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# The Research of the Closed Fuzzy Matroids

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***Abstract.*** The closed fuzzy matroid is a subsystem of the fuzzy matroid theory. Some important conclusions, ideas and methods of the closed fuzzy matroid are briefly introduced and commented in this paper. We try to let readers to know the latest development and results of the closed fuzzy matroids.

***Keywords:*** Matroids, Independent set, Dependent set, Fuzzy matroids, Closed fuzzy matroids, Rank

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## 1. Introduction

Matroids was firstly proposed by Whitney [1] in 1935 and it provides a useful approach for linking many fundamental ideas of graph theory, linear algebra, lattices and etc. ([2], [3], [4]). Fuzzy matroids introduced by Goetchel and Voxman in the late 1980s [5]. Some scholars improved fuzzy matroid theory ([9], [10], [11], [12], [17], [18], [19], [20], [21], [22], [23]), So fuzzy matroid theory research has made great progress. And now, closed fuzzy matroids is at the emerging stage, many scholars are full of research interests. Therefore, the purpose of this article is to review the research progress of closed fuzzy matroids in a brief introduction in recent year, which is to let readers to better understand the latest research results of the closed fuzzy matroids.

Firstly, we introduce some basic knowledge of matroids.

**Definition 1.1.** ([2],[3],[4]) Let $E$ be a finite set and let $I$ be a nonempty family of subsets of $E$ satisfying:

(I1) $\varnothing \in I$;

(I2) If $X \in I$ , and $Y \subseteq X$, then $Y \in I$;

(I3) If $X, Y \in I$, and $|X| < |Y|$(where $|X|$ denotes the cardinality of $X$, then there exists $W \in I$ such that $X \subset W \subseteq X \cup Y$.

Then we call the pair $M = (E, I)$ a (crisp) matroid, $I$ a system of independent sets on $E$. For each subset $X \subseteq E$ is called an independent set of $M$ if $X \in I$, otherwise, $X$ is called a dependent set of $M$. Any member of $I$ that has maximal cardinality is called a basis of $M$. That is $B$ is a basis of $M$ if $B \in I$, but there isn't a $B' \supset B$, such that $B' \in I$. If $C \notin I$, but for any $C' \subset C$ and $C' \in I$, Then $C$ called $C$ is the circuit of M.

**Definition 1.2.** [5] If $E$ is a finite set, then a fuzzy set $\mu$ on $E$ is a mapping $\mu : E \to [0, 1]$. We denote the family of all fuzzy sets on $E$ by $F(E)$.

If $\mu, \nu \in F(E)$, then we will use the following notation in the course of this paper [1].

$\text{supp}\mu = \{x \in E \,|\mu(x) > 0\}$,
$m(\mu) = \inf \{\mu(x) \,|x \in \text{supp}\mu\}$,

$R^+(\mu) = \{\mu(x) \,|\mu(x) > 0, \forall x \in E\ \}$,
$C_r(\mu) = \{x \in E \,|\mu(x) \geq r\ \}$, where $r \in [0, 1]$,
$\mu \vee \nu = \max\{\mu, \nu\}, \mu \wedge \nu = \min\{\mu, \nu\}$.

If $\mu, \nu \in F(E)$, then we write $\mu \leq \nu$ if $\mu(x) \leq \nu(x)$ for each $x \in E$.

We write $\mu < \nu$ if

(1) $\mu \leq \nu$;
(2) $\mu(x) < \nu(x)$ for some $x \in E$.

If $E$ is a finite set, we denote the "cardinality" of a fuzzy set $\mu \in F(E)$ by $|\mu| = \sum\limits_{x \in E} \mu(x)$. We say that $\mu$ is an elementary fuzzy set if $|R^+(\mu)| = 1$.

To facilitate understanding the following, we note three fuzzy sets $\mu\backslash\backslash_a$, $\mu||_{\nu^b}$ and $\mu||_{r^c}$ as follows(where $\forall a \in \text{supp}\mu, \forall b \in \text{supp}\nu, \forall c, x \in E$):

$$(\mu\backslash\backslash_a)(x) := \begin{cases} \mu(x), x \neq a, \\ 0, x = a. \end{cases}$$
$$(\mu||_{\nu^b})(x) := \begin{cases} \mu(x), x \neq b, \\ \nu(b), x = b. \end{cases}$$
$$(\mu||_{r^c})(x) := \begin{cases} \mu(x), x \neq c, \\ r, x = c. \end{cases}$$

**Definition 1.3.** **([5],[6])** Let $E$ be a finite set and that $\psi \subseteq F(E)$ is a nonempty family of fuzzy sets satisfying:

$(F\psi 1)$ If $\mu \in \psi, \nu \in F(E)$, and $\mu < \nu$, then $\nu \in \psi$.

$(F\psi 2)$ If $\mu, \nu \in \psi$ and $|\text{supp}\mu| < |\text{supp}\nu|$, then there exists $\omega \in \psi$ such that

(a) $\mu < \omega \leq \mu \vee \nu$.
(b) $m(\omega) \geq \min\{m(\mu), m(\nu)\}$.

Then the pair $M = (E, \psi)$ is called a fuzzy matroids on $E$, and $\psi$ is the family of independent fuzzy sets of $M$. Let $\mu \in F(E)$, we say that $\mu$ is a independent fuzzy set of $M$ if $\mu \in \psi$, otherwise, $\mu$ is a dependent fuzzy set of $M$. A fuzzy basis of $M$ is a maximal member $\mu$ in $\psi$(where $\mu$ is said to be maximal in $\psi$ if whenever $\nu \in \psi$ and $\mu \leq \nu$ then $\mu = \nu$).

## 2. Sufficent And Necessary Condition Of The Closed Fuzzy Matroids.

The closed fuzzy matroid plays an important role in the study of fuzzy matroids. We firstly give the definition of closed fuzzy matroid after we introduce the sufficient and necessary condition of the closed fuzzy matroids.

**Theorem 2.1.** **([5],[13])** Let $M = (E, \psi)$ is a fuzzy matroids. For each $r$, $0 < r \leq 1$, let $\mathrm{M}_r = (E, \mathrm{I}_r)$ be a crisp matroids on $E$ (where $\mathrm{I}_r = \{C_r(\mu)|\forall \mu \in \psi\}$. Since $E$ is a finite set, there is at most a finite number of matroids that can be defined on $E$. Thus there is a finite sequence $0 = r_0 < r_1 < r_2 < \cdots < r_n \leq 1$ such that

(i) $r_0 = 0, r_n \leq 1$,
(ii) If $0 < s \leq r_n$, let $I_s \neq \{\phi\}$; if $s > r_n$, let $I_s = \{\phi\}$,
(iii)If $s, t \in (r_i, r_{i+1}]$,then $\mathrm{I}_s = \mathrm{I}_t, i = 0, 1, \cdots, n-1$,
(iv) If $r_i < s < r_{i+1} < t < r_{i+2}$, then $\mathrm{I}_t \subset \mathrm{I}_s, i = 0, 1, \cdots, n-2$.

The sequence $0 = r_0 < r_1 < r_2 < \cdots < r_n \leq 1$(or$(r_1, r_2, \cdots, r_n)$ is called the fundamental sequence of $M$. For any $i (i = 1, 2, \cdots, n)$, let $\bar{r_i} = \frac{1}{2}(r_{i-1} + r_i)$, then the decreasing sequence of matroids $\mathrm{M}_{\overline{r_1}} \supset \mathrm{M}_{\overline{r_2}} \supset \cdots \supset \mathrm{M}_{\overline{r_n}}$ is called the $M$-induced matroids sequence of $M = (E, \psi)$.

**Theorem 2.2.** **([5],[13])** Let $M = (E, \psi)$ is a fuzzy matroids. Suppose that $r_0 < r_1 < r_2 < \cdots < r_n$ is the fundamental sequence of $M = (E, \psi)$ . Then $M$ is closed if $\mathrm{I}_r = \mathrm{I}_{r_{i+1}}$(where $\mathrm{I}_r = \{C_r(\mu)|\forall \mu \in \psi\}$) whenever $r_i < r \leq r_{i+1} (i = 1, 2, \cdots, n-1)$.

**Definition 2.3.** ([6],[14]) Let $M = (E, \psi)$, A fuzzy basis for a fuzzy matroids $M$ is a maximal member $\mu$ in $\psi$(where $\mu$ is said to be maximal in $\psi$ if whenever $\nu \in \psi$ and $\mu \leq \nu$ then $\mu = \nu$).

**Theorem 2.4.** ([6],[14]) Let $M = (E, \psi)$ is a fuzzy matroids. $M$ is a closed fuzzy matroids if and only if for any $\mu \in \psi$ there exists a fuzzy basis $\nu \in \psi$, such that $\mu \leq \nu$.

The Closed fuzzy matroids is based on the GV fuzzy matroids, limits a basic conditions, for any $r_i < r \leq r_{i+1}(0 \leq i \leq n-1)$ , there are $I_r = I_{r_{i+1}}$. Thus closed fuzzy matroids is GV fuzzy matroids constraints promotion.

## 3. Some Conclusion For The regular Fuzzy Matroids.

**Definition 3.1.** [6] Let $M = (E, \psi)$ be a fuzzy matroids with the fundamental sequence $0 = r_0 < r_1 < r_2 < \cdots < r_n \leq 1$. $M = (E, \psi)$ is said to be regular if whenever $r_i < r_j$ and $B$ is a basis of $\mathrm{M}_{r_i} = (E, \mathrm{I}_{r_i})$, there is a basis $A$ of $\mathrm{M}_{r_j} = (E, \mathrm{I}_{r_j})$ such that $A \subseteq B$.

**Theorem 3.2.** [6] Let $M = (E, \psi)$ be a closed fuzzy matroids. $M = (E, \psi)$ is a closed regular fuzzy matroids if and only if all bases of $M = (E, \psi)$ have the same cardinality.

Obviously, if $M$ is closed regular matroids, then it must meet the requirements of the above definition 3.1. Closed fuzzy matroids $M$ is closed regular matroids, which is on the basis of the definition 3.1 , and there have the same cardinality for the basis $A, B$ of $M = (E, \psi)$.

**Theorem 3.3.** [6] Let $M = (E, \psi)$ be a closed regular fuzzy matroids with the fundamental sequence $0 = r_0 < r_1 < r_2 < \cdots < r_n \leq 1$. Let $\mu \in F(E)$. If $\mu$ is a fuzzy basis of $M$, then $R^+(\mu) = \{r_1, r_2, \cdots, r_n\}$. Moreover, for any $i(i = 1, 2, \cdots, n)$, $C_{r_i}(\mu)$ is a basis of $(E, \mathrm{I}_{r_i})$.

**Theorem 3.4.** [15]] Let $M = (E, \psi)$ be a closed regular fuzzy matroids. If $\mu$ is a fuzzy basis of $M$ and $R^+(\mu) = \{r_1, r_2, \cdot\cdot\cdot, r_n\}$(where $0 < r_1 < \cdots < r_n \leq 1$), then $0, r_1, \cdot\cdot\cdot, r_n$ is the fundamental sequence of $M = (E, \psi)$.

**Definition 3.5.** [8] Let $M = (E, \psi)$ be a fuzzy matroids. The rank of $M = (E, \psi)$ is a mapping $\rho : F(E) \to [0, +\infty)$ defined by function of $\rho(\mu) = \sup\{|\nu| \,|\nu \leq \mu, \nu \in \psi\}$ (where $|\nu| = \sum\limits_{x \in E} \nu(x)$).

**Theorem 3.6.** [15] Let $M = (E, \psi)$ be a closed regular fuzzy matroids with the fundamental sequence $0 = r_0 < r_1 < r_2 < \cdots < r_n \leq 1$. And the $M$-induced matroids sequence is $\mathrm{M}_{r_1} \supset \mathrm{M}_{r_2} \supset \cdots \supset \mathrm{M}_{r_n}$(where for $i = 1, 2, \cdots, n$, $\mathrm{M}_{r_i} = (E, \mathrm{I}_{r_i})$. $\mu$ is a fuzzy basis of $M$, then

(1) $|\mathrm{supp}\mu| = \rho(M_{r_1})$;

(2) $|\mathrm{supp}\mu| \geq n$;

(3) If $|\mathrm{supp}\mu| = n$, then $|\mu| = \sum\limits_{i=1}^{n} r_i$.

**Theorem3.7.** [15] Let $M = (E, \psi)$ be a closed fuzzy matroids with the fundamental sequence $0 = r_0 < r_1 < r_2 < \cdots < r_n \leq 1$.

(1)If for any fuzzy basis $\mu$, there have $|\mathrm{supp}\mu| = n$ and $R^+(\mu) = \{r_1, r_2, \cdot\cdot\cdot, r_n\}$, then $M = (E, \psi)$ is the closed regular fuzzy matroids;

(2)Let $\mu, \nu$ be any fuzzy basis of $M = (E, \psi)$, suppose that $A_i = \{x \in E \,|\mu(x) = r_i\}$, $B_i = \{x \in E \,|\nu(x) = r_i\}(1 \leq i \leq n)$,if $|A_i| = |B_i|$, then $M = (E, \psi)$ is the regular fuzzy matroids.

Theorem 3.6 is discussed from the perspective of the rank function to form a closed regular fuzzy matroids which has a series of conclusions; Theorem 3.7 is in fact deeply promotion and specific from Theorem 3.2, which helps us understand the details of the closed regular fuzzy matroids .

**4. Basis And Circuits Of The Closed Fuzzy Matroids.**

Goetschel and Voxman studied a series of theorems and conclusions in their the literature [5-12], such as the fuzzy basis, fuzzy circuits. In recent years, Li Yonghong who has studied the closed fuzzy matroids basis and circuits[13-18].

**Theorem 4.1.** [13] Let $M = (E, \psi)$ be a fuzzy matroids with the fundamental sequence $0 = r_0 < r_1 < \cdots < r_n \leq 1$. Let $\mu \in F(E)$. If $\mu$ is a basis of $M$, for any $r \in R^+(\mu)$, we have $C_r(\mu) \in I_1$ and for any $r_1 \leq r \leq m(\mu)$, we have $C_r(\mu) = C_{m(\mu)}(\mu)$ is a basis of $(E, \mathrm{I}_r)$.

**Theorem 4.2.** [13] Let $M = (E, \psi)$ be a closed fuzzy matroids on $E$ with the fundamental sequence $0 = r_0 < r_1 < \cdots < r_n \leq 1$. Suppose further that $\mathrm{M}_{r_1} \supset \mathrm{M}_{r_2} \supset \cdots \supset \mathrm{M}_{r_n}$ (where $\mathrm{M}_{r_i} = (E, \mathrm{I}_{r_i})$ $i = 1, 2, \cdots, n$) is the $M$-induced matroids sequence of $M$. Let $\mu \in F(E)$. Then $\mu$ is a fuzzy basis of $M$ if and only if

(i) $R^+(\mu) \subseteq \{r_1, r_2, \cdots, r_n\}$;

(ii) For each $r \in R^+(\mu)$, we have $C_r(\mu) \in \mathrm{I}_r$,and $C_r(\mu) = C_{m(\mu)}(\mu)$ for each $r, r_1 \leq r \leq m(\mu)$;

(iii) Suppose that $\mathrm{I}_{m(\mu)} = \mathrm{I}_{r_k}$.Let $A_k =$supp$\mu$ be a base of crisp matroids $(E, \mathrm{I}_{r_k})$.If there exists a maximal subset $A_i$ of $A_{i-1}$ in $\mathrm{I}_{r_i}(i = k+1, k+2, \cdots, n)$ such that $A_n \subseteq A_{n-1} \subseteq \cdots \subseteq A_{k+1} \subseteq A_k$, then for each $x \in A_n$, we have $\mu(x) = r_n$. And for each $x \in A_i \backslash A_{i+1}$,$i = k+1, k+2, \cdots, n-1$, we have $\mu(x) = r_i$.

**Corollary 4.3.** [13] Let $M = (E, \psi)$ be a closed fuzzy matroids on $E$ with the fundamental sequence $0 = r_0 < r_1 < \cdots < r_n \leq 1$.$\mu \in F(E)$.If

(1) $R^+(\mu) = \{r_n\}$;

(2) $C_{r_n}(\mu)$ is the basis of $(E, I_{r_i})(1 \leq i \leq n)$.

Then $\mu$ is the fuzzy basis of $M$.

**Theorem 4.4.** [7] Let $M = (E, \psi)$ be fuzzy matroids, $\mu \in F(E)$ and $R^+(\mu) = \{\beta_1, \beta_2, \cdot\cdot\cdot, \beta_k\}$(where $\beta_1 < \beta_2 < \cdots < \beta_k$). Then $\mu$ is a fuzzy circuits of $M$ if and only if

(1) $C_{\beta_1}(\mu)$ is the circuits of $(E, I_{\beta_1})$;

(2) $C_{\beta_i}(\mu) \in I_{\beta_i}, 2 \leq i \leq k$.

**Theorem 4.5.** [14] Let $M = (E, \psi)$ be a fuzzy matroids on $E$ with the fundamental sequence $0 = r_0 < r_1 < \cdots < r_n \leq 1$,$\mu \in F(E)$, then $\mu$ is the elementary fuzzy circuits of $M$, if and only if $|R^+(\mu)| = 1$, and $C_{m(\mu)}$ is the circuits of $(E, I_{m(\mu)})$.

**Theorem 4.6.** [14] Let $M = (E, \psi)$ be a closed fuzzy matroids on $E$ with the fundamental sequence $0 = r_0 < r_1 < \cdots < r_n \leq 1$,$\mu \in F(E)$. If $m(\mu) \leq r_1$, then $\mu$ is the elementary fuzzy circuits of $M$,if and only if $|R^+(\mu)| = 1$ and $C_{m(\mu)}$ is the circuits of $(E, I_{r_1})$.

**Theorem 4.7.** [14] Let $M = (E, \psi)$ be a closed fuzzy matroids on $E$ with the fundamental sequence $0 = r_0 < r_1 < \cdots < r_n \leq 1$,$\mu \in F(E)$. If $\mu$ is the fuzzy circuits of $M$, then for any $\beta \in R^+(\mu)$, we have $\beta \leq r_n$.

The following example is according to the theorem 4.7. We will from some fuzzy sets of the closed fuzzy matroids to calculate its a fuzzy circuits.

**Example 4.8.** [14] Suppose that $E = \{1, 2, 3, 4\}, I_{\frac{1}{2}} = \{\varnothing, \{1\}, \{2\}, \{3\}, \{4\}, \{1, 2\}, \{1, 3\}, \{1, 4\}\}$, $I_1 = \{\varnothing,\{1\}, \{3\}, \{1, 3\}\}$. Then $(E, I_{\frac{1}{2}})$and$(E, I_1)$ are matroids, and

$I_{\frac{1}{2}} \supset I_1$. Let $I_r = \begin{cases} I_{\frac{1}{2}}, 0 < r \leq \frac{1}{2}, \\ I_1, \frac{1}{2} < r \leq 1. \end{cases}$ and $\psi = \{\mu \in F(E)\,|C_r(\mu) \in I_r, 0 < r \leq 1\}$, According to the relevant theorem of the literature ([5], [6]), $M = (E, \psi)$ is a closed fuzzy matroids with the fundamental sequence $r_0 = 0, r_1 = \frac{1}{2}, r_2 = 1$, $I_{\frac{1}{2}} \supset I_1$ is the $M$-induced matroids sequence of $M$. Let fuzzy sets be

$$\mu(x) = \begin{cases} \frac{1}{2}, x = 1, \\ \frac{1}{2}, x = 2, \\ \frac{2}{3}, x = 3, \\ 1, x = 4. \end{cases}$$

$\mu$ is a dependent fuzzy sets of $M = (E, \psi)$, because of supp$\mu = C_{\frac{1}{2}}(\mu) \notin I_{\frac{1}{2}}$. $\mu = \{1, 2, 3, 4\}$ is support set of $\mu$, and $A = \{2, 3\}$, $B = \{2, 4\}$ is subset of supp$\mu$ , which is circuits of $I_{\frac{1}{2}}$. For $A = \{2, 3\}$, setting fuzzy set $\nu$ because of $\{3\} \in I_1$, such that

$$\nu(x) = \begin{cases} 0, x = 1, \\ \frac{1}{2}, x = 2, \\ \frac{2}{3}, x = 3, \\ 0, x = 4. \end{cases}$$

Because of $R^+(\nu) = \{\frac{1}{2}, \frac{2}{3}\}$, supp$\nu = C_{\frac{1}{2}}(\nu) = \{2, 3\}$ is circuits of $I_{\frac{1}{2}}$. $C_{\frac{2}{3}}(\nu) = \{3\} \in I_1, \nu \leq \mu$, according to theorem 4.7, $\nu$ is fuzzy circuits of $M = (E, \psi)$. For $B = \{2, 4\}$, setting fuzzy set $\omega$, because of $\{4\} \notin I_1$, such that

$$\omega(x) = \begin{cases} 0, x = 1, \\ \frac{1}{2}, x = 2, \\ 0, x = 3, \\ \frac{1}{2}, x = 4. \end{cases}$$

Because of $R^+(\omega) = \{\frac{1}{2}\}$, $\omega = C_{\frac{1}{2}}(\omega) = \{2, 4\}$ circuits of $I_{\frac{1}{2}}$. According to theorem 4.6, we know that $\omega$ is the elementary fuzzy circuits of $M = (E, \psi)$.

**Theorem 4.9.** [14] Let $M = (E, \psi)$ be a closed fuzzy matroids on $E$ with the fundamental sequence $0 = r_0 < r_1 < \cdots < r_n \leq 1$,$\mu \in F(E)$, then $\mu$ is fuzzy circuits of $M$ if and only if which satisfies the following conditions:

(1) $C_{m(\mu)}$ is fuzzy circuits of $(E, I_{r_1})$;

(2) For $\beta \in R^+(\mu), \beta > m(\mu)$, then $\beta \leq r_n$ and exists $i(1 \leq i \leq n)$ such that $C_\beta(\mu) \in I_{r_i}$.

The above theorems and related examples of closed fuzzy matroids which discuss the necessary and sufficient condition of fuzzy circuits are important contents of fuzzy matroids. This will be conducive to fuzzy matroid from basic research to applied research.

## 5. Basis And Circuits Of The Closed Regular Fuzzy Matroids.

Li Yonghong and some other scholars continue to research for the bases and the circuits of the closed regular fuzzy matroids on the basis of the closed fuzzy matroids.

**Definition 5.1.** **([7], [16])** Let $M = (E, \psi)$ be a fuzzy matroids, then $\mu \in F(E)$ is a fuzzy circuits of $M$, if $\mu \notin \psi$, but for $a \in$supp$\mu$ , always exists $\mu \backslash\backslash a \in \psi$.

**Theorem 5.2.** [7] Let $M = (E, \psi)$ be a fuzzy matroids, $\mu \in F(E)$, and $R^+(\mu) = \{\beta_1, \beta_2, \cdots, \beta_k\}$(where $\beta_1 < \beta_2 < \cdots < \beta_k$, then $\mu$ is a fuzzy circuits if and only if which satisfies the following:

(1) $C_{\beta_1}(\mu)$ is the circuits of matroids $(E, I_{\beta_1})$;

(2) $C_{\beta_j}(\mu) \in I_{\beta_j}, 2 \leq j \leq k$.

**Theorem 5.3.** [16] Let $M = (E, \psi)$ be a closed regular fuzzy matroids with the fundamental sequence $r_0 < r_1 < \cdots < r_n$, $\mu \in \psi$, $\mu$ is a basis of $M$. For $b \in E\backslash \operatorname{supp}\mu$, then $\mu||_{a^b}$ is the fuzzy circuits of $M$ if and only if $0 < a \leq r_1$ and $C_a(\mu||_{a^b})$ is the circuits of $(E, I_a)$ , where $(\mu||_{a^b})(x) = \begin{cases} \mu(x), x \neq b, \\ a, x = b. \end{cases}$

**Theorem 5.4.** [16] Let $M = (E, \psi)$ be a closed regular fuzzy matroids with the fundamental sequence $r_0 < r_1 < \cdots < r_n$, $\mu \in \psi$, then $\mu$ is a basis of $M$ if and only if $R^+(\mu) = \{r_1, r_2, \cdots, r_n\}$, and $C_{r_i}(\mu)$ is the basis of $(E, I_{r_i})$(where $(i = 1, 2 \cdots n)$.

**Theorem 5.5.** [16] Let $M = (E, \psi)$ be a closed regular fuzzy matroids with the fundamental sequence $r_0 < r_1 < \cdots < r_n$, $\mu$ is the fuzzy circuits of $M$ and $R^+(\mu) = \{r_1, r_2, \cdots, r_n\}$, $a \in \operatorname{supp}\mu$, then $\mu\backslash\backslash_a$ is the fuzzy basis of $M$ if and only if which satisfies the following:

(1) $\mu(a) = r_1$;

(2) $R^+(\mu\backslash\backslash_a) = \{r_1, r_2 \cdots, r_n\}$;

(3) $C_{r_1}(\mu\backslash\backslash_a)$ is the basis of $(E, I_{r_i})$ and $C_{r_j}(\mu)$ is the basis of $(E, I_{r_j})$(where for all $(E, I_{r_j})$).

This section is two parts of knowledge which puts the basis and the circuits of the closed regular fuzzy matroids together, rather than one single study, which has a good reference for future research on fuzzy matroids.

## 6. The Rank Of Fuzzy Circuits In Closed(Regular)Fuzzy Matroids.

This part is to associate with fuzzy basis, fuzzy circuits, rank function, to get some good conclusions, which greatly broadens the closed (regular) fuzzy matroids research.

With the previous knowledge of the background of this article, we quickly understand the following theorem.

**Theorem 6.1.** [17] Let $M = (E, \psi)$ be a closed fuzzy matroids on $E$ with the fundamental sequence $0 = r_0 < r_1 < r_2 < \cdots < r_n \leq 1$. Suppose further that $\mathrm{M}_{r_1} \supset \mathrm{M}_{r_2} \supset \cdots \supset \mathrm{M}_{r_n}$(where $\mathrm{M}_{r_i} = (E, \mathrm{I}_{r_i})$, $i = 1, 2, \cdots, n$ ) is the $M$-induced matroids sequence of $M$. $\rho$ is rank function. If $\mu$ is a fuzzy circuits of $M$, and $\operatorname{supp}\mu$ is the circuits of $M_{r_1} = (E, I_{r_1})$, then $\rho(\mu) = |\mu| - m(\mu)$.

**Theorem 6.2.** [17] Let $M = (E, \psi)$ be a closed fuzzy matroids on $E$ with the fundamental sequence $0 = r_0 < r_1 < r_2 < \cdots < r_n \leq 1$. $\rho$ is rank function.suppose that $\mu_1, \mu_2$ are the fuzzy circuits of $M$, and $\omega_1, \omega_2$ are the fuzzy basis of $M$, if $\omega_1 \subseteq \mu_1, \omega_2 \subseteq \mu_2$, then $\rho(\mu_1) = \rho(\mu_2)$.

For the Theorem 6.2, we have a knowledge of the closed regular fuzzy matroids, which has the same cardinality, to understand this theorem is not difficult.


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