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Delay-partitioning approach to stability criteria for T-S fuzzy systems with time-varying delay

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Abstract. This paper is concerned with the stability criteria of T-S fuzzy systems with time-varying delay by delay-partitioning approach. Based on Finsler's lemma, LMI approach and an appropriate augmented LKF established in the framework of state vector augmentation, some tighter bounding inequalities such as Seuret-Wirtinger's integral inequality and Peng-Park's integral inequality are employed to deal with (time-varying) delay-dependent integral items. Therefore, less conservative delay-dependent stability criteria are obtained in terms of LMIs, which can be solved efficiently with the Matlab LMI toolbox. Finally, one numerical example is provided to show that the proposed conditions are less conservative than existing ones.

Keywords: Delay-partitioning approach, Linear matrix inequalities (LMIs), Lyapunov-Krasovskii functional (LKF), Stability, Time-varying delay, T-S fuzzy systems.

AMS Mathematics Subject Classification (2010): 93C10, 93D20

1. Introduction

Takagi-Sugeno (T-S) fuzzy model was first introduced in [1], then much effort has been made in the stability analysis and control synthesis of such a model during the past two decades, due to the fact that it can combine the flexibility of fuzzy logic theory and fruitful linear system theory into a unified framework to approximate complex nonlinear systems [2, 3]. On the other hand, as a source of instability and deteriorated performance, time-delay often occurs in many dynamic systems such as biological systems, chemical processes,communication networks and so on. Therefore, stability analysis for T-S fuzzy systems with time-delay has received more interest in recent years, see, e.g., [4, 5, 6] and references therein.

As long as the recent techniques adopted in the stability analysis of T-S fuzzy systems with time-varying delay are concerned, the most famous is the delay-partitioning approach [7, 8, 9, 10, 11]. It has been proved that less conservative results may be expected with the increasing delay-partitioning segments [7, 10]. Recently, by dividing the delay interval into two uniform segments, [11] obtained the less conservative results than those in [5, 13] for T-S fuzzy systems with time-varying delay. More recently, on the basis of delay-partitioning approach and Peng-Park's integral inequality established by reciprocally convex approach, [10] has developed less conservative stability criteria than those in [5, 9, 11] for the uncertain T-S fuzzy systems with interval time-varying delay. Most recently, a novel LKF is established via the delay-decomposition method, then by means of employing the reciprocally convex approach, [7] has achieved less conservative results than those in [5, 10, 14, 15, 16, 17, 18] for the uncertain T-S fuzzy systems with time-varying delay. However, when revisiting this problem, we find that the aforementioned works still leave plenty of room for improvement.

This paper will develop new stability criteria of T-S fuzzy systems with time-varying delay by means of delay-partitioning approach and Finsler's lemma. Based on a modified augmented-LKF, less conservative stability criteria are obtained by employing Seuret-Wirtinger's integral inequality and Peng-Park's integral inequality to deal with (time-varying) delay-dependent integral items. Finally, one numerical example is provided to show the merits of the proposed results.

Notations. Through this paper, $\mathbb{R}^n$ and $\mathbb{R}^{n\times m}$ denote, respectively, the n-dimensional Euclidean space and the set of all $n \times m$ real matrices; the notation $A > (\geq) B$ means that $A - B$ is positive (semi-positive) definite; I (0) is the identity (zero) matrix with appropriate dimension; A^{T} denotes the transpose; $\mathrm{He}(A)$ represents the sum of A and A^{T}; $\|\bullet\|$ denotes the Euclidean norm in $\mathbb{R}^n$; "*" denotes the elements below the main diagonal of a symmetric block matrix; $C([-\tau,0],\mathbb{R}^n)$ is the family of continuous functions ϕ from interval $[-\tau,0]$ to $\mathbb{R}^n$ with the norm $\|\phi\|_\tau = \sup_{-\tau\leq\theta\leq 0}\|\phi(\theta)\|$; let $x_t(\theta) = x(t+\theta),\ \theta \in [-\tau, 0]$.

2. Problem formulation

In this section, a class of T-S fuzzy systems with time-varying delay is concerned. For each $i = 1, 2, \cdots, r$ (r is the number of plant rules), the ith rule of this T-S fuzzy model is represented as follows:

Plant Rule i: IF $\theta_1(t)$ **is** M_{i1}, $\theta_2(t)$ **is** M_{i2}, $\cdots$, $\theta_p(t)$ **is** M_{ip}, **THEN**

$$\left\{\begin{array}{l} \dot{x}(t) = A_i x(t) + A_{di} x(t-\tau(t)), \quad t \geq 0 \\ x(t) = \phi(t), \qquad t \in [-\tau, 0], \end{array}\right. \tag{2.1}$$

where $\theta_1(t), \theta_2(t), \cdots, \theta_p(t)$ are the premise variables, and each $M_{il}(i = 1, 2, \cdots, r; l = 1, 2, \cdots, p)$ is a fuzzy set; $x(t) \in \mathbb{R}^n$ is the state vector; $\phi(t) \in C([-\tau,0],\mathbb{R}^n)$ is the initial function; A_i and A_{di} are constant real matrices with appropriate dimensions; the delay $\tau(t)$ is a time-varying functional satisfying

$$0 \leq \tau(t) \leq \tau, \tag{2.2}$$

$$\dot{\tau}(t) < \mu, \tag{2.3}$$

where τ and μ are constants assumed to exist.

By a center-average defuzzier, product inference and singleton fuzzifier, the dynamic fuzzy model in (2.1) can be represented by

$$\left\{\begin{array}{l} \dot{x}(t) = \sum\limits_{i=1}^{r} h_i(\theta(t))\{A_i x(t) + A_{di} x(t-\tau(t))\}, \\ x(t) = \phi(t), \qquad t \in [-\tau, 0], \end{array}\right. \tag{2.4}$$

where

$$h_i(\theta(t)) = \frac{\prod_{l=1}^{p} M_{il}(\theta_l(t))}{\sum_{i=1}^{r}\prod_{l=1}^{p} M_{il}(\theta_l(t))}, \quad i = 1, \cdots, r, \tag{2.5}$$

in which $M_{il}(\theta_l(t))$ is the grade of membership of $\theta_l(t)$ in M_{il}, and $\theta(t) = (\theta_1(t), \cdots, \theta_r(t))$; By definition, the fuzzy weighting functions $h_i(\theta(t))$ satisfy $h_i(\theta(t)) \geq 0$ and $\sum_{i=1}^{r} h_i(\theta(t)) =$

1. For notational simplicity, h_i is used to represent $h_i(\theta(t))$ in the following description.

Before proceeding, recall the following lemmas which will be used throughout the proofs.

Lemma 1 (Finsler's lemma) [19]. Let $\zeta \in \mathbb{R}^n$, $\Phi = \Phi^{\mathrm{T}} \in \mathbb{R}^{n\times m}$, and $B \in \mathbb{R}^{m\times n}$ such that $rank(B) < n$. Then the following statements are equivalent:
(i) $\zeta^{\mathrm{T}}\Phi\zeta < 0,\ \forall\ B\zeta = 0,\ \zeta \neq 0$;
(ii) ${B^{\perp}}^{\mathrm{T}}\Phi B^{\perp} < 0$;
(iii) $\exists\ Y \in \mathbb{R}^{n\times m}:\ \Phi + \mathrm{He}(YB) < 0$.

Lemma 2 (Peng-Park's integral inequality)[10, 12]. For any matrix $\begin{bmatrix} Z & S \\ * & Z \end{bmatrix} \geq 0$, positive scalars τ and $\tau(t)$ satisfying $0 < \tau(t) < \tau$, vector function $\dot{x} : [-\tau, 0] \to \mathbb{R}^n$ such that the concerned integrations are well defined, then

$$-\tau \int_{t-\tau}^{t} \dot{x}^{\mathrm{T}}(s)Z\dot{x}(s)ds \leq \varpi^{\mathrm{T}}(t)\Omega\varpi(t),$$

where

$$\varpi(t) = [x^{\mathrm{T}}(t), x^{\mathrm{T}}(t-\tau(t)), x^{\mathrm{T}}(t-\tau)]^{\mathrm{T}},$$
$$\Omega = \begin{bmatrix} -Z & Z - S & S \\ * & -2Z + \mathrm{He}(S) & -S + Z \\ * & * & -Z \end{bmatrix}.$$

Lemma 3 (Seuret-Wirtinger's integral inequality) [20]. For any matrix $Z > 0$, the following inequality holds for all continuously differentiable function $x : [\alpha, \beta] \to \mathbb{R}^n$:

$$\int_{\alpha}^{\beta} \dot{x}^{\mathrm{T}}(s)Z\dot{x}(s)ds \geq \frac{1}{\beta - \alpha}\nu^{\mathrm{T}}(t)\widetilde{\Omega}\nu(t),$$

where

$$\nu(t) = \begin{bmatrix} x(\beta) \\ x(\alpha) \\ \frac{1}{\beta-\alpha}\int_{\alpha}^{\beta} x(s)ds \end{bmatrix}, \quad \widetilde{\Omega} = \begin{bmatrix} 4Z & 2Z & -6Z \\ * & 4Z & -6Z \\ * & * & 12Z \end{bmatrix}.$$

3. Main results

This section aims to develop a novel stability criteria for fuzzy system (2.4) with time-varying delay by delay-partitioning approach.

For any integer $m \geq 1$, define $\delta = \frac{\tau}{m}$, then $[0, \tau]$ can be divided into m segments, i.e.,

$$[0,\tau] = \textstyle\bigcup_{j=1}^{m}[(j-1)\delta, j\delta]. \tag{3.1}$$

For notational simplification, motivated by [10], let

$$\begin{cases} e_s = [\underbrace{0,\cdots,0}_{s-1}, I, \underbrace{0,\cdots,0}_{m-s+4}]^{\mathrm{T}},\ s = 1,\cdots,m+4 \\ \\ \zeta(t) = [x^{\mathrm{T}}(t-\tau(t)),\ \zeta_1^{\mathrm{T}}(t),\ x^{\mathrm{T}}(t-m\delta),\ \frac{1}{\delta}\int_{t-\delta}^{t} x^{\mathrm{T}}(s)ds,\ \dot{x}(t)]^{\mathrm{T}}, \end{cases} \tag{3.2}$$

where

$$\zeta_1(t) = [x^{\mathrm{T}}(t), x^{\mathrm{T}}(t-\delta), x^{\mathrm{T}}(t-2\delta), \cdots, x^{\mathrm{T}}(t-(m-1)\delta)]^{\mathrm{T}}.$$

Based on Lyapunov-Krasovskii stability theorem [21], we firstly state the following stability criterion for the system (2.4).

Theorem 1. Given a positive integer m, scalars $\tau \geq 0$, μ, and $\delta = \frac{\tau}{m}$, then the nominal system (2.4) with a time-delay $\tau(t)$ satisfying (2.2) and (2.3) is asymptotically stable if there exist symmetric positive matrices $P = \begin{bmatrix} P_1 & P_2 \\ * & P_3 \end{bmatrix}$, $R_l = \begin{bmatrix} R_{1l} & R_{2l} \\ * & R_{3l} \end{bmatrix}$, $X = [X_{ij}]_{m\times m} \triangleq \begin{bmatrix} X_{11} & \cdots & X_{1m} \\ \vdots & \ddots & \vdots \\ * & \cdots & X_{mm} \end{bmatrix}$, Q_j, Z_0, Z_j, and any matrices Y and $S_{ij}(i = 1,\cdots,r; j = 1,\cdots,m; l = 1,\cdots,m-1)$ with appropriate dimensions, such that the following LMIs hold for $i = 1,\cdots,r$ and $k = 1,\cdots,m$:

$$\Theta(i,k) + \mathrm{He}(Y\Gamma_i) < 0, \tag{3.3}$$

$$\Psi(i,k) = \begin{bmatrix} Z_k & S_{ik} \\ * & Z_k \end{bmatrix} \geq 0, \tag{3.4}$$

where

$$\Gamma_i = A_i e_2^{\mathrm{T}} + A_{di} e_1^{\mathrm{T}} - e_{m+4}^{\mathrm{T}},$$

$$\Theta(i,k) = \sum_{j=0}^{3} \Theta_j + \Theta_4(k) + \Theta_5(i,k) + e_{m+4}\bar{Z}e_{m+4}^{\mathrm{T}},$$

$$\Theta_0 = \begin{bmatrix} e_2^{\mathrm{T}} \\ e_3^{\mathrm{T}} \\ e_{m+3}^{\mathrm{T}} \end{bmatrix}^{\mathrm{T}} \begin{bmatrix} -4Z_0 & -2Z_0 & 6Z_0 \\ * & -4Z_0 & 6Z_0 \\ * & * & -12Z_0 \end{bmatrix} \begin{bmatrix} e_2^{\mathrm{T}} \\ e_3^{\mathrm{T}} \\ e_{m+3}^{\mathrm{T}} \end{bmatrix},$$

$$\Theta_1 = \mathrm{He}\left\{ \begin{bmatrix} e_2^{\mathrm{T}} \\ \delta e_{m+3}^{\mathrm{T}} \end{bmatrix}^{\mathrm{T}} \begin{bmatrix} P_1 & P_2 \\ * & P_3 \end{bmatrix} \begin{bmatrix} e_{m+4}^{\mathrm{T}} \\ e_2^{\mathrm{T}} - e_3^{\mathrm{T}} \end{bmatrix} \right\},$$

$$\Theta_2 = \begin{bmatrix} e_2^{\mathrm{T}} \\ e_3^{\mathrm{T}} \\ \vdots \\ e_{m+1}^{\mathrm{T}} \end{bmatrix}^{\mathrm{T}} X \begin{bmatrix} e_2^{\mathrm{T}} \\ e_3^{\mathrm{T}} \\ \vdots \\ e_{m+1}^{\mathrm{T}} \end{bmatrix} - \begin{bmatrix} e_3^{\mathrm{T}} \\ e_4^{\mathrm{T}} \\ \vdots \\ e_{m+2}^{\mathrm{T}} \end{bmatrix}^{\mathrm{T}} X \begin{bmatrix} e_3^{\mathrm{T}} \\ e_4^{\mathrm{T}} \\ \vdots \\ e_{m+2}^{\mathrm{T}} \end{bmatrix},$$

$$\Theta_3 = \sum_{j=1}^{m-1} \left(\begin{bmatrix} e_{j+1}^{\mathrm{T}} \\ e_{j+2}^{\mathrm{T}} \end{bmatrix}^{\mathrm{T}} R_j \begin{bmatrix} e_{j+1}^{\mathrm{T}} \\ e_{j+2}^{\mathrm{T}} \end{bmatrix} - \begin{bmatrix} e_{j+2}^{\mathrm{T}} \\ e_{j+3}^{\mathrm{T}} \end{bmatrix}^{\mathrm{T}} R_j \begin{bmatrix} e_{j+2}^{\mathrm{T}} \\ e_{j+3}^{\mathrm{T}} \end{bmatrix} \right),$$

$$\Theta_4(k) = \sum_{j=1}^{k-1} \left[e_{j+1}Q_j e_{j+1}^{\mathrm{T}} - e_{j+2}Q_j e_{j+2}^{\mathrm{T}} \right] + e_{k+1}Q_k e_{k+1}^{\mathrm{T}} - (1-\mu)e_1 Q_k e_1^{\mathrm{T}},$$

$$\Theta_5(i,k) = \begin{bmatrix} e_{k+1}^{\mathrm{T}} \\ e_1^{\mathrm{T}} \\ e_{k+2}^{\mathrm{T}} \end{bmatrix}^{\mathrm{T}} \begin{bmatrix} -Z_k & Z_k - S_{ik} & S_{ik} \\ * & -2Z_k + \mathrm{He}(S_{ik}) & Z_k - S_{ik} \\ * & * & -Z_k \end{bmatrix} \begin{bmatrix} e_{k+1}^{\mathrm{T}} \\ e_1^{\mathrm{T}} \\ e_{k+2}^{\mathrm{T}} \end{bmatrix} + \sum_{j=1, j\neq k}^{m} \begin{bmatrix} e_{j+1}^{\mathrm{T}} \\ e_{j+2}^{\mathrm{T}} \end{bmatrix}^{\mathrm{T}} \begin{bmatrix} -Z_j & Z_j \\ * & -Z_j \end{bmatrix} \begin{bmatrix} e_{j+1}^{\mathrm{T}} \\ e_{j+2}^{\mathrm{T}} \end{bmatrix},$$

with $\bar{Z} = \delta^2 \sum_{j=0}^{m} Z_j$.

Proof. For any $t \geq 0$, there should exist an integer $k \in \{1,2,\cdots,m\}$, such that

$\tau(t) \in [(k-1)\delta,\ k\delta]$. Then, choose the following Lyapunov-Krasovskii functional candidate:

$$V(t, x_t) = \sum_{i=1}^{5} V_i(x_t), \tag{3.5}$$

where

$$V_1(x_t) = \eta_0^{\mathrm{T}}(t) P \eta_0(t),$$

$$V_2(x_t) = \int_{t-\delta}^{t} \zeta_1^{\mathrm{T}}(s) X \zeta_1(s) ds,$$

$$V_3(x_t) = \sum_{j=1}^{m-1} \int_{t-\delta}^{t} \eta_j^{\mathrm{T}}(s) R_j \eta_j(s) ds,$$

$$V_4(x_t) = \sum_{j=1}^{k-1} \int_{t-j\delta}^{t-(j-1)\delta} x^{\mathrm{T}}(s) Q_j x(s) ds + \int_{t-\tau(t)}^{t-(k-1)\delta} x^{\mathrm{T}}(s) Q_k x(s) ds,$$

$$V_5(x_t) = \sum_{j=1}^{m} \delta \int_{-j\delta}^{-(j-1)\delta} \int_{t+\theta}^{t} \dot{x}^{\mathrm{T}}(s) Z_j \dot{x}(s) ds d\theta + \delta \int_{-\delta}^{0} \int_{t+\theta}^{t} \dot{x}^{\mathrm{T}}(s) Z_0 \dot{x}(s) ds d\theta,$$

with

$$\eta_0(t) = [x^{\mathrm{T}}(t), \int_{t-\delta}^{t} x^{\mathrm{T}}(s) ds]^{\mathrm{T}},$$

$$\eta_j(s) = [x^{\mathrm{T}}(s-(j-1)\delta), x^{\mathrm{T}}(s-j\delta)]^{\mathrm{T}},\ j = 1, \cdots, m-1.$$

Taking derivative of $V(t, x_t)$ along the trajectory of the system (2.4) yields:

$$\dot{V}(t, x_t) = \sum_{i=1}^{5} \dot{V}_i(x_t). \tag{3.6}$$

where

$$\dot{V}_1(x_t) = 2\eta_0^{\mathrm{T}}(t) P \dot{\eta}_0(t) = \zeta^{\mathrm{T}}(t) \Theta_1 \zeta(t), \tag{3.7}$$

$$\dot{V}_2(x_t) = \zeta_1^{\mathrm{T}}(t) X \zeta_1(t) - \zeta_1^{\mathrm{T}}(t-\delta) X \zeta_1(t-\delta) = \zeta^{\mathrm{T}}(t) \Theta_2 \zeta(t), \tag{3.8}$$

$$\dot{V}_3(x_t) = \sum_{j=1}^{m-1} [\eta_j^{\mathrm{T}}(t) R_j \eta_j(t) - \eta_j^{\mathrm{T}}(t-\delta) R_j \eta_j(t-\delta)] = \zeta^{\mathrm{T}}(t) \Theta_3 \zeta(t), \tag{3.9}$$

$$\dot{V}_4(x_t) \le \zeta^{\mathrm{T}}(t) \Theta_4(k) \zeta(t), \tag{3.10}$$

(3.11)

$$\dot{V}_5(x_t) = \dot{x}^{\mathrm{T}}(t) \bar{Z} \dot{x}(t) - \delta \int_{t-\delta}^{t} \dot{x}^{\mathrm{T}}(s) Z_0 \dot{x}(s) ds - \delta \sum_{j=1}^{m} \int_{t-j\delta}^{t-(j-1)\delta} \dot{x}^{\mathrm{T}}(s) Z_j \dot{x}(s) ds.$$

For the case of $\tau(t) \notin [(k-1)\delta,\ k\delta]$ and $\tau(t) \in [(k-1)\delta,\ k\delta],\ 1 \le k \le m$, applying Jensen's inequality and Lemma 2 to deal with the last integral item in

(3.11), respectively, it can be deduced for $\begin{bmatrix} Z_k & \widehat{S}_k \\ * & Z_k \end{bmatrix} \geq 0$ (where $\widehat{S}_k = \sum_{i=1}^{r} h_i S_{ik}$) that

(3.12)
$$\begin{aligned}
&-\delta \sum_{j=1}^{m} \int_{t-j\delta}^{t-(j-1)\delta} \dot{x}^{\mathrm{T}}(s) Z_j \dot{x}(s) ds \\
&\leq \sum_{j=1, j\neq k}^{m} \nu_1^{\mathrm{T}}(t) \begin{bmatrix} -Z_j & Z_j \\ * & -Z_j \end{bmatrix} \nu_1(t) + \varpi_1^{\mathrm{T}}(t) \begin{bmatrix} -Z_k & Z_k - \widehat{S}_k & \widehat{S}_k \\ * & -2Z_k + \mathrm{He}(\widehat{S}_k) & Z_k - \widehat{S}_k \\ * & * & -Z_k \end{bmatrix} \varpi_1(t) \\
&= \sum_{i=1}^{r} h_i \zeta^{\mathrm{T}}(t) \Theta_5(i,k) \zeta(t),
\end{aligned}$$

where $\nu_1(t) = \left[x^{\mathrm{T}}(t-(j-1)\delta), x^{\mathrm{T}}(t-j\delta)\right]^{\mathrm{T}}$, $\varpi_1(t) = [x^{\mathrm{T}}(t-(k-1)\delta), x^{\mathrm{T}}(t-\tau(t)), x^{\mathrm{T}}(t-k\delta)]^{\mathrm{T}}$.

On the other hand, it follows from Lemma 3 that

$$-\delta \int_{t-\delta}^{t} \dot{x}^{\mathrm{T}}(s) Z_j \dot{x}(s) ds \leq \zeta^{\mathrm{T}}(t) \Theta_0 \zeta(t). \tag{3.13}$$

By (3.6)-(3.13), the following inequality holds

$$\dot{V}(t, x_t) \leq \sum_{i=1}^{r} h_i \zeta^{\mathrm{T}}(t) \Theta(i,k) \zeta(t), \tag{3.14}$$

where $\Theta(i,k)$ are defined in Theorem 1.

In what follows, the system (2.4) with the augmented vector $\zeta(t)$ can be rewritten as:

$$0 = \sum_{i=1}^{r} h_i \Gamma_i \zeta(t)$$

where Γ_i $(i = 1, 2, \cdots, r)$ are defined in Theorem 1.

Therefore, the asymptotic stability conditions for the system (2.4) can be represented by

$$\begin{aligned}
&\sum_{i=1}^{r} h_i \zeta^{\mathrm{T}}(t) \Theta(i,k) \zeta(t) < 0, \\
&\textit{subject to}: \quad 0 = \sum_{i=1}^{r} h_i \Gamma_i \zeta(t).
\end{aligned} \tag{3.15}$$

By Finsler's lemma, for any matrix Y with appropriate dimension, the conditions in (3.15) are equivalent to

$$\sum_{i=1}^{r} h_i \zeta^{\mathrm{T}}(t) [\Theta(i,k) + \mathrm{He}(Y\Gamma_i)] \zeta(t) < 0. \tag{3.16}$$

Then, it follows from (3.14), (3.15), (3.16) and LMIs (3.3) that $\dot{V}(t, x_t) < 0$. Therefore, by Lyapunov-Krasovskii stability theorem [21], the system (2.4) with any delay $\tau(t)$ satisfying (2.2) and (2.3) is globally asymptotically stable. This completes the proof. □

Remark 1. Based on delay-partitioning approach, the new LKF (3.5) is different from those in [5, 7, 10, 14, 23] on account of the $[X_{ij}]_{m\times m}$-dependent integral item is considered. By employing such a $[X_{ij}]_{m\times m}$-dependent LKF, less conservative results can be achieved, which will be demonstrated later by numerical example.

Remark 2. The tighter bounding inequality, i.e., Peng-Park's integral inequality (Lemma 2), is employed to effectively estimate the time-varying delay-dependent integral term $-\delta\int_{t-k\delta}^{t-(k-1)\delta}\dot{x}^{\mathrm{T}}(s)Z_k\dot{x}(s)ds$. Therefore, significant improvement in both computational efficiency and performance behavior may be expected while inheriting the advantages of delay-partitioning method.

Remark 3. In the proof of Theorem 1, motivated by [7], some fuzzy-weighting matrices $\widehat{S}_k=\sum_{i=1}^{r}h_iS_{ik}$ are introduced to consider the relationships of the T-S fuzzy models, which will lead to less conservative results.

By means of Finsler's lemma, one can eliminate free variables which were used in zero equalities. Theorem 1 is based on the form of (iii) in Lemma 1. From Lemma 1, one can check that the ${B^{\perp}}^{\mathrm{T}}\Phi B^{\perp}<0$ is equivalent to the existence of Y such that $\Phi+\mathrm{He}(YB)<0$ holds. Therefore, we will propose another stability criterion based on the form of (ii) in Lemma 1, i.e. the following corollary.

Corollary 1. Given a positive integer m, scalars $\tau\geq 0$, μ, and $\delta=\frac{\tau}{m}$, then the T-S system (2.4) with a time-delay $\tau(t)$ satisfying (2.2) and (2.3) is asymptotically stable if there exist symmetric positive matrices $P=\begin{bmatrix} P_1 & P_2\\ * & P_3\end{bmatrix}$, $X=[X_{ij}]_{m\times m}$, Z_0, Z_j, $R_l=\begin{bmatrix} R_{1l} & R_{2l}\\ * & R_{3l}\end{bmatrix}$ and any matrices S_{ij} $(i=1,\cdots,r;\ j=1,\cdots,m; l=1,\cdots,m-1)$ with appropriate dimensions, such that the following LMIs hold for $i=1,\cdots,r$ and $k=1,\cdots,m$:

$$\Gamma_i^{\perp\mathrm{T}}\Theta(i,k)\Gamma_i^{\perp}<0, \tag{3.17}$$

$$\Psi(i,k)\geq 0, \tag{3.18}$$

where $\Theta(i,k)$, Γ_i and $\Psi(i,k)$ are defined in Theorem 1.

Finally, in the case of the time-varying delay $\tau(t)$ being non-differentiable or unknown $\dot{\tau}(t)$, setting $Q_k=0$ $(Q_j\neq 0, j=1,\cdots,k-1)$ in Theorem 1, one has the following corollary.

Corollary 2. Given a positive integer m, scalars $\tau\geq 0$, μ, and $\delta=\frac{\tau}{m}$, then the T-S system (2.4) with a time-delay $\tau(t)$ satisfying (2.2) is asymptotically stable if there exist symmetric positive matrices $P=\begin{bmatrix} P_1 & P_2\\ * & P_3\end{bmatrix}$, $X=[X_{ij}]_{m\times m}$, Z_0, Z_j, $R_l=\begin{bmatrix} R_{1l} & R_{2l}\\ * & R_{3l}\end{bmatrix}$ and any matrices Y and S_{ij} $(i=1,\cdots,r;\ j=1,\cdots,m; l=1,\cdots,m-1)$ with appropriate dimensions, such that the following LMIs hold for $i=1,\cdots,r$ and $k=1,\cdots,m$:

$$\widetilde{\Theta}(i,k)+\mathrm{He}(Y\Gamma_i)<0, \tag{3.19}$$

$$\Psi(i,k)\geq 0, \tag{3.20}$$

where

$$\widetilde{\Theta}(i,k) = \sum_{j=0}^{3} \Theta_j + \widetilde{\Theta}_4(k) + \Theta_5(i,k) + e_{m+4}\bar{Z}e_{m+4}^{\mathrm{T}}$$

with $\Gamma_i,\ \Psi(i,k),\ \Theta_0,\cdots,\Theta_3,\ \Theta_5(i,k)$ and $\bar{Z}$ are defined in Theorem 1 and $\widetilde{\Theta}_4(k) = \sum_{j=1}^{k-1}\left[e_{j+1}Q_j e_{j+1}^{\mathrm{T}} - e_{j+2}Q_j e_{j+2}^{\mathrm{T}}\right]$.

4. Numerical example

This section gives one example to demonstrate the effectiveness of the proposed approach. For comparisons, we study the T-S fuzzy system (2.4) with fuzzy rules investigated in recent publications [5, 7, 10, 14, 16, 18].

Example 1. Consider the following T-S fuzzy systems with time-varying delay and the following rules [7, 10]:

$$R^1: \text{ If } \theta(t) \text{ is } \pm\,\pi/2, \text{ then } x(t) = A_1x(t) + A_{d1}x(t-\tau(t));$$

$$R^2: \text{ If } \theta(t) \text{ is } 0, \text{ then } x(t) = A_2x(t) + A_{d2}x(t-\tau(t)).$$

where

$$A_1 = \begin{bmatrix} -2 & 0 \\ 0 & -0.9 \end{bmatrix},\ A_{d1} = \begin{bmatrix} -1 & 0 \\ -1 & -1 \end{bmatrix},$$

$$A_2 = \begin{bmatrix} -1 & 0.5 \\ 0 & -1 \end{bmatrix},\ A_{d2} = \begin{bmatrix} -1 & 0 \\ 0.1 & -1 \end{bmatrix}.$$

The membership functions for above rules 1 and 2 are $h_1(\theta(t)) = \sin^2(\theta(t)),\ h_2(\theta(t)) = \cos^2(\theta(t))$, where $\theta(t) = x_1(t)$.

For different μ, the Maximum allowable delay bounds of the time-varying delay computed by Theorem 1 with $m = 3, 4$ are listed in Table 1. For comparison, the upper bounds obtained by the conditions in [5, 7, 14, 16, 17] are also tabulated in Table 1. It is clear that the method proposed in this paper is less conservative than those in [5, 7, 14, 16, 17]. It is also concluded that the conservatism is gradually reduced with the increase of m.

TABLE 1. Maximum bounds of τ for different μ: Example 1

μ	0	0.1	≥ 1
Lien et al. [5]	1.59	1.48	0.83
Li et al.[16]	1.59	1.48	0.98
Liu et al. [17]	1.59	1.49	1.26
Kwon et al. [14]	1.66	1.53	1.27
Zeng et al. [7] (m=3)	2.00	1.81	1.36
Theorem 1 (m=3)	2.33	2.17	1.64
Theorem 1 (m=4)	2.49	2.33	1.83

5. Conclusion

In this paper, new stability criteria for T-S fuzzy systems with time-varying delay have been investigated by delay-partitioning approach, Finsler's lemma and LMI approach. A modified augmented-LKF is established in the framework of state vector augmentation. Then, by virtue of employing some tighter bounding inequalities

(Seuret-Wirtinger's integral inequality and Peng-Park's integral inequality) to deal with (time-varying) delay-dependent integral items, none of any useful time-varying items are arbitrarily ignored, therefore, less conservative results can be expected. At last, the effectiveness and merits of the theoretical results has been demonstrated by one numerical example.

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