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The Stability of a Class of System of the Predatory Functional Responses with Holling-III Style

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Abstract. Aclass of system of the predatory functional responses with Holling-III style and Logistic growth rate is considered in this paper. Then singular point of the system is obtained. Finally, the sufficient conditions of partial and global stability of the positive equilibrium of the system are obtained.

Keywords: Holling-III style; Logistic growth rate, partial and global stability

1. Introduction

In ecology, lots of single population models had been investigated. However, in nature, population is not dependent rather than co-exist with other population. They cooperate or compete between each other that is coming into being the complex ecosystem [1]. In this paper, we discuss a model of ecosystem with two populations. And the paper is organized as follows. In section 2, we give the model which we research. In section3, the partial and global stability have been investigated. Section 4 contains conclusions.

2. The model

In this paper, we discuss the model as follows:

$$\begin{cases} \dfrac{dx}{dt} = x(k - Ax) - \dfrac{C_1 x^2 y}{1+Bx^2} \\ \dfrac{dy}{dt} = \dfrac{C_2 x^2 y}{1+Bx^2} - Dy \end{cases} \tag{1}$$

where x , y respectively present the density of prey populationand predator population, and $k, A, B, C_i, D_i \ (i = 1, 2)$ are positive constant.

3. Stability of system

(i) Simplifying the system parameter [2]

Let $x_1 = ax, y_1 = by, t_1 = lt, \dfrac{B}{a^2} = 1, \dfrac{D}{l} = 1, \dfrac{C_1}{lab} = 1$

We can get the system from (1)

$$\begin{cases} \dfrac{dx_1}{dt_1} = x_1\left(\dfrac{K}{D} - \dfrac{A}{D\sqrt{B}}x_1\right) - \dfrac{x_1^2 y_1}{1+x_1^2} \\ \dfrac{dy_1}{dt_1} = \dfrac{\dfrac{C_1}{DB}x_1^2 y_1}{1+x_1^2} - y_1 \end{cases} \tag{2}$$

Then we let $a_1 = \dfrac{K}{D}, a_2 = \dfrac{A}{\sqrt{B}D}, a_3 = \dfrac{C_2}{DB}$,

so we have $\begin{cases} \dfrac{dx_1}{dt_1} = x_1\left(a_1 - a_2 x_1\right) - \dfrac{x_1^2 y_1}{1+x_1^2} \\ \dfrac{dy_1}{dt_1} = \dfrac{a_3 x_1^2 y_1}{1+x_1^2} - y_1 \end{cases}$ (3)

(ii) Calculating the singular point [3].

The system has three singular points:

$E_0\,(0,0)$, $E_1\left(\dfrac{a_1}{a_2},0\right)$, $E^*\left(\dfrac{1}{\sqrt{a_3-1}}, \dfrac{a_1 a_3}{\sqrt{a_3-1}} - \dfrac{a_2 a_3}{a_3-1}\right)$

where the third one should satisfy $a_3 > 1$ and $a_1\sqrt{a_3-1} - a_2 > 0$.

(iii) Partialstability of the singular point

The coefficient matrix of (3) around singular points is $J(E) = \begin{pmatrix} P_x & P_y \\ Q_x & Q_y \end{pmatrix}$,

where $P_x = \left(a_1 - a_2 x - \dfrac{xy}{1+x^2}\right) - \dfrac{x(1-x^2)y}{(1+x^2)^2} - a_2 x = a_1 - 2a_2 x - \dfrac{2xy}{(1+x^2)^2}$

$P_y = -\dfrac{x^2}{1+x^2}$

$Q_x = \dfrac{2a_3 xy}{(1+x^2)^2}$

$Q_y = \dfrac{a_3 x^2}{1+x^2} - 1$

(a) $E_0 = (0 \quad 0)$ $J(E_0) = \begin{pmatrix} a_1 & 0 \\ 0 & -1 \end{pmatrix}$

Characteristic roots are $\lambda_1 = a_1 > 0,\ \lambda_2 = -1 > 0,$ so E_0 is the saddle point.

(b) $E_1 = \left(\dfrac{a_1}{a_2}, 0\right)$

$$J(E_1)=\begin{pmatrix} -a_1 & -\dfrac{\left(\dfrac{a_1}{a_2}\right)^2}{1+\left(\dfrac{a_1}{a_2}\right)^2} \\ 0 & a_3-\dfrac{a_3}{1+\left(\dfrac{a_1}{a_2}\right)^2}-1 \end{pmatrix}$$

$$\lambda_1=-a_1<0 \quad \lambda_2=a_3-1-\frac{a_2^2a_3}{a_2^2+a_1^2}=a_3-\frac{a_1^2}{a_1^2+a_2^2}-1$$

Discussing the positive and negative: if it is positive, E_1 isthe saddle point; otherwise E_1 is the stable point.

(1) When $a_3-1<\dfrac{1}{\left(\dfrac{a_1}{a_2}\right)^2}$,that is when $\dfrac{a_1}{a_2}<\dfrac{1}{\sqrt{a_3-1}}$, E_1 is the stable point, that is E_1 is a symptotically stable.

(2) When $\dfrac{a_1}{a_2}>\dfrac{1}{\sqrt{a_3-1}}$, E_1 is not stable.

(c) $E^*=\left(\dfrac{1}{\sqrt{a_3-1}},\dfrac{a_1a_3}{\sqrt{a_3-1}}-\dfrac{a_2a_3}{a_3-1}\right)$

We can get the characteristic equation:

$$\lambda^2-\frac{\left(a_1-2a_2x^*\right)x^{*2}-a_1}{1+x^{*2}}\lambda+\frac{2y^*}{a_3{}^2x^{*3}}=0,$$

and $\lambda_{1,2}=\dfrac{Tr\left(E^*\right)\pm\sqrt{Tr^2\left(E^*\right)-4\det\left(E^*\right)}}{2}$, where $Tr\left(E^*\right)<0$.

Then putting E^* in $Tr\left(E^*\right)<0$, we can get $\dfrac{a_1}{a_2}\sqrt{a_3-1}\left(2-a_3\right)>2$.

If $\dfrac{a_1}{a_2}>\dfrac{1}{\sqrt{a_3-1}}$, we can have

(1) $\dfrac{a_1}{a_2}\sqrt{a_3-1}\left(2-a_3\right)>2$, E^* is not stable;

(2) $\dfrac{a_1}{a_2}\sqrt{a_3-1}\left(2-a_3\right)<2$, E^* is stable.

(iv) Global stability of the singular point

In order to investigate the global stability of the singular point, we should study the limit cycle of system first.

Now we construct the outer boundary of the limit cycle which is $l_1, l_2,\ l_3$ and l_4, where

$l_1 : x = \dfrac{a_1}{a_2}$; $l_2 : y + x - k = 0$; $l_3 : x = 0$; $l_4 : y = 0$.

$(a)\ l_1 : x = \dfrac{a_1}{a_2}$

When $y > 0$, we can get $\dfrac{dx}{dt}|_{x=\frac{a_1}{a_2}} = -\dfrac{xy}{1+x^2} < 0$. So the path is going through l_1 from its right to its left.

$(b)\ l_2 : y + x - k = 0$

We can get $\dfrac{dl}{dt}|_{l=0} = x(a_1 - a_2x) - \dfrac{x^2y}{1+x^2} + \dfrac{a_3x^2y}{1+x^2} - y$

$= x(a_1 - a_2x) - \dfrac{x^2y}{1+x^2} + \dfrac{a_3x^2y}{1+x^2} - x - k$

When k is large enough, $\dfrac{dl}{dt}|_{l=0} < 0$ can be got.

So the path is going through l_2 from its upper right to its lower left.

l_3 and l_4 are both the path. So we can get a region G which have outer boundary of l_1, l_2 , l_3 and l_4. There is no singular point without E^*. $O(0,0)$ and $E_1(x_1,0)$ which are the singular points on ∂G are the saddle points. So the path G cannot get in G from forward direction. The system at least has one stablelimit cycle which contains E^* in G.

4. Conclusion

In this paper, we discuss a model of ecosystem with two populations with Holling-III style and Logistic growth rate. And we investigate the stability of thesystem of the predatory functional responses by discussing characteristic roots. Lastly, the sufficient conditions of the partial and global stability have been had.

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