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Research on Feedback Control for a Kind of Chaotic Finance System

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Abstract. Two feedback control schemes are presented for a chaotic financial system in this paper. The system is controlled to any equilibrium points, and N identical chaotic systems are achieved synchronization. Based on Routh-Hurwitz stability criteria, the first method analyzes the characteristic polynomial of the corresponding matrix, and then designs the linear feedback controller. By using the stability theory of linear time-invariant system, the linear feedback controllers and nonlinear feedback controllers of the second method are designed directly. And they realize the chaos control and chaos synchronization, respectively. For the design of the controllers, this technique has no need to design Lyapunov function. Numerical simulations show the validity and feasibility of the proposed methods.

Keywords: chaotic finance system; feedback control; lower (upper) triangular matrix; synchronization control

1. Introduction

Research on the generation and control of chaotic financial system has become an important subject in the application of chaos. A dynamic model of financial system which is composed of four sub-blocks: production, money, stock and labor force, and its nonlinear dynamical characteristics, are received great attention by scholars in the past several decades [1-3]. Based on this, a wide variety of approaches have been proposed for the control and synchronization of the system including speed feedback control and linear feedback control [4-6], linear and nonlinear feedback synchronization [7], hybrid feedback synchronization control and a method via special matrix structure [8], realizing projective synchronization of two n-dimensional chaotic fractional-order systems via lower (upper) triangular matrix [9], and so on. However, the study above mainly control the system to one fixed balance point, or synchronize two identical chaotic systems.

This paper will control the chaotic financial system to any equilibrium point, and discuss the synchronization of N identical chaotic systems. The rest of paper is organized as the follows. In Section 2, the mathematical model of the chaotic financial system is introduced. Two of feedback control schemes are applied to control the system to any equilibrium point, and the simulation results are given in Section 3. In Section 4, we discuss the synchronization of N identical systems. Section 5 contains conclusions.

2. Mathematical model of a chaotic financial system

Since chaos phenomenon in financial field is founded in 1985, it has huge impacts on the western mainstream economics. There is chaos in economic and financial system; this means that the system itself has intrinsic instability [3]. Some long-term dynamical behaviors of the model are irregular and extreme sensitive to initial values and parameter variation.

The nonlinear chaotic financial system can be described by the following differential equations [10]:

$$\begin{cases} \dot{x} = z + (y-a)x \\ \dot{y} = 1 - by - x^2 \\ \dot{z} = -x - cz \end{cases} . \tag{1}$$

where x , y , z represent the interest rate, investment demand and price index, respectively. The parameter a is the saving, b is the per-investment cost, c is the elasticity of demands of commercials. And they are positive constants.

We can obtain the equilibrium points of system (1) as follows. For more details, see [1-3].

(1) When $c-b-abc>0$, the system(1) has three equilibrium points: $p_0\left(0,1/b,0\right)$, $p_{1,2}(\pm\sqrt{\delta},(1+ac)/c,\mp\sqrt{\delta}/c)$, in which $\delta=(c-b-abc)/c$. According to different eigenvalues of Jacobian matrix at the equilibrium points, then classify them as follows.

(1) When $c-b-abc>0$, equilibrium point p_0 is always a saddle point and has stable structure;

(2) When $c+a-1/b>0$, $(c^2+bc-1)(bc^2+2c-3b-2abc)-2c^2(c-b-abc)$ >0 , then the points p_1 and p_2 are always locally stable[11].

(3) When $c-b-abc\le 0$, the system (1) has a unique equilibrium point $p_0\left(0,1/b,0\right)$.

3. Feedback control

Consider the controlled financial system as follows:

$$\dot{\boldsymbol{x}} = f\ (\boldsymbol{x}) + \boldsymbol{u}. \tag{2}$$

where $\boldsymbol{x}\in R^n$ is state vector, $f\ (\boldsymbol{x})\in R^n$ is nonlinear continuous differentiable function, $\boldsymbol{u}\in R^n$ is external input control vector. Suppose $\boldsymbol{x}=\boldsymbol{0}$ is an equilibrium point of system (2).

When the origin is the equilibrium point of nonlinear system, we study its stability by studying the stability of the equilibrium point of the linear system. Otherwise, the asymptotic stability of equilibrium point of nonlinear systems is consistent with the approximate linear system (It is got by approximately linearizing the nonlinear system in a small field of equilibrium point) [12]. Because of the linear system can be stabilized, the characteristic of ergodicity of chaotic dynamical system guarantees its track to enter into the field of equilibrium point eventually. We can apply feedback control law to stable the original system. And the track is oriented to intended orbit [13]. Let $\boldsymbol{x}\in R^n$ be a state vector and B be a $n\times n$ matrix. The system (2) can be linearization approximately in the vicinity of equilibrium point as follows [12]:

$$\dot{\boldsymbol{x}} = A\ \boldsymbol{x}+\boldsymbol{u}\,.$$

where $A=\dfrac{\partial}{\partial \boldsymbol{x}}f$. Our aim is to design a controller, so that the linear system is asymptotically stable at the equilibrium point. That is to say, the original nonlinear system is also asymptotically stable at the equilibrium point.

3.1. Design controller based on the characteristic polynomial

According to the theory of feedback control method above, in this section, we approximately linearize the nonlinear system (1) in a small field of equilibrium point firstly. And then we design the control law based on stability theory of linear time-invariant system and Routh-Hurwitz stability criteria.

Lemma 3.1. [14] If $\boldsymbol{x}=\boldsymbol{0}$ is one equilibrium point of the following linear system

$$\dot{\boldsymbol{x}} = C\ \boldsymbol{x},\ \boldsymbol{x}(\boldsymbol{0})=\boldsymbol{x}_{\boldsymbol{0}},\ \boldsymbol{t}\geq\boldsymbol{0}\,.$$

then $\boldsymbol{x}=\boldsymbol{0}$ is asymptotically stable if and only if the matrix C must have all eigenvalues with negative real parts.

Lemma 3.2. [15] (Routh-Hurwitz stability criteria) Assume the n-order algebraic equation with constant coefficients as $a_0\lambda^n+a_1\lambda^{n-1}+\cdots+a_{n-1}\lambda+a_n=0,\ (a_0>0)$, it has all roots with negative real parts if and only if all the sequential principal minors of matrix $D=D(a_{ij})$ are positive, where

$$a_{ij}=a_{2i-j},\ (i,\ j=1,2,...,n;\ a_k=0,\ \text{if}\ k<0\ \text{or}\ k>n\ )\,.$$

$$D=\begin{pmatrix} a_1 & a_0 & 0 & 0 & \cdots & 0 \\ a_3 & a_2 & a_1 & a_0 & \cdots & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ a_{2n-1} & a_{2n-2} & a_{2n-3} & a_{2n-4} & \cdots & a_n \end{pmatrix}.$$

Especially, for algebraic equation of 2 order $a_0\lambda^2+a_1\lambda+a_2=0$, $(a_0>0)$, it has all roots with negative real parts if and only if the condition $\begin{cases} a_1>0 \\ a_2>0 \end{cases}$ is satisfied.

According to lemma 3.1, the stability of system (1) can be transformed into studying the eigenvalues of corresponding matrix.

Suppose the Jacobian matrix of the system (1) at $p_0\left(0,1/b,0\right)$ is A_0, we can yield the approximately linear system of system (1) in a neighborhood of equilibrium point as follows

$$\begin{pmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \end{pmatrix} = A_0 \begin{pmatrix} x \\ y \\ z \end{pmatrix} \tag{3}$$

where $A_0 = \begin{pmatrix} 1/b-a & 0 & 1 \\ 0 & -b & 0 \\ -1 & 0 & -c \end{pmatrix}$.

Let $\det(\lambda E - A_0) = 0$, we have

$$(\lambda+b)[\lambda^2+(a+c-1/b)\lambda+ac-c/b+1]=0. \tag{4}$$

Matrix A_0 has a negative eigenvalue, i.e. $\lambda=-b$. In order to make matrix A_0 has all eigenvalues with negative real parts, just need the following formula to have two roots with negative real parts.

$$\lambda^2+(a+c-1/b)\lambda+ac-c/b+1=0. \tag{5}$$

Based on Routh-Hurwitz stability criteria, we have $\begin{cases} a+c-1/b>0 \\ ac-c/b+1>0 \end{cases}$.

By analyzing the expression of system (3) and the structure of matrix $\lambda E - A_0$, we can control the first equation or the third one or the first and third ones of system (3).

Firstly, we control the first equation of system (3). To change the coefficient of λ, and further satisfy the conditions of stability of the system, we take the controller $\boldsymbol{u}_0 = (k_0 x, 0, 0)$, then (3) is described by

$$\begin{cases} \dot{x} = (1/b-a+k_0)x+z \\ \dot{y} = -by \\ \dot{z} = -x-cz \end{cases}. \tag{6}$$

We rewrite system (6) as follows

$$\begin{pmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \end{pmatrix} = A_0' \begin{pmatrix} x \\ y \\ z \end{pmatrix}.$$

where $A_0' = \begin{pmatrix} 1/b-a+k_0 & 0 & 1 \\ 0 & -b & 0 \\ -1 & 0 & -c \end{pmatrix}$.

In order to make matrix A_0' has all eigenvalues with negative real parts, and based on Routh-Hurwitz stability criteria, let

$$\begin{cases} a+c-1/b-k_0>0 \\ ac-k_0c-c/b+1>0 \end{cases}. \tag{7}$$

Then we get

(1) When $c \le 1$, the solution of inequality (7) is $k_0 < a + c - 1/b$.

(2) When $c > 1$, the solution of inequality (7) is $k_0 < a + 1/c - 1/b$.

So we obtain the theorem 3.1as follows.

Theorem 3.1. Let $\boldsymbol{u}_0 = (k_0 x, 0, 0)$ be the controller, where $k_0 < a + c - 1/b$ (when $c \le 1$) , or $k_0 < a + 1/c - 1/b$ (when $c > 1$) , x is the interest rate of system (1). The controlled system (1) is asymptotically stable at the equilibrium point $p_0\left(0, 1/b, 0\right)$.

Inappropriate combination of the parameters of system is the root of the emergence of chaos of economic system. It is possible to make the system tended to chaos and out of control, or may make the system stalled rigid state [10]. When $0 < a < 6.42$, $6.61 < a < 7.02$, $b = 0.1, c = 1$, system (1) occurs chaos [3]. But according to the relevant economic knowledge, the elasticity of demand c satisfying $c = 1$ is rarely seen in real life. Therefore, to make the system (1) occur chaos and the parameter values had practical significance, in this paper, all simulation experiments of chaos control take $a = 0.9, b = 0.2, c = 1.2$ [8] (when $c = 1.2 > 1$, meaning the demand is flexible, product with this attribute is mostly durable goods) , take simulation time as $t \in [0, 50]$, the initial value is taken as $(x(0), y(0), z(0)) = (3,1,5)$. When $k_0 = -4$, simulated result of theorem 3.1 is shown in Figure 1.

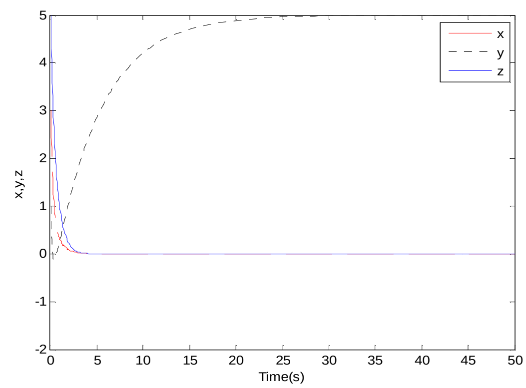

Figure 1: State variables of system (1) vary with time under the controller $\boldsymbol{u}_0$

Remark 3.1. With similar method, we can control the third equation or the first and third ones of system (1). But this method is inconformity for high-dimensional systems and is difficult to control the system to complex equilibrium point, such as the equilibrium points $p_{1,2}$ of system (1).

3.2. Design controller via lower (upper) triangular matrix structure

In order to design the controller, we firstly linearize the system (1) in the vicinity of the equilibrium point approximately. Then explore the structure of the coefficient matrix of the linear system, and design the controller based on direct-design idea. Change the coefficient matrix of the system into lower (upper) triangular matrix, and its diagonal elements are negative. Whereby, the system is controlled to the different equilibrium points.

3.2.1. Being stabled to the equilibrium point p_0

We transform the stability of system (1) into studying the structure of the coefficient matrix, which has all eigenvalues with negative real parts under the controller.

Theorem 3.2. Let $\boldsymbol{u}_1 = (k_1 x - z, 0, 0)$ be the controller, where $k_1 < a - 1/b$, x, z are the interest rate and price index of system (1), respectively. System (1) is settled to the equilibrium point $p_0\left(0, 1/b, 0\right)$ under the controller $\boldsymbol{u}_1$.

Proof: The approximately linear system of system (1) in a small field of equilibrium point p_0 is system (3), and its coefficient matrix is

$$A_0 = \begin{pmatrix} 1/b - a & 0 & 1 \\ 0 & -b & 0 \\ -1 & 0 & -c \end{pmatrix}.$$

In order to make coefficient matrix A_0 to be lower triangular matrix, besides its diagonal elements are negative. We design the controller $\boldsymbol{u}_1 = (k_1 x - z, 0, 0)$, and the controlled system (3) becomes

$$\begin{pmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \end{pmatrix} = A_1 \begin{pmatrix} x \\ y \\ z \end{pmatrix}. \tag{8}$$

where $A_1 = \begin{pmatrix} 1/b - a + k_1 & 0 & 0 \\ 0 & -b & 0 \\ -1 & 0 & -c \end{pmatrix}.$

So when $k_1 < a - 1/b$, A_1 is lower triangular matrix, besides its diagonal elements are all negative. And it has all eigenvalues with negative real parts. According to lemma 3.1, system (8) is asymptotically stabilized to the equilibrium point $p_0\left(0, 1/b, 0\right)$. Namely system (1) is asymptotically stabilized to the equilibrium point $p_0\left(0, 1/b, 0\right)$ under the controller $\boldsymbol{u}_1$.

Remark 3.2. The parameter k_1 of the controller $\boldsymbol{u}_1$ has many choices, otherwise the smaller value of k_1 is, the shorter the time of x tending to 0 will be.

3.2.2. Being stabled to the equilibrium point p_1

When $a=0.9$, $b=0.2$, $c=1.2$, equilibrium point p_1 and p_2 are local unstable [11]. To control the chaotic system (1) to unstable equilibrium point p_1, theorem 3.3 designs the controller $\boldsymbol{u}_2$.

Theorem 3.3. Let $\boldsymbol{u}_2 = (k_2 x - \sqrt{\delta} y - z, 0, 0)$ be the controller, in which $k_2 > 1/c$, and x, y, z are the interest rate, investment demand and price index of system (1), respectively. System (1) is driven to the equilibrium point $p_1(\sqrt{\delta}, (1+ac)/c, -\sqrt{\delta}/c)$ under the controller $\boldsymbol{u}_2$.

Proof: In a neighborhood of equilibrium point p_1, system (1) can be linearized approximately as follows

$$\begin{pmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \end{pmatrix} = A_2 \begin{pmatrix} x \\ y \\ z \end{pmatrix}. \tag{9}$$

where $A_2 = \begin{pmatrix} 1/c & \sqrt{\delta} & 1 \\ -2\sqrt{\delta} & -b & 0 \\ -1 & 0 & -c \end{pmatrix}$.

So we design the controller $\boldsymbol{u}_2 = (k_2 x - \sqrt{\delta} y - z, 0, 0)$, and the controlled system (9) becomes

$$\begin{pmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \end{pmatrix} = A_2' \begin{pmatrix} x \\ y \\ z \end{pmatrix}.$$

where $A_2' = \begin{pmatrix} 1/c + k_2 & 0 & 0 \\ -2\sqrt{\delta} & -b & 0 \\ -1 & 0 & -c \end{pmatrix}$.

When $k_2 < -1/c$, A_2' is lower triangular matrix. Moreover, its diagonal elements are negative. Therefore, system (1) is asymptotically stable to equilibrium point p_1 under the controller $\boldsymbol{u}_2$.

Remark 3.3. With similar method, we can change the target matrix into upper triangular matrix and settle system (1) to the equilibrium point p_2. The method doesn't need to use Lyapunov function and the Routh-Hurwitz stability criteria. And the structure of the controller is simple.

Remark 3.4. Change the goal matrix into upper triangular matrix or lower triangular matrix, although the principle is same, but the control factors are different for the financial system. When the goal matrix is changed into lower triangular matrix, we need to adjust two factors of interest rates, investment demand and price indices of system (1) at least. But when we change the target matrix into upper triangular matrix, we can simply adjust the interest rate of system (1) to achieve the purpose. Therefore, based on the different actual situation, we can choose to change the target matrix into an upper triangular matrix or lower triangular matrix.

4. Synchronization control of identical chaotic financial systems

Chaotic finance system (1) is sensitive dependence on initial conditions. Even if two identical systems, while the difference of their trajectories increases gradually with time when the initial values have minimal change. Achieving for the synchronization of identical systems with different initial values has great of significance. In this section, we still use the guidelines of triangular matrix to design the control law, and synchronize N identical chaotic financial systems with different initial values.

Consider the following N identical chaotic financial systems

$$\begin{cases} \dot{x}_i = z_i + (y_i - a)x_i \\ \dot{y}_i = 1 - by_i - x_i^2 \\ \dot{z}_i = -x_i - cz_i \end{cases}, i = 1, 2, 3..., N.$$

where $x_i, y_i, z_i, i = 1, 2, 3..., N$ are interest rates, investment demand, the price index of the i-th system, respectively.

We define the drive system and $N-1$ the controlled response systems as follows, respectively [16].

$$\begin{cases} \dot{x}_1 = z_1 + (y_1 - a)x_1 \\ \dot{y}_1 = 1 - by_1 - x_1^2 \\ \dot{z}_1 = -x_1 - cz_1 \end{cases}.$$

$$\begin{cases} \dot{x}_2 = z_2 + (y_2 - a)x_2 + u(x_2, x_1) \\ \dot{y}_2 = 1 - by_2 - x_2^2 + u(y_2, y_1) \\ \dot{z}_2 = -x_2 - cz_2 + u(z_2, z_1) \end{cases}.$$

$$\vdots$$

$$\begin{cases} \dot{x}_N = z_N + (y_N - a)x_N + u(x_N, x_1) \\ \dot{y}_N = 1 - by_N - x_N^2 + u(y_N, y_1) \\ \dot{z}_N = -x_N - cz_N + u(z_N, z_1) \end{cases}.$$

Let us define the error systems between response systems and drive system as

$$e_{i1} = x_i - x_1, e_{i2} = y_i - y_1, e_{i3} = z_i - z_1, i = 2, 3..., N. \tag{10}$$

Our purpose is to design the appropriate controllers $\boldsymbol{u}(\boldsymbol{x}_i, \boldsymbol{x}_1)$ for the response systems such that the error systems (10) are asymptotically stable to origin. Namely the synchronization of N identical chaotic systems are realized, that is

$$\lim_{t\to\infty}\|\boldsymbol{e_i}\| = \lim_{t\to\infty}\|\boldsymbol{x_i}-\boldsymbol{x}_1\| = 0 ..$$

where $\boldsymbol{x}_1=(x_1,y_1,z_1)$, $\boldsymbol{e_i}=(e_{i1},e_{i2},e_{i3})$, $\boldsymbol{x_i}=(x_i,y_i,z_i)$, $i=2,3...,N$

Use the ideas of dimensionality reduction, consider the subsystem of the error systems as follows firstly

$$\begin{cases}\dot{e}_{i1}=x_iy_i-x_1y_1-ae_{i1}+e_{i3}\\ \dot{e}_{i2}=-be_{i2}-x_i^2+x_1^2\\ \dot{e}_{i3}=-e_{i1}-ce_{i3}\end{cases}, i=2,3...,N. \tag{11}$$

It is difficult to approximately linearize the error systems in the field of equilibrium points. So we design controller to make the system (11) to be a linear system. And we take the controller $\boldsymbol{u}'\left(\boldsymbol{x_i},\boldsymbol{x}_1\right)=(x_1y_1-x_iy_i,\, x_i^2-x_1^2,\,0)$, then the subsystem of the error system (11) under the controller $\boldsymbol{u}'\left(\boldsymbol{x_i},\boldsymbol{x}_1\right)$ becomes

$$\begin{cases}\dot{e}_{i1}=-ae_{i1}+e_{i3}\\ \dot{e}_{i2}=-be_{i2}\\ \dot{e}_{i3}=-e_{i1}-ce_{i3}\end{cases}, i=2,3...,N. \tag{12}$$

Suppose the coefficient matrix of linear system (12) is

$$C_i'=\begin{pmatrix}-a & 0 & 1\\ 0 & -b & 0\\ -1 & 0 & -c\end{pmatrix}.$$

If we take the controller $\boldsymbol{u}''\left(\boldsymbol{x_i},\boldsymbol{x}_1\right)=(z_1-z_i,\,0,\,0)$, system (12) becomes

$$\begin{cases}\dot{e}_{i1}=-ae_{i1}\\ \dot{e}_{i2}=-be_{i2}\\ \dot{e}_{i3}=-e_{i1}-ce_{i3}\end{cases}, i=2,3...,N. \tag{13}$$

The coefficient matrix of linear system (13) is lower triangular matrix, and it has all eigenvalues with negative real parts.

Then the error system is $\dot{\boldsymbol{e}}_{\boldsymbol{i}}=C_i\,\boldsymbol{e_i}$, $i=2,3...,N$, namely

$$\dot{\boldsymbol{e}}=\begin{pmatrix}\dot{\boldsymbol{e}}_2\\ \dot{\boldsymbol{e}}_3\\ \vdots\\ \dot{\boldsymbol{e}}_N\end{pmatrix}=\begin{pmatrix}C_2 & \cdots & 0\\ \vdots & \ddots & \vdots\\ 0 & \cdots & C_N\end{pmatrix}\begin{pmatrix}\boldsymbol{e}_2\\ \boldsymbol{e}_3\\ \vdots\\ \boldsymbol{e}_N\end{pmatrix}. \tag{14}$$

where $$C_i=\begin{pmatrix}-a & 0 & 0\\ 0 & -b & 0\\ -1 & 0 & -c\end{pmatrix},\ C=\begin{pmatrix}C_2 & \cdots & 0\\ \vdots & \ddots & \vdots\\ 0 & \cdots & C_N\end{pmatrix}.$$

So C is also lower triangular matrix, and its diagonal elements are negative. Therefore, it has all eigenvalues with negative real parts. According to lemma3. 1, the error system (14) are asymptotically stable to origin under the controller

$$\boldsymbol{u}\left(\boldsymbol{x}_i,\boldsymbol{x}_1\right)=\left(\boldsymbol{u}'\left(\boldsymbol{x}_i,\boldsymbol{x}_1\right)+\boldsymbol{u}''\left(\boldsymbol{x}_i,\boldsymbol{x}_1\right)\right)=(x_1y_1-x_iy_i+z_1-z_i,\,x_i^2-x_1^2,\,0).$$

Based on the analysis above, parameter a,b,c may take any values, and N identical chaotic financial systems are realized synchronization. Besides, each controller of response systems has exactly the same structure, and easy to implement. For example, when $N=3$, and let

$$\boldsymbol{e}_1=\begin{pmatrix}e_{11}\\e_{12}\\e_{13}\end{pmatrix}=\begin{pmatrix}x_3-x_2\\y_3-y_2\\z_3-z_2\end{pmatrix}.$$

So the another error system is obtained by subtracting one controlled response system form another. So we have

$$\dot{\boldsymbol{e}}_1=\begin{pmatrix}-a&0&0\\0&-b&0\\-1&0&-c\end{pmatrix}\begin{pmatrix}e_{11}\\e_{12}\\e_{13}\end{pmatrix}. \tag{15}$$

And we take $a=0.9, b=0.2, c=0.5$ [17] (when the elasticity of demands of commercials c satisfies $0<c=0.5<1$, it belongs to the lack of flexibility, and the goods with this attribute mostly are necessities) . Take $t\in[0,50]$, $\left(x_1(0),\,y_1(0),\,z_1(0)\right)=(3,1,5)$, $\left(x_2(0),\,y_2(0),\,z_2(0)\right)=(6,8,6)$, and $\left(x_3(0),\,y_3(0),\,z_3(0)\right)=(3,3,4)$. The state variables e_{11},e_{12},e_{13} vary with time t are shown in Figure 2.

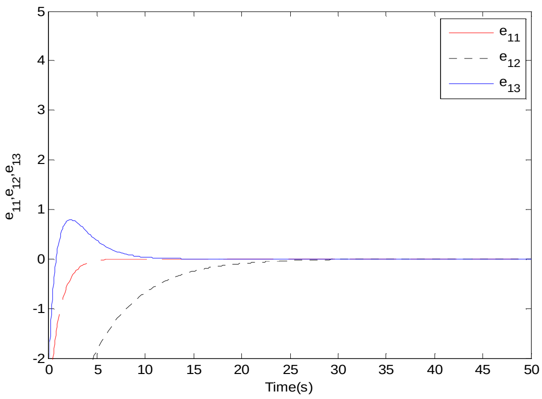

Figure 2: State variables of system (15) vary with time t under the controller $\boldsymbol{u}$

5. Conclusion

Matrix theory in mathematics is applied to chaotic control and synchronization of the financial system in the paper. We have proposed state feedback control law, and have realized control and synchronization of a chaotic financial system. The theoretical analysis and numerical simulations verify the validity and feasibility of those methods. And the synchronization of different chaotic systems will be further studied.

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