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Approximation of Markov Integrated Semigroup Induced by Dual Markov Branching Process and its Application to Signal Processing

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Abstract. In this paper, some conditions and properties of approximation of Markov integrated semigroup induced by dual Markov branching process are discussed ; Theorem 1 and Theorem 2 are obtained. Finally their application to signal processing is studied.

Keywords: Approximation, Markov Integrated Semigroup, Dual Markov Branching Process,Signal Processing, $q-$matrix,Transition Function.

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1. Introduction

A one-dimensional Markov branching process is a continuous-time Markov chain (briefly, CTMC)[1] on a countable state space $E = \{0, 1, 2, ...\}$whose stochastic evolution is governed by the branching property.Markov branching process q-matrix is given by

$$\tilde{q}_{ij} = \begin{cases} ib_{j-i+1} & j \geq i-1 \\ 0 & \text{otherwise} \end{cases} \tag{1.1}$$

where $b_k \geq 0, k \neq 1, and \sum_{k\geq 0} b_k \leq 0$.

Consider the Dual branching q-matrix Q which is a continuous-time Markov chains on the state space $E = Z_+ = \{0, 1, 2, \cdots\}$ and the q-matrix $Q = (q_{ij}, i, j \in E)$: Dual Markov branching process(DMBP)[2] $X(t)$ is a continuous-time Markov chain on the state space $Z^+ = \{0, 1, 2, \cdots\}$, where the q-matrix[3,4] $Q = \{q_{ij}; i, j \in Z^+\}$, is given by

$$q_{ij} = \begin{cases} ia_{i-j+1} - (j+1)a_{i-j} & i \geq j-1 \\ 0 & \text{otherwise} \end{cases} \tag{1.2}$$

where $a_k = \sum_{j=0}^{k} b_j, k \geq 0, b_j$ is the sequence defined by branching q-Matrix $\tilde{Q}.a_0 \leq 0 \quad a_1 \geq a_2 \geq \cdots \geq a_k \geq \cdots \geq 0$.

The following are from [8]:

1. The Dual branching q-matrix Q generates a positive once integrated semigroup of contractions $T(t) = (T_{ij}(t); i, j \in Z^+)$ on l_∞ if and only if Q is zero-exit. And $(T'_{ij}(t))$ is exactly its Q-function $F(t) = (f_{ij}(t))$.

2. The integrated semigroup $T(t)$ generated by Dual branching q-matrix is a Markov integrated semigroup.

Let us consider a measured state space (X, A), where A is the σ-algebra on X.Our aim in this paper is to discuss some conditions and properties of approximation of Markov integrated semigroup induced by dual Markov branching process and its application to signal processing.

For more unmentioned notations and preliminary ,we refer to the References of this paper.

2. Theories and Proofs

Approximation problem is a very important one on Markov process and has been widely applied in many fields.It is well known that Markov integrated semigroup can be approximated by an uniformly convergence integrated semigroup.

First,we have:

Theorem 1. *Let $Q = (q_{ij}, i, j \in E)$and ${}_nQ = ({}_nq_{ij}, i, j \in E)$are both FRR q-matrices ,let $F(t) = (f_{ij}(t))$ and ${}_nF(t) = ({}_nf_{ij}(t))$ are both FRR transition functions, $\Phi(\lambda) = (\phi_{ij}(\lambda), i, j \in E)$ and ${}_n\Phi(\lambda) = ({}_n\phi_{ij}(\lambda), i, j \in E)$ are both their corresponding resolvent functions respectively,then the following are equivalent:*

(i) for any $i, j \in E$, ${}_nq_{ij} \to q_{ij}$ as $n \longrightarrow \infty$;

(ii) for any $i, j \in E$and $t \geq 0$, ${}_nf_{ij}(t) \to f_{ij}(t)$as $n \longrightarrow \infty$;

(iii) for any $i, j \in E$and $\lambda > 0$, ${}_n\phi_{ij}(\lambda) \to \phi_{ij}(\lambda)$as $n \longrightarrow \infty$.

Proof. (i)⇒(ii):We known that: ${}_n\Omega_x \to \Omega_x$, ${}_nQ$ and Q are both FRR q-matrices. Letting $x = e_j = (0, 0, \cdots, 1, 0, \cdots)^T$,for a fixed i_0, we can obtain:

$$\begin{aligned} ||{}_n\Omega e_j - \Omega e_j||_E &= ||{}_nQe_j - Qe_j||_E \\ &= ||({}_nq_{ij})_i - (q_{ij})_i||_E \\ &= ||({}_nq_{ij})_i - (q_{ij})_i|| \\ &= \sup_{i \in E} |{}_nq_{ij} - q_{ij}| \\ &= |{}_nq_{i_0j} - q_{i_0j}| \to 0 \end{aligned}$$

for any$\lambda > 0$.

$$\begin{aligned} 0 &\leq \int_0^{+\infty} e^{-2\lambda t}|{}_nf_{ij}(t) - f_{ij}(t)|dt \\ &\leq \int_0^{+\infty} e^{-2\lambda t}|{}_nf_{i_0j}(t) - f_{ij}(t)|dt \\ &\leq |{}_nq_{ij} - q_{ij}| \\ &\leq |{}_nq_{i_0j} - q_{i_0j}| \to 0 \end{aligned}$$

By the squeeze theorem for limit of functions ,we must get ${}_nf_{ij}(t) \to f_{ij}(t)$as $n \longrightarrow \infty$.

(ii)⇒(iii):It can be completed easily by Lebesgue's theorem on control and convergence.

(iii)⇒(i):Because ${}_nF(t) = ({}_nf_{ij}(t))$and $F(t) = (f_{ij}(t))$ are both FRR transition functions,${}_nF(t)$and $F(t)$ are both positive strong continuous contraction semigroup.

Letting their generators are ${}_nA$ and A respectively,then we know that resolvent functions $\Phi(\lambda) = (\phi_{ij}(\lambda), i, j \in E)$ and ${}_n\Phi(\lambda) = ({}_n\phi_{ij}(\lambda), i, j \in E)$ are just resolvent equation of operators ${}_nA$ and A respectively,namelyú$\frac{1}{4}0 < \lambda \in \rho({}_nA) = \rho(A)$ and $R(\lambda :_n A) =_n \Phi(\lambda) \quad R(\lambda : A) = \Phi(\lambda)$.

We will prove that for any $\lambda > 0, {}_n\Phi(\lambda)x \to \Phi(\lambda)x$ as $n \longrightarrow \infty$. Since ${}_nf_{ij}(t)$ and $f_{ij}(t)$ are both transition functions by [1],${}_n\phi_{ij}(\lambda)$ and $\phi_{ij}(\lambda)$ are both FRR resolvent functions,there must exist the state i_0,such that

$$\sup_{i\in E} |{}_n\phi_{ij}(\lambda) - \phi_{ij}(\lambda)| = |{}_n\phi_{i_0j}(\lambda) - \phi_{i_0j}(\lambda)|$$

Because span $\{e_j, j \in E\}$is dense and

$$\begin{aligned} ||{}_n\Phi(\lambda) - \Phi(\lambda)|| &\leq ||{}_n\Phi(\lambda)|| + ||\Phi(\lambda)|| \\ &= \sup_{i\in E}\sum_{j\in E} |{}_n\phi_{ij}(\lambda)| \\ &\quad + \sup_{i\in E}\sum_{j\in E} |\phi_{ij}(\lambda)| \\ &\leq \sup_{i\in E}\sum_{j\in E} |{}_n\phi_{i_0j}(\lambda)| \\ &\quad + \sup_{i\in E}\sum_{j\in E} |\phi_{i_0j}(\lambda)| \\ &\leq \frac{4}{\lambda} \end{aligned}$$

so ${}_n\Phi(\lambda) - \Phi(\lambda)$is bounded .By the corresponding theorem in [2],we obtain :

$${}_nF(t) \to F(t) \qquad t \geq 0 \ n \longrightarrow \infty$$

$$\begin{aligned} \sup_{i\in E} |{}_nq_{ij} - q_{ij}| &= |{}_nq_{i_0j} - q_{i_0j}| \\ &\leq |{}_nF(t) - F(t)| \to 0 \end{aligned}$$

The proof is completed. □

Theorem 2. *Suppose that $T^n_{(t)}(n \in N)$ is a series of MIS,$P^n_{(t)}$ is the corresponding transition function of $T^n_{(t)}$, A_n is the generator of $P^n_{(t)}$ in l_1.If there exists λ_0 such that $Re\lambda_0 > 0$ and the range of operator $R(\lambda_0)$ is dense in l_1,then there exists a unique operator A: A will generate a contraction c_0 semigroup $P_{(t)}$.If $T_{(t)}$ is the corresponding Markov integrated semigroup,then $T^n_{(t)}x \to T_{(t)}x(t \geq 0, x \in l_\infty)$ as $n \longrightarrow \infty$.*

Proof. By [3,Theorem 4.4],we know that there exists a unique operator A:generating a contraction c_0 semigroup $P_{(t)}$. $P^n_{(t)}x \to P_{(t)}x, t \geq 0, x \in l_1$.

Letting $P_{ij}(t) = \langle e_iP_{(t)}, e_j\rangle$,we obtain that $P_{(t)} = (P_{ij}(t), i, j \in E)$ is a transition function ,so we have

$$\langle \int_0^\infty e^{-\lambda t} e_iP_{(t)}dt, t\rangle = \langle e_i, \lambda \int_0^\infty e^{-\lambda t}T_{(t)}dt\rangle \tag{2.1}$$

$$\langle e_iP_{(t)}, e_j\rangle = \int_0^\infty e^{-\lambda t}\langle e_iP_{(t)}, e_j\rangle dt = \int_0^\infty e^{-\lambda t}P_{ij}(t)dt = \langle e_i, T_{(t)}e_j\rangle \tag{2.2}$$

So $T^n_{(t)}x \to T_{(t)}x(t \geq 0, x \in l_\infty)$ as $n \longrightarrow \infty$.
The proof is completed. □

3. Applications

Basic morphological transformation in one-dimensional discrete signal processing is defined as: let the original signal $f(n)$ be a discrete function ,it is defined on $E = \{0, 1, 2, \cdots\}$;otherwise defining the sequence structure element $g(n)$ as a discrete function on $G = \{0, 1, 2, \cdots M - 1\}$.

By hypothesis the image,morphological filtering is to filter out signal structural elements through morphological transformation than the structural elements of noise.Expansion operator removes the negative pulse and signal positive pulse.Corrosion Operators remove the Positive Pulse and smooth the negative Pulse, the operator removes the positive pulse (see[9,10,11]) .Closed trading Remove the negative pulse counting and retained the Positive Pulse.Control the inverter output to Bench spindle speed.We now relate the local groups we have defined to the theory of approximation.

Example 1. *Suppose $P(s)$ denote the probability of each digital input signals, $P(A \to B)$ denote the integral of conditional probability density in the interval[a,b].We get:*

$$P(A \to B) = \int_A^B P(s)ds$$

Because the digital signal is entered with equal probability:

$$\begin{aligned} 0 &\leq \int_0^{+\infty} p(t)|_n f_{ij}(t) - f_{ij}(t)|dt \\ &\leq \int_0^{+\infty} p(t)|_n f_{i_0 j}(t) - f_{i_0 j}(t)|dt \\ &\leq |_n q_{i_0 j} - q_{i_0 j}| \to 0 \end{aligned}$$

So the digital signal is very stable and can be observed easily.It supplies a different angle to study signal processing.

Example 2. *Since 1980s,HMM is successfully applied to speech recognition and text recognition.In recent years, people try to apply it to the mobile communication of multiuser detection and bioinformatics of DNA sequence comparisons.The important factor to affect the pattern recognition system recognition rate is non-standard input signal. In order to improve the robustness of the recognition system,we need to construct an adaptive signal on the input mode, anti-jamming hidden Markov model. Characteristics of HMM system are determined by its three characteristics parameter vectors completely.*

We have

$$\begin{aligned} P(X|\lambda) &= P(X, \Sigma Q|\lambda) = \Sigma P(Q) P(X|Q, \lambda) \\ &= \Sigma P(Q) \prod_{t=1}^{T} P(x_t) \\ &= \Sigma P(Q) \end{aligned}$$

4. Conclusion

We have obtained some conditions and properties of approximation of Markov integrated semigroup induced by dual Markov branching process:Theorem 1 and Theorem 2. We study its application to Signal Processing and have obtained more valuable results.The results can help us consider Signal Processing from a new angle.In broadband noise, the binary PAM signal through the non-linear double-steady-state stochastic resonance receiver from the transmission can be met. Further research on non-steady state probability density of the system can better improve these results.

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