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New Operations on Intuitionistic Fuzzy Multisets

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Abstract. In this paper we proposed some new operations on intuitionistic fuzzy multisets (IFMSs), deduced some theorems with respect to the algebra of IFMSs and modal operators on IFMSs.

Keywords: algebra, intuitionistic fuzzy sets, intuitionistic fuzzy multisets, modal operator, operations

AMS Mathematics Subject Classifications (2010): 03E72

1. Introduction

The concept of intuitionistic fuzzy sets (IFSs) was introduced by Atanassov [2] as a generalization of fuzzy set proposed earlier by Zadeh [12]. Shyamal *et al.* [1] studied distance of intuitionistic fuzzy set and discussed interval valued intuitionistic fuzzy set. Some further studies in this direction can be seen in [13-21]. Shinoj and Sunil [8] proposed the concept of intuitionistic fuzzy multisets (IFMSs); theoretical views of IFMSs and some applications were given in [4-7, 9-10].

In this research, we introduce some new operations on IFMSs as an extension of the works in [3, 11] and deduce some new results in IFMSs.

2. Concise note of intuitionistic fuzzy multisets

Definition 1. [8] Let X be a nonempty set. An IFMS A drawn from X is characterized by two functions: "count membership" of A denoted as CM_A and "count non-membership" of A denoted as CN_A given respectively by $CM_A: X \longrightarrow Q$ and $CN_A: X \longrightarrow Q$ where Q is the set of all crisp multisets drawn from the unit interval [0,1] s.t. for each $x \in X$, the membership sequence is defined as a decreasingly ordered sequence of elements in $CM_A(x)$ and it is denoted as $(\mu_A^1(x), \mu_A^2(x), \ldots, \mu_A^n(x))$, where $\mu_A^1(x) \geq \mu_A^2(x) \geq \ldots \geq \mu_A^n(x)$ whereas the corresponding non-membership sequence of elements in $CN_A(x)$ is denoted by $(\nu_A^1(x), \nu_A^2(x), \ldots, \nu_A^n(x))$ *s.t.* $0 \leq \mu_A^i(x) + \nu_A^i(x) \leq 1$ for every $x \in X$ and $i = 1, \ldots, n$. This means, an IFMS A is defined as; $A = \{\langle x, CM_A(x), CN_A(x)\rangle : x \in X\}$ or $A = \{\langle x, \mu_A^i(x), \nu_A^i(x)\rangle : x \in X\}$, *for* $i = 1, \ldots, n$.

For each IFMS A in X, $\pi_A^i(x) = 1 - \mu_A^i(x) - \nu_A^i(x)$ is the intuitionistic fuzzy multisets index or hesitation margin of x in A. The hesitation margin $\pi_A^i(x)$ for each $i = 1, \ldots, n$ is the degree of non-determinancy of $x \in X$, to the set A and $\pi_A^i(x) \in$

$[0,1]$. Similarly, $\pi_A^i(x)$as in IFS, is the function that expresses lack of knowledge of whether $x \in A$ or $x \notin A$.

In general, an IFMS A is given as $A = \{\langle x, \mu_A^i(x), \nu_A^i(x), \pi_A^i(x)\rangle : x \in X\}$, *or* $\{\langle x, \mu_A^i(x), \nu_A^i(x), 1 - \mu_A^i(x) - \nu_A^i(x)\rangle : x \in X\}$, or $\{\langle x, \mu_A^i(x), 1 - \mu_A^i(x) - \pi_A^i(x), \pi_A^i(x)\rangle : x \in X\}$, *or* $\{\langle x, 1 - \nu_A^i(x) - \pi_A^i(x), \nu_A^i(x), \pi_A^i(x)\rangle : x \in X\}$ since $\mu_A^i(x) + \nu_A^i(x) + \pi_A^i(x) = 1$ for each $i = 1, . . . , n$.

Definition 2. We define IFMS alternatively. Let X be nonempty set. An IFMS A drawn from X is given as $A = \{\langle \mu_A^1(x), \ldots, \mu_A^n(x), \ldots, \nu_A^1(x), \ldots, \nu_A^n(x), \ldots\rangle : x \in X\}$ where the functions $\mu_A^i(x), \nu_A^i(x) : X \to [0,1]$ define the belongingness degrees and the non-belongingness degrees of A in X *s.t.* $0 \leq \mu_A^i(x) + \nu_A^i(x) \leq 1$ for $i = 1, \ldots$ If the sequence of the membership functions and non-membership (belongingness functions and non-belongingness functions) have only n-terms (i.e. finite), n is called the '*dimension*' of A. Consequently $A = \{\langle \mu_A^1(x), \ldots, \mu_A^n(x), \nu_A^1(x), \ldots, \nu_A^n(x)\rangle : x \in X\}$ for $i = 1, \ldots, n$. when no ambiguity arises, we define $A = \{\langle \mu_A^i(x), \nu_A^i(x)\rangle : x \in X\}$ for $i = 1, \ldots, n$.

Definition 3 (modal operators). [7]Let X be nonempty. If A is an IFMS drawn from X, then;

(i) $A = \{\langle x, \mu_A^i(x)\rangle : x \in X\} = \{\langle x, \mu_A^i(x), 1 - \mu_A^i(x)\rangle : x \in X\}$

(ii) $\Diamond A = \{\langle x, 1 - \nu_A^i(x)\rangle : x \in X\} = \{\langle x, 1 - \nu_A^i(x), \nu_A^i(x), \rangle : x \in X\}$, for each $i = 1,2, \ldots, n$.

Operations on intuitionistic fuzzy multisets [8]

For any two IFMSs A and B drawn from X, the following operations hold. Let $A = \{\langle x, \mu_A^i(x), \nu_A^i(x)\rangle : x \in X\}$and $B = \{\langle x, \mu_B^i(x), \nu_B^i(x)\rangle : x \in X\}$, for each $i = 1,2, \ldots, n$.

1. Complement: $A^c = \{\langle x, \nu_A^i(x), \mu_A^i(x)\rangle : x \in X\}$
2. Union: $A \cup B = \{\langle x, max\,(\mu_A^i(x), \mu_B^i(x)), min\,(\nu_A^i(x), \nu_B^i(x))\rangle : x \in X\}$
3. Intersection: $A \cap B = \{\langle x, min\,(\mu_A^i(x), \mu_B^i(x)), max\,(\nu_A^i(x), \nu_B^i(x))\rangle : x \in X\}$
4. Addition: $A \oplus B = \{\langle x, \mu_A^i(x) + \mu_B^i(x) - \mu_A^i(x)\mu_B^i(x), \nu_A^i(x)\,\nu_B^i(x)\rangle : x \in X\}$
5. Multiplication: $A \otimes B = \{\langle x, \mu_A^i(x)\mu_B^i(x), \nu_A^i(x) + \nu_B^i(x) - \nu_A^i(x)\,\nu_B^i(x)\rangle : x \in X\}$

Algebraic laws in intuitionistic fuzzy multisets [6]

Let A, B and C be IFMSs in X, then the following algebra follow:

1. Complementary law: $(A^c)^c = A$
2. Idempotent laws: $(i)\ A \cup A = A \quad (ii)\ A \cap A = A$
3. Commutative laws: $(i)\ A \cup B = B \cup A\ (ii)\ A \cap B = B \cap A$
   $(iii) A \oplus B = B \oplus A\ (iv) A \otimes B = B \otimes A$
4. Associative laws: $(i) (A \cup B) \cup C = A \cup (B \cup C)$
   $(ii)\ (A \cap B) \cap C = A \cap (B \cap C)$
   $(iii) A \oplus (B \oplus C) = (A \oplus B) \oplus C$
   $(iv)\ A \otimes (B \otimes C) = (A \otimes B) \otimes C$

5. Distributive laws: $(i) A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$
$(ii)\ A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$
$(iii)\ A \oplus (B \cup C) = (A \oplus B) \cup (A \oplus C)$
$(iv) A \oplus (B \cap C) = (A \oplus B) \cap (A \oplus C)$
$(v)\ A \otimes (B \cup C) = (A \otimes B) \cup (A \otimes C)$
$(vi) A \otimes (B \cap C) = (A \otimes B) \cap (A \otimes C)$
6. De Morgan's laws: $(i)\ (A \cup B)^c = A^c \cap B^c$ $(ii)\ (A \cap B)^c = A^c \cup B^c$
$(iii)(A \oplus B)^c = A^c \otimes B^c (iv)(A \otimes B)^c = A^c \oplus B^c$
7. Absorption laws: $(i)\ A \cap (A \cup B) = A$ $(ii)\ A \cup (A \cap B) = A$

Note: Distributive laws hold for both right and left hands.

3. New operations on intuitionistic fuzzy multisets

For any two IFMSs A and B drawn from X, the following new operations hold. Let $A = \{\langle x, \mu_A^i(x), \nu_A^i(x)\rangle : x \in X\}$ and $B = \{\langle x, \rangle : x \in X\}$, for each $i = 1,2,\dots,n$.

1. $A@B = \{\langle x, \frac{1}{2}(\mu_A^i(x) + \mu_B^i(x)), \frac{1}{2}(\nu_A^i(x) + \nu_B^i(x))\rangle : x \in X\}$
2. $A\$B = \{\langle x, (\mu_A^i(x)\mu_B^i(x))^{\frac{1}{2}}, (\nu_A^i(x)\nu_B^i(x))^{\frac{1}{2}}\rangle : x \in X\}$
3. $A\#B = \{\langle x, \frac{2\mu_A^i(x)\mu_B^i(x)}{\mu_A^i(x)+\mu_B^i(x)}, \frac{2\nu_A^i(x)\nu_B^i(x)}{\nu_A^i(x)+\nu_B^i(x)}\rangle : x \in X\}$
4. $A * B = \{\langle x, \frac{\mu_A^i(x)+\mu_B^i(x)}{2(\mu_A^i(x)\mu_B^i(x)+1)}, \frac{\nu_A^i(x)+\nu_B^i(x)}{2(\nu_A^i(x)\nu_B^i(x)+1)}\rangle : x \in X\}$.

Note that for convenient seek, we may write $A = \{\langle x, \mu_A^i, \nu_A^i \rangle : x \in X\}$ in place of $A = \{\langle x, \mu_A^i(x), \nu_A^i(x)\rangle : x \in X\}$.

Theorem1. Let A, B and C be IFMSs in X, then; (i) $A@B = B@A$ (ii) $A\$B = B\$A$ $(i) A\#B = B\#A$ (iv) $A * B = B * A$ (v) $\overline{A@B} = A@B$ (vi) $\overline{A\$B} = A\$B$

(vii) $\overline{A\#B} = A\#B$ (viii) $\overline{A * B} = A * B$

Proof:

$(i) A@B = \{\langle x, \frac{1}{2}(\mu_A^i(x) + \mu_B^i(x)), \frac{1}{2}(\nu_A^i(x) + \nu_B^i(x))\rangle : x \in X\}$

$= \{\langle x, \frac{1}{2}(\mu_B^i(x) + \mu_A^i(x)), \frac{1}{2}(\nu_B^i(x) + \nu_A^i(x))\rangle : x \in X\} = B@A$

(ii) $A\$B = \{\langle x, (\mu_A^i(x)\mu_B^i(x))^{\frac{1}{2}}, (\nu_A^i(x)\nu_B^i(x))^{\frac{1}{2}}\rangle : x \in X\}$

$= \{\langle x, (\mu_B^i(x)\mu_A^i(x))^{\frac{1}{2}}, (\nu_B^i(x)\nu_A^i(x))^{\frac{1}{2}}\rangle : x \in X\} = B\$A$

(iii) $A\#B = \{\langle x, \frac{2\mu_A^i(x)\mu_B^i(x)}{\mu_A^i(x)+\mu_B^i(x)}, \frac{2\nu_A^i(x)\nu_B^i(x)}{\nu_A^i(x)+\nu_B^i(x)}\rangle : x \in X\}$

$$= \left\{\langle x, \frac{2\mu_B^i(x)\mu_A^i(x)}{\mu_B^i(x) + \mu_A^i(x)}, \frac{2\nu_B^i(x)\nu_A^i(x)}{\nu_B^i(x) + \nu_A^i(x)}\rangle : x \in X\right\} = B\#A$$

(iv) $A * B = \{\langle \mathrm{x}, \frac{\mu_A^i(x)+\mu_B^i(x)}{2(\mu_A^i(x)\mu_B^i(x)+1)}, \frac{\nu_A^i(x)+\nu_B^i(x)}{2(\nu_A^i(x)\nu_B^i(x)+1)} \rangle : x \in \mathrm{X}\}$

$$= \left\{\langle \mathrm{x}, \frac{\mu_B^i(x)+\mu_A^i(x)}{2(\mu_B^i(x)\mu_A^i(x)+1)}, \frac{\nu_B^i(x)+\nu_A^i(x)}{2(\nu_B^i(x)\nu_A^i(x)+1)} \rangle : x \in \mathrm{X}\right\} = \mathrm{B} * \mathrm{A}$$

$(v)\ \overline{A@B} = \{\langle \mathrm{x}, \frac{1}{2}(\nu_A^i(x)+\nu_B^i(x)), \frac{1}{2}(\mu_A^i(x)+\mu_B^i(x))\rangle : x \in \mathrm{X}\}$

$= \{\langle \mathrm{x}, \frac{1}{2}(\mu_A^i(x)+\mu_B^i(x)), \frac{1}{2}(\nu_A^i(x)+\nu_B^i(x))\rangle : x \in \mathrm{X}\} = A@B$

(vi) $\overline{A\$B} = \{\langle \mathrm{x}, (\nu_A^i(x)\nu_B^i(x))^{\frac{1}{2}}, (\mu_A^i(x)\mu_B^i(x))^{\frac{1}{2}}\rangle : x \in \mathrm{X}\}$

$= \{\langle \mathrm{x}, (\mu_A^i(x)\mu_B^i(x))^{\frac{1}{2}}, (\nu_A^i(x)\nu_B^i(x))^{\frac{1}{2}}\rangle : x \in \mathrm{X}\} = A\$B$

(vii) $\overline{A\#B} = \{\langle \mathrm{x}, \frac{2\nu_A^i(x)\nu_B^i(x)}{\nu_A^i(x)+\nu_B^i(x)}, \frac{2\mu_A^i(x)\mu_B^i(x)}{\mu_A^i(x)+\mu_B^i(x)}\rangle : x \in \mathrm{X}\}$

$$= \left\{\langle \mathrm{x}, \frac{2\mu_A^i(x)\mu_B^i(x)}{\mu_A^i(x)+\mu_B^i(x)}, \frac{2\nu_A^i(x)\nu_B^i(x)}{\nu_A^i(x)+\nu_B^i(x)}\rangle : x \in \mathrm{X}\right\} = A\#B$$

(viii) $\overline{A * B} = \left\{\langle \mathrm{x}, \frac{\nu_A^i(x)+\nu_B^i(x)}{2(\nu_A^i(x)\nu_B^i(x)+1)}, \frac{\mu_A^i(x)+\mu_B^i(x)}{2(\mu_A^i(x)\mu_B^i(x)+1)} \rangle : x \in \mathrm{X}\right\}$

$= \left\{\langle \mathrm{x}, \frac{\mu_A^i(x)+\mu_B^i(x)}{2(\mu_A^i(x)\mu_B^i(x)+1)}, \frac{\nu_A^i(x)+\nu_B^i(x)}{2(\nu_A^i(x)\nu_B^i(x)+1)} \rangle : x \in \mathrm{X}\right\} = \mathrm{A} * \mathrm{B}.$

Theorem 2. Let A, B and C be IFMSs in X, then;
(i) $A@(B \cap C) = (A@B) \cap (A@C)$
(ii) $A@(B \cup C) = (A@B) \cup (A@C)$(iii) $A\#(B \cap C) = (A\#B) \cap (A\#C)$
(iv) $A\#(B \cup C) = (A\#B) \cup (A\#C)$(v) $A\$(B \cap C) = (A\$B) \cap (A\$C)$
(vi) $A\$(B \cup C) = (A\$B) \cup (A\$C)$.
Proof:
(i) $B \cap C = \{\langle x, min\ (\mu_B^i, \mu_C^i), max\ (\nu_B^i, \nu_C^i)\rangle : x \in X\}$

$A@(B \cap C) = \{\langle \mathrm{x}, \frac{\mu_A^i + \min(\mu_B^i, \mu_C^i)}{2}, \frac{\nu_A^i + \max(\nu_B^i, \nu_C^i)}{2}\rangle : \mathrm{x} \in \mathrm{X}\}$
$= \left\{\langle \mathrm{x}, \frac{1}{2}(\mu_A^i + \mu_B^i), \frac{1}{2}(\nu_A^i + \nu_B^i)\rangle : x \in \mathrm{X}\right\} \cap \left\{\langle \mathrm{x}, \frac{1}{2}(\mu_A^i + \mu_C^i), \frac{1}{2}(\nu_A^i + \nu_C^i)\rangle : x \in \mathrm{X}\right\}$
$= (A@B) \cap (A@C)$.
(ii) $B \cup C = \{\langle x, max\ (\mu_B^i, \mu_C^i), min\ (\nu_B^i, \nu_C^i)\rangle : x \in X\}$

$A@(B \cup C) = \{\langle \mathrm{x}, \frac{\mu_A^i + \max(\mu_B^i, \mu_C^i)}{2}, \frac{\nu_A^i + \min(\nu_B^i, \nu_C^i)}{2}\rangle : \mathrm{x} \in \mathrm{X}\}$

$=\left\{\langle \mathrm{x}, \frac{1}{2}\left(\mu_A^i + \mu_B^i\right), \frac{1}{2}\left(\nu_A^i + \nu_B^i\right)\rangle : x \in \mathrm{X}\right\} \cup \left\{\langle \mathrm{x}, \frac{1}{2}\left(\mu_A^i + \mu_C^i\right), \frac{1}{2}\left(\nu_A^i + \nu_C^i\right)\rangle : x \in \mathrm{X}\right\}$

$= (A@B) \cup (A@C)$

(iii) $B \cap C = \{\langle x, min\ (\mu_B^i, \mu_C^i),\ max\ (\nu_B^i, \nu_C^i)\rangle : x \in X\}$

$A\#(B \cap C) = \{\langle \mathrm{x}, \frac{2\mu_A^i \min(\mu_B^i, \mu_C^i)}{\mu_A^i + \min(\mu_B^i, \mu_C^i)}, \frac{2\nu_A^i \max(\nu_B^i, \nu_C^i)}{\nu_A^i + \max(\nu_B^i, \nu_C^i)}\rangle : \mathrm{x} \in \mathrm{X}\}$
$= \{\langle \mathrm{x}, \frac{2\mu_A^i \mu_B^i}{\mu_A^i + \mu_B^i}, \frac{2\nu_A^i \nu_B^i}{\nu_A^i + \nu_B^i}\rangle : x \in \mathrm{X}\} \cap \{\langle \mathrm{x}, \frac{2\mu_A^i \mu_C^i}{\mu_A^i + \mu_C^i}, \frac{2\nu_A^i \nu_C^i}{\nu_A^i + \nu_C^i}\rangle : x \in \mathrm{X}\}$
$= (A\#B) \cap (A\#C)$
(iv) $A\#(B \cup C) = \{\langle \mathrm{x}, \frac{2\mu_A^i \max(\mu_B^i, \mu_C^i)}{\mu_A^i + \max(\mu_B^i, \mu_C^i)}, \frac{2\nu_A^i \min(\nu_B^i, \nu_C^i)}{\nu_A^i + \min(\nu_B^i, \nu_C^i)}\rangle : \mathrm{x} \in \mathrm{X}\}$
$= \{\langle \mathrm{x}, \frac{2\mu_A^i \mu_B^i}{\mu_A^i + \mu_B^i}, \frac{2\nu_A^i \nu_B^i}{\nu_A^i + \nu_B^i}\rangle : x \in \mathrm{X}\} \cup \{\langle \mathrm{x}, \frac{2\mu_A^i \mu_C^i}{\mu_A^i + \mu_C^i}, \frac{2\nu_A^i \nu_C^i}{\nu_A^i + \nu_C^i}\rangle : x \in \mathrm{X}\}$
$= (A\#B) \cup (A\#C)$
(v) $A\$(B \cap C) = \{\langle \mathrm{x}, (\mu_A^i \min(\mu_B^i, \mu_C^i))^{\frac{1}{2}}, (\nu_A^i \max(\nu_B^i, \nu_C^i))^{\frac{1}{2}}\rangle : x \in \mathrm{X}\}$
$= \{\langle \mathrm{x}, (\mu_A^i \mu_B^i)^{\frac{1}{2}}, (\nu_A^i \nu_B^i)^{\frac{1}{2}}\rangle : x \in \mathrm{X}\} \cap \{\langle \mathrm{x}, (\mu_A^i \mu_C^i)^{\frac{1}{2}}, (\nu_A^i \nu_C^i)^{\frac{1}{2}}\rangle : x \in \mathrm{X}\}$
$= (A\$B) \cap (A\$C)$
(vi) $A\$(B \cup C) = \{\langle \mathrm{x}, (\mu_A^i \max(\mu_B^i, \mu_C^i))^{\frac{1}{2}}, (\nu_A^i \min(\nu_B^i, \nu_C^i))^{\frac{1}{2}}\rangle : x \in \mathrm{X}\}$
$= \{\langle \mathrm{x}, (\mu_A^i \mu_B^i)^{\frac{1}{2}}, (\nu_A^i \nu_B^i)^{\frac{1}{2}}\rangle : x \in \mathrm{X}\} \cup \{\langle \mathrm{x}, (\mu_A^i \mu_C^i)^{\frac{1}{2}}, (\nu_A^i \nu_C^i)^{\frac{1}{2}}\rangle : x \in \mathrm{X}\}$
$= (A\$B) \cup (A\$C)$.

Theorem 3. Let A, B and C be IFMSs in X, then; (i) $\Box(A@B) = \Box A@\Box B$ (ii) $\Box(A\$B) = \Box A\$\Box B$
(iii) $\Box(A\#B) = \Box A\#\Box B$ (iv) $\Diamond(A@B) = \Diamond A@\Diamond B$ (v) $\Diamond(A\$B) = \Diamond A\$\Diamond B$ (vi) $\Diamond(A\#B) = \Diamond A\#\Diamond B$
Proof:
(i) $A@B = \{\langle \mathrm{x}, \frac{1}{2}(\mu_A^i + \mu_B^i), \frac{1}{2}(\nu_A^i + \nu_B^i)\rangle : x \in \mathrm{X}\}$
$\Box(A@B) = \{\langle \mathrm{x}, \frac{1}{2}(\mu_A^i + \mu_B^i)\rangle : x \in \mathrm{X}\} = \Box A@\Box B$
(ii) $A\$B = \{\langle \mathrm{x}, (\mu_A^i \mu_B^i)^{\frac{1}{2}}, (\nu_A^i \nu_B^i)^{\frac{1}{2}}\rangle : x \in \mathrm{X}\}$
$\Box(A\$B) = \{\langle \mathrm{x}, \left(\mu_A^i \mu_B^i\right)^{\frac{1}{2}}\rangle : x \in \mathrm{X}\right\} = \Box A\$\Box B$
(iii) $A\#B = \{\langle \mathrm{x}, \frac{2\mu_A^i \mu_B^i}{\mu_A^i + \mu_B^i}, \frac{2\nu_A^i \nu_B^i}{\nu_A^i + \nu_B^i}\rangle : x \in \mathrm{X}\}$
$\Box(A\#B) = \{\langle \mathrm{x}, \frac{2\mu_A^i \mu_B^i}{\mu_A^i + \mu_B^i}\rangle : x \in \mathrm{X}\} = \Box A\#\Box B$
(vi) Let us take the hesitation margin to be zero,

$$A@B = \{\langle \mathrm{x}, \frac{1}{2}(1 - \nu_A^i + 1 - \nu_B^i), \frac{1}{2}(1 - \mu_A^i + 1 - \mu_B^i)\rangle : x \in \mathrm{X}\}$$

$$\lozenge(A@B) = \left\{\langle x, \frac{1}{2}\left(1-\nu_A^i+1-\nu_B^i\right)\rangle : x \in X\right\} = \lozenge A@\lozenge B$$

(v) Let us take the hesitation margin to be zero,

$$A\$B = \{\langle x, ([1-\nu_A^i][1-\nu_B^i])^{\frac{1}{2}}, ([1-\mu_A^i][1-\mu_B^i])^{\frac{1}{2}}\rangle : x \in X\}$$

$$\lozenge(A\$B) = \{\langle x, \left([1-\nu_A^i][1-\nu_B^i]\right)^{\frac{1}{2}}\rangle : x \in X\right\} = \lozenge A\$\lozenge B$$

(vi) Let us take the hesitation margin to be zero,

$$A\#B = \{\langle x, \frac{2[1-\nu_A^i][1-\nu_B^i]}{1-\nu_A^i+1-\nu_B^i}, \frac{2[1-\mu_A^i][1-\mu_B^i]}{1-\mu_A^i+1-\mu_B^i}\rangle : x \in X\}$$

$$\lozenge(A\#B) = \{\langle x, \frac{2[1-\nu_A^i][1-\nu_B^i]}{1-\nu_A^i+1-\nu_B^i}\rangle : x \in X\} = \lozenge A\#\lozenge B.$$

4. Conclusion

We conclude that $@, \$, \#$ and $*$ are not associative. Again, $@, \$$ and $\#$ are idempotent and $*$ is not. Let us see these:

$$A@A = \left\{\langle x, \frac{1}{2}\left(\mu_A^i+\mu_A^i\right), \frac{1}{2}\left(\nu_A^i+\nu_A^i\right)\rangle : x \in X\right\} = \left\{\langle x, \frac{1}{2}\left(2\mu_A^i\right), \frac{1}{2}\left(2\nu_A^i\right)\rangle : x \in X\right\}$$

$$= \left\{\langle x, \mu_A^i, \nu_A^i\rangle : x \in X\right\} = A$$

$$A\$A = \{\langle x, (\mu_A^i\mu_A^i)^{\frac{1}{2}}, \nu_A^i\nu_A^i)^{\frac{1}{2}}\rangle : x \in X\} = \{\langle x, ([\mu_A^i]^2)^{\frac{1}{2}}, ([\nu_A^i]^2)^{\frac{1}{2}}\rangle : x \in X\}$$

$$= \left\{\langle x, \mu_A^i, \nu_A^i\rangle : x \in X\right\} = A$$

$$A\#A = \left\{\langle x, \frac{2\mu_A^i\mu_A^i}{\mu_A^i+\mu_A^i}, \frac{2\nu_A^i\nu_A^i}{\nu_A^i+\nu_A^i}\rangle : x \in X\right\} = \left\{\langle x, \frac{2[\mu_A^i]^2}{2\mu_A^i}, \frac{2[\nu_A^i]^2}{2\nu_A^i}\rangle : x \in X\right\}$$

$$= \left\{\langle x, \mu_A^i, \nu_A^i\rangle : x \in X\right\} = A.$$

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