The \((a, b)\)-KA E-Banhatti Indices of Graphs

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Abstract. In this study, we propose the first and second modified E-Banhatti indices and the first and second \((a, b)\)-KA E-Banhatti indices of a graph. We compute the first and second \((a, b)\)-KA E-Banhatti indices for \(HC_5C_7[p, q]\) nanotubes. We also establish some other E-Banhatti indices directly as a special case of these indices for some special values of \(a\) and \(b\).

Keywords: first and second modified E-Banhatti indices, first and second \((a, b)\)-KA E-Banhatti indices, nanotube

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1. Introduction

Let \(G\) be a finite, simple, connected graph. Let \(V(G)\) be the vertex set and \(E(G)\) be the edge set of \(G\). The degree \(d_G(u)\) of a vertex \(u\) is the number of vertices adjacent to \(u\). For undefined terms and notations, we refer [1].

A graph index is a numerical parameter mathematically derived from the graph structure. The graph indices have their applications in various disciplines of Science and Technology [2].

Kulli [3] defined the Banhatti degree of a vertex \(u\) of a graph \(G\) as

\[
B(u) = \frac{d_G(e)}{n - d_G(u)},
\]

where \(|V(G)| = n\) and the vertex \(u\) and edge \(e\) are incident in \(G\).

Kulli introduced the first and second E-Banhatti indices in [3] and they are defined as

\[
EB_1(G) = \sum_{u \in V(G)} [B(u) + B(v)], \quad EB_2(G) = \sum_{u \in V(G)} B(u)B(v).
\]

The first and second hyper E-Banhatti indices were proposed by Kulli in [4] and they are defined as

\[
HEB_1(G) = \sum_{u \in V(G)} [B(u) + B(v)]^2, \quad HEB_2(G) = \sum_{u \in V(G)} [B(u)B(v)]^2.
\]

Kulli [5] proposed the E-Banhatti Nirmali index of a graph \(G\) is
The modified E-Banhatti Nirmala index \([5]\) of a graph \(G\) is
\[ \text{mEBN}(G) = \sum_{u \neq v \in E(G)} \frac{1}{\sqrt{B(u) + B(v)}}. \]

The E-Banhatti Sombor index \([6]\) of a graph \(G\) is
\[ \text{EBS}(G) = \sum_{u \neq v \in E(G)} \sqrt{B(u)^2 + B(v)^2}. \]

Kulli \([6]\) introduced the modified E-Banhatti Sombor index of a graph \(G\) and it is defined as
\[ \text{mEBS}(G) = \sum_{u \neq v \in E(G)} \frac{1}{\sqrt{B(u)^2 + B(v)^2}}. \]

Kulli \([7]\) introduced the product connectivity E-Banhatti index and the reciprocal product connectivity E-Banhatti index of a graph \(G\) and they are defined as
\[ PEB(G) = \frac{1}{\sqrt{B(u)B(v)}}, \quad \text{RPEB}(G) = \sum_{u \neq v \in E(G)} \sqrt{B(u)B(v)}. \]

In \([8]\), the FE-Banhatti index of a graph \(G\) is defined as
\[ FEB(G) = \sum_{u \neq v \in E(G)} \left[ B(u)^2 + B(v)^2 \right]. \]

We introduce the first and second modified E-Banhatti indices of a graph \(G\) and they are defined as
\[ \text{mEB}_1(G) = \sum_{u \neq v \in E(G)} \frac{1}{B(u) + B(v)}, \quad \text{mEB}_2(G) = \sum_{u \neq v \in E(G)} \frac{1}{B(u)B(v)}. \]

Motivated by the work on E-Banhatti indices, we introduce the first and second \((a, b)\)-KA E-Banhatti indices of a graph and they are defined as
\[ KAB_{a,1}(G) = \sum_{u \neq v \in E(G)} \left[ B(u)^a + B(v)^a \right]^b, \quad KAB_{a,2}(G) = \sum_{u \neq v \in E(G)} \left[ B(u)^a \cdot B(v)^a \right]^b, \]
where \(a\) and \(b\) are real numbers.

Recently, some graph indices were studied in \([9, 10, 11, 12]\).
This paper determines the first and second \((a, b)\)-KA E-Banhatti indices for \(HC_5C_7[p, q]\) nanotubes.

2. Observations
We observe the following
(1) \( EB_1(G) = KAB_{1,1}^{1}(G) \) \hspace{1cm} (2) \( HEB_1(G) = KAB_{1,2}^{1}(G) \)
(3) \( EBN(G) = KAB_{1,2}^{1}(G) \) \hspace{1cm} (4) \( \text{mEBN}(G) = KAB_{1,-1}^{1}(G) \)
(5) \( EBS(G) = KAB_{2,-1}^{1}(G) \) \hspace{1cm} (6) \( \text{mEBS}(G) = KAB_{1,-2}^{1}(G) \)
(7) \( FEB(G) = KAB_{2,1}^{1}(G) \) \hspace{1cm} (8) \( \text{mEB}_1(G) = KAB_{1,-1}^{1}(G) \)
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Furthermore, we also see that

\begin{align*}
(1) \quad \text{\(EB_2(G) = KAB_{1,1}^2(G)\)} & \quad \text{\(HEB_2(G) = KAB_{1,2}^2(G)\)} \\
(2) \quad \text{\(PEB(G) = KAB_{1,1}^2(G)\)} & \quad \text{\(RPEB(G) = KAB_{1,2}^2(G)\)} \\
(3) \quad \text{\(EB_2(G) = KAB_{1,1}^2(\frac{1}{2})\)} & \quad \text{\(HEB_2(G) = KAB_{1,2}^2(\frac{1}{2})\)} \\
(4) \quad \text{\(EB_2(G) = KAB_{1,1}^2(\frac{1}{2})\)} & \quad \text{\(HEB_2(G) = KAB_{1,2}^2(\frac{1}{2})\)} \\
(5) \quad \text{\(EB_2(G) = KAB_{1,1}^2(\frac{1}{2})\)} & \quad \text{\(HEB_2(G) = KAB_{1,2}^2(\frac{1}{2})\)}
\end{align*}

3. \(HC_5C_7[p, q]\) Nanotubes

We consider \(HC_5C_7[p, q]\) nanotubes, see Figure 1.

![Figure 1: 2-D lattice of \(HC_5C_7[8, 4]\) nanotube](image-url)

The graphs of a nanotube \(HC_5C_7[p, q]\) have \(4pq\) vertices and \(6pq - p\) edges are shown in the above graph. Let \(G = HC_5C_7[p, q]\).

In \(G\), there are two types of edges as follows:

\[
E_1 = \{uv \in E(G) \mid d(u) = 2, d(v) = 3\}, \quad |E_1| = 4p.
\]

\[
E_2 = \{uv \in E(G) \mid d(u) = d(v) = 3\}, \quad |E_2| = 6pq - 5p.
\]

Therefore, in \(G\), we obtain that \(\{B(u), B(v) : uv \in E(NHPX[m, n])\}\) has two Banhatti edge set partitions.

\[
BE_1 = \{uv \in E(G) \mid B(u) = \frac{3}{4pq - 2}, B(v) = \frac{3}{4pq - 3}\}, \quad |BE_1| = 4p.
\]

\[
BE_2 = \{uv \in E(G) \mid B(u) = B(v) = \frac{4}{4pq - 3}\}, \quad |BE_2| = 6pq - 5p.
\]

We calculate the first \((a, b)\)-KA E-Banhatti index of a nanotube \(HC_5C_7[p, q]\) as follows:

**Theorem 1.** Let \(G = HC_5C_7[p, q]\) be a nanotube. Then

\[
KAB_{a,b}^1(G) = 4p \left[ \left( \frac{3}{4pq - 2} \right)^a + \left( \frac{3}{4pq - 3} \right)^a \right]^b + 2^b \left( 6pq - 5p \right) \left( \frac{4}{4pq - 3} \right)^{ab}.
\]

**Proof:** From definition and by cardinalities of the Banhatti edge partition of \(G\), we obtain

\[
KAB_{a,b}^1(G) = \sum_{uv \in E(G)} \left[ B(u)^a + B(v)^a \right]^b
\]
By solving the above equation, we get the desired result.

We obtain the following results by using Theorem 1.

**Corollary 1.1.** Let \( G = HC_5 C_2[p, q] \) be a nanotube. Then

(i) \( [3] \) \( EB_1(G) = KAB_{1,1}^{1,1}(G) = \frac{12p(8pq - 5)}{(4pq - 2)(4pq - 3)} + \frac{8(6pq - 5p)}{(4pq - 3)} \).

(ii) \( HEB_1(G) = KAB_{1,2}^{1,1}(G) = \frac{36p(8pq - 5)^2}{(4pq - 2)^2(4pq - 3)^2} + \frac{64(6pq - 5p)}{(4pq - 3)^2} \).

(iii) \( EBN(G) = KAB_{1/2}^{1,1}(G) = \frac{4\sqrt{3}p\sqrt{(8pq - 5)}}{(4pq - 2)(4pq - 3)} + \frac{2\sqrt{2}(6pq - 5p)}{(4pq - 3)} \).

(iv) \( mEBN(G) = KAB_{1/2}^{1,1}(G) = \frac{4p\sqrt{(4pq - 2)(4pq - 3)}}{2\sqrt{2}} + \frac{(6pq - 5p)\sqrt{(4pq - 3)}}{2\sqrt{2}} \).

(v) \( EBS(G) = KAB_{2/2}^{1,1}(G) = \frac{12p\sqrt{(4pq - 2)^2 + (4pq - 3)^2}}{(4pq - 2)(4pq - 3)} + \frac{4\sqrt{2}(6pq - 5p)}{(4pq - 3)} \).

(vi) \( mEBS(G) = KAB_{2/2}^{1,1}(G) = \frac{4p(4pq - 2)(4pq - 3)}{3(4pq - 2)^2 + (4pq - 3)^2} + \frac{(6pq - 5p)(4pq - 3)}{4\sqrt{2}} \).

(vii) \( FEB(G) = KAB_{2/2}^{1,1}(G) = \frac{36p[(4pq - 2)^2 + (4pq - 3)^2]}{(4pq - 2)^2(4pq - 3)^2} + \frac{32(6pq - 5p)}{(4pq - 3)^2} \).

(viii) \( mE_B(G) = KAB_{1/2}^{1,1}(G) = \frac{4p(4pq - 2)(4pq - 3)}{3(8pq - 5)} + \frac{(6pq - 5p)(4pq - 3)}{8} \).

We calculate the second \((a, b)\)-KA E-Banhatti index of a nanotube \( HC_5 C_2[p, q] \) as follows:

**Theorem 2.** Let \( G = HC_5 C_2[p, q] \) be a nanotube. Then

\[
KAB_{a,b}^2(G) = 4p \left( \frac{9^a}{(4pq - 2)(4pq - 3)} \right)^b + (6pq - 5p) \left( \frac{4}{(4pq - 3)} \right)^{2ab}.
\]

**Proof:** From the definition and by cardinalities of the Banhatti edge partition of \( G \), we obtain

\[
KAB_{a,b}^2(G) = \sum_{uv \in E(G)} \left[ B(u)^a \times B(v)^b \right].
\]
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\[
= 4p \left[ \frac{3}{4pq - 2} \right]^a \left[ \frac{3}{4pq - 3} \right]^b \left( 6pq - 5p \right) \left[ \frac{4}{4pq - 3} \right]^a \left[ \frac{4}{4pq - 3} \right]^b
\]

By solving the above equation, we get the desired result.

We obtain the following results by using Theorem 1.

**Corollary 2.1.** Let \(G = HC_5C_7[p,q]\) be a nanotube. Then

(i) \([3]\) \(EB_2(G) = KAB_{1,1}^2(G) = \frac{36p}{(4pq - 2)(4pq - 3)} + \frac{16(6pq - 5p)}{(4pq - 3)^2}\).

(ii) \(HEB_2(G) = KAB_{1,2}^2(G) = \frac{324p}{(4pq - 2)^3(4pq - 3)^2} + \frac{256(6pq - 5p)}{(4pq - 3)^4}\).

(iii) \(PEB(G) = KAB_{1/2}^2(G) = \frac{4p\sqrt{(4pq - 2)(4pq - 3)}}{3} + \frac{(6pq - 5p)(4pq - 3)}{4}\).

(iv) \(m^mRPEB(G) = KAB_{1/2}^2(G) = \frac{12p}{\sqrt{(4pq - 2)(4pq - 3)}} + \frac{4(6pq - 5p)}{(4pq - 3)}\).

(v) \(m^mEB_2(G) = KAB_{1,-1}^2(G) = \frac{4p(4pq - 2)(4pq - 3)}{9} + \frac{(6pq - 5p)(4pq - 3)^2}{16}\).

4. **Conclusion**

In this study, we have defined the first and second modified E-Banhatti indices and the first and second \((a, b)\)-KA E-Banhatti indices of a graph. The first and second \((a, b)\)-KA E-Banhatti indices and some other E-Banhatti indices for particular values of \(a\) and \(b\) for \(HC_5C_7[p,q]\) nanotubes have been determined.

**REFERENCES**

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