t)), g(F_{ABLx_{2n},LLx_{2n}}(t)), g(F_{STx_{2n+1},Mx_{2n+1}}(t)), \tfrac{1}{2}(g(F_{ABLx_{2n},Mx_{2n+1}}(t)) + g(F_{STx_{2n+1},LLx_{2n}}(t)))\}).$$

Letting $n \to \infty$, we get

$$g(F_{Lz,z}(t)) \leq \phi(\max\{g(F_{Lz,z}(t)), g(F_{Lz,Lz}(t)), g(F_{z,z}(t)), \tfrac{1}{2}(g(F_{Lz,z}(t)) + g(F_{z,Lz}(t)))\})$$
$$= \phi(g(F_{Lz,z}(t))),$$

which implies that $g(F_{Lz,z}(t)) = 0$ by Lemma 2.1 and so we have $Lz = z$.

Now, using steps 5-7 gives us $Mz = STz = Sz = Tz = z$.

Step 9. As $M(X) \subset AB(X)$, there exists $w \in X$ such that $z = Mz = ABw$.

Putting $x = w$ and $y = x_{2n+1}$ for $t > 0$ in (3.1.5), we get

$$g(F_{Lw,Mx_{2n+1}}(t)) \leq \phi(\max\{g(F_{ABw,STx_{2n+1}}(t)), g(F_{ABw,Lw}(t)), g(F_{STx_{2n+1},Mx_{2n+1}}(t)), \tfrac{1}{2}(g(F_{ABw,Mx_{2n+1}}(t)) + g(F_{STx_{2n+1},Lw}(t)))\}).$$

Letting $n \to \infty$, we get

$$g(F_{Lw,z}(t)) \leq \phi(\max\{g(F_{z,z}(t)), g(F_{z,Lw}(t)), g(F_{z,z}(t)), \tfrac{1}{2}(g(F_{z,z}(t)) + g(F_{z,Lw}(t)))\})$$
$$= \phi(g(F_{Lw,z}(t))),$$

which implies that $g(F_{Lw,z}(t)) = 0$ by Lemma 2.1 and so we have $Lw = z$.

Thus, we have $Lw = z = ABw$.

As (L,AB) is semi-compatible, so $L(AB)x_{2n} \to ABz$ and

as L is continuous, $L^2x_{2n} \to Lz$ and $L(AB)x_{2n} \to Lz$.

Since limit of a sequence is unique, so $ABz = Lz$.

Also, $Bz = z$ follows from step 4.

Thus, $Az = Bz = Lz = z$ and we obtain that z is the common fixed point of the six maps in this case also.

Step 10. (Uniqueness) Let u be another common fixed point of A, B, S, T, L and M; then $Au = Bu = Su = Tu = Lu = Mu = u$.

Putting $x = z$ and $y = u$ for $t > 0$ in (3.1.5), we get

$$g(F_{Lz,Mu}(t)) \le \phi(\max\{g(F_{ABz,STu}(t)), g(F_{ABz, Lz}(t)), g(F_{STu, Mu}(t)), \tfrac{1}{2}(g(F_{ABz, Mu}(t)) + g(F_{STu, Lz}(t)))\}).$$

Letting $n \to \infty$, we get

$$g(F_{z,u}(t)) \le \phi(\max\{g(F_{z,u}(t)), g(F_{z, z}(t)), g(F_{u, u}(t)), \tfrac{1}{2}(g(F_{z, u}(t)) + g(F_{u, z}(t)))\}) = \phi(g(F_{z,u}(t))),$$

which implies that $g(F_{z,u}(t)) = 0$ by Lemma 2.1 and so we have $z = u$.

Therefore, z is a unique common fixed point of A, B, S, T, L and M.

This completes the proof.

Remark 3.1. If we take B = T = I, the identity map on X in theorem 3.1, then the condition (3.1.2) is satisfied trivially and we get

Corollary 3.1. Let A, S, L, M : $X \to X$ be mappings satisfying the condition :

(3.1.11) $L(X) \subseteq S(X)$, $M(X) \subseteq A(X)$;

(3.1.12) Either A or L is continuous;

(3.1.13) the pair (L, A) is semi-compatible and (M, S) is occasionally weakly compatible;

(3.1.14) $g(F_{Lx,My}(t)) \le \phi(\max\{g(F_{Ax,Sy}(t)), g(F_{Ax, Lx}(t)), g(F_{Sy, My}(t)), \tfrac{1}{2}(g(F_{Ax, My}(t)) + g(F_{Sy, Lx}(t)))\})$

for all $t > 0$, where a function $\phi : [0,+\infty) \to [0,+\infty)$ satisfies the condition (Φ).

Then A, S, L and M have a unique common fixed point in X.

Remark 3.2. In view of remark 3.1, corollary 3.1 is a generalization of the result of Cho et al. [2] and theorem 3.1 is a generalization of the result of Sharma et al. [14] in the sense that condition of compatibility of the pairs of self maps has been restricted to weakly semi-compatible and occasionally weakly compatible self maps and only one of the mappings of the first pair is needed to be continuous.

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