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Computation of Reduced Kulli-Gutman Sombor Index of Certain Networks

V.R.Kulli

Department of Mathematics,
Gulbarga University, Gulbarga 585106, India
e-mail:vrkulli@gmail.com

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Abstract. In this paper, we introduce the reduced Kulli-Gutman Sombor index of a graph. Also we compute the Kulli-Gutman Sombor index and the reduced Kulli-Gutman Sombor index of oxide and honeycomb networks.

Keywords: Kulli-Gutman Sombor index, reduced Kulli-Gutman Sombor index, graph, network.

AMS Mathematics Subject Classification (2010): 05*C*05, 05*C*07, 05*C*09, 05*C*92

1. Introduction

Let $G = (V, E)$ be a finite, simple connected graph. The degree $d_G(u)$ of a vertex u is the number of vertices adjacent to u. The degree of an edge $e = uv$ in G is defined by $d_G(e) = d_G(u)+d_G(v) – 2$. We refer to [1] for undefined term and notation.

The first and second Banhatti indices of a graph G were introduced by Kulli in [2], and they are defined as

$$B_1(G) = \sum_{ue}\left[d_G(u) + d_G(e)\right], \qquad B_2(G) = \sum_{ue} d_G(u)\, d_G(e).$$

where ue means that the vertex u and edge e are incident in G.

Recently, some topological indices were studied, for example, in [3, 4, 5].

The Kulli-Gutman Sombor index was introduced by Kulli et al. in [6], defined it as

$$KG(G) = \sum_{ue} \sqrt{d_G(u)^2 + d_G(e)^2}.$$

For definition, see also [7, 8].

We can express the Kulli-Gutman Sombor index as

$$KG(G) = \sum_{uv\in E(G)} \left[\sqrt{d_G(u)^2 + \left(d_G(u)+d_G(v)-2\right)^2} + \sqrt{d_G(v)^2 + \left(d_G(u)+d_G(v)-2\right)^2}\right]$$

Recently, some Sombor indices were studied, for example, in [9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21].

We introduce the reduced Kulli-Gutman Sombor index of a graph G and it is defined as

$$KG_{red}(G)=\sum_{ue}\sqrt{\left(d_G(u)-1\right)^2+\left(d_G(e)-2\right)^2}.$$

We can express the reduced Kulli-Gutman Sombor index as

$$KG_{red}(G)=\sum_{uv\in E(G)}\left[\sqrt{\left(d_G(u)-1\right)^2+\left(d_G(u)+d_G(v)-4\right)^2}+\sqrt{\left(d_G(v)-1\right)^2+\left(d_G(u)+d_G(v)-4\right)^2}\right].$$

In this paper, we determine the Kulli-Gutman Sombor index and the reduced Kulli-Gutman Sombor index for oxide networks and honeycomb networks.

2. Results for some standard graphs

Proposition 1. If G is r-regular with n vertices and $r \geq 2$, then

$$KG_{red}(G)=nr\sqrt{5r^2-18r+17}.$$

Proof: Let G is r-regular with n vertices and $r \geq 2$ and $\frac{nr}{2}$ edges. Every edge of G is incident with r edges. Then $d_G(e) = 2r - 2$.

$$KG_{red}(G)=\sum_{ue}\sqrt{\left(d_G(u)-1\right)^2+\left(d_G(e)-2\right)^2}$$

$$=\frac{nr}{2}\left(\sqrt{(r-1)^2+(2r-2-2)^2}+\sqrt{(r-1)^2+(2r-2-2)^2}\right) \quad =nr\sqrt{5r^2-18r+17}.$$

Corollary 1.1. Let C_n be a cycle with $n \geq 3$ vertices. Then $KG_{red}(C_n)=2n.$

Corollary 1.2. Let K_n be a complete graph with $n \geq 3$ vertices. Then

$$KG_{red}(K_n)=n(n-1)\sqrt{5n^2-28n+40}.$$

3. Results for Oxide networks

Oxide networks are of vital importance in the study of silicate networks. An oxide network of dimension five denotes an oxide network of dimension shown in Figure 1.

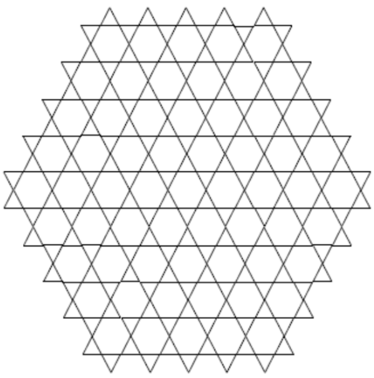

Figure 1: Oxide network of dimension 5

Let G be the graph of an oxide network OX_n with $9n^2 + 3n$ vertices and $18n^2$ edges. In OX_n, there are two types of edges based on the degree of end vertices of each edge as follows:

$E_1=\{uv \in E(G) \mid d_G(u)=2, d_G(v)=4\}$, $|E_1|=12n$,

$E_2=\{uv \in E(G) \mid d_G(u)=4, d_G(v)=4\}$, $|E_2|=18n^2 - 12n$

Theorem 1. If G is the graph of an oxide network OX_n, then

$$KG(OX_n)=72\sqrt{13}n^2+24\left(\sqrt{5}+2\sqrt{2}-2\sqrt{13}\right)n.$$

Proof: Let G be the graph of an oxide network OX_n. Then

$$KG(OX_n)=\sum_{uv\in E(OX_n)}\left[\sqrt{d_G(u)^2+\left(d_G(u)+d_G(v)-2\right)^2}+\sqrt{d_G(v)^2+\left(d_G(u)+d_G(v)-2\right)^2}\right]$$

$$=12n\left(\sqrt{2^2+4^2}+\sqrt{4^2+4^2}\right)+\left(18n^2-12n\right)\left(\sqrt{4^2+6^2}+\sqrt{4^2+6^2}\right).$$

After simplification, we obtain the desired result.

Theorem 2. If G is the graph of an oxide network OX_n, then

$$KG_{red}(OX_n)=180n^2+12\left(\sqrt{5}+\sqrt{13}-10\right)n.$$

Proof: Let G be the graph of an oxide network OX_n. Then

$$KG_{red}(OX_n)=\sum_{uv\in E(OX_n)}\left[\sqrt{\left(d_G(u)-1\right)^2+\left(d_G(u)+d_G(v)-4\right)^2}+\sqrt{\left(d_G(v)-1\right)^2+\left(d_G(u)+d_G(v)-4\right)^2}\right]$$

$$=12n\left(\sqrt{1^2+2^2}+\sqrt{3^2+2^2}\right)+\left(18n^2-12n\right)\left(\sqrt{3^2+4^2}+\sqrt{3^2+4^2}\right)$$

gives the desired result after simplification.

4. Results for Honeycomb networks

Honeycomb networks are very useful in Computer Graphics and in Chemistry. An honeycomb network of dimension n is denoted by HC_n, where n is the number of hexagons between central and boundary hexagon. A honeycomb network of dimension four is shown in Figure 2.

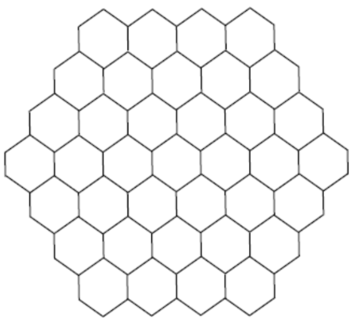

Figure 2: Honeycomb network of dimension 4

Let G be the graph of a honeycomb network HC_n with $6n^2$ vertices and $9n^2 - 3n$ edges. In HC_n, there are three types of edges based on the degree of end vertices of each edge as follows:

$E_1=\{uv \in E(G) \mid d_G(u)=2, d_G(v)=2\}$, $|E_1|=6$,
$E_2=\{uv \in E(G) \mid d_G(u)=2, d_G(v)=3\}$, $|E_2|=12n-12$
$E_3=\{uv \in E(G) \mid d_G(u)=3, d_G(v)=3\}$, $|E_2|=9n^2 - 15n + 6$.

Theorem 3. If G is the graph of a honeycomb network HC_n, then

$$KG(HC_n)=90n^2+\left(12\sqrt{13}+36\sqrt{2}-150\right)n-12\sqrt{13}-12\sqrt{2}+60.$$

Proof: Let G be the graph of a honeycomb network HC_n. Then

$$KG(HC_n)=\sum_{uv\in E(HC_n)}\left[\sqrt{d_G(u)^2+\left(d_G(u)+d_G(v)-2\right)^2}+\sqrt{d_G(v)^2+\left(d_G(u)+d_G(v)-2\right)^2}\right]$$

$$=6\left(\sqrt{2^2+2^2}+\sqrt{2^2+2^2}\right)+(12n-12)\left(\sqrt{2^2+3^2}+\sqrt{3^2+3^2}\right)$$

$$+\left(9n^2-15n+6\right)\left(\sqrt{3^2+4^2}+\sqrt{3^2+4^2}\right)$$

gives the desired result after simplification.

Theorem 4. If G is the graph of a honeycomb network HC_n, then

$$KG_{red}(HC_n)=36\sqrt{2}n^2+\left(\sqrt{5}-4\sqrt{2}\right)12n+\left(1+\sqrt{2}-\sqrt{5}\right)12.$$

Proof: Let G be the graph of a honeycomb network HC_n. Then

$$KG_{red}(HC_n)=\sum_{uv\in E(HC_n)}\left[\sqrt{\left(d_G(u)-1\right)^2+\left(d_G(u)+d_G(v)-4\right)^2}+\sqrt{\left(d_G(v)-1\right)^2+\left(d_G(u)+d_G(v)-4\right)^2}\right]$$

$$=6\left(\sqrt{1^2+0^2}+\sqrt{1^2+0^2}\right)+(12n-12)\left(\sqrt{1^2+1^2}+\sqrt{2^2+1^2}\right)$$

$$+\left(9n^2-15n+6\right)\left(\sqrt{2^2+2^2}+\sqrt{2^2+2^2}\right).$$

After simplification, we obtain the desired result.

6. Conclusion

In this paper, we have defined the reduced Kulli-Gutman Sombor index of a graph. Furthermore, the Kulli-Gutman Sombor index and the reduced Kulli-Gutman Sombor index for oxide and honeycomb networks are computed.

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