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Performance Evaluation of Spatial Modulation via Numerical Simulations

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Abstract. Spatial Modulation (SM) is a recently developed low-complexity Multiple-Input Multiple-Output scheme that jointly uses antenna indices and a conventional constellation set to convey information. Furthermore, many developed SM systems has been proposed to mitigate the limitations of basic SM systems. In this paper, different types of spatial modulation techniques have been evaluated via simulations. Basic spatial modulation, differential spatial modulation (DSM) and space-time block coded spatial modulation (STBC-SM) which combines spatial modulation and space-time block coding (STBC) are both simulated in time-invariant channel environment and in time-varying channel environment. The simulation results are compared to each other in terms of Bit Error Rate (BER) performance, leading to some interesting observations and useful conclusions. First, STBC-SM system has taken advantage of the benefits of SM and STBC both while avoiding their drawback, resulting in the best BER performance among the three communication systems. Moreover, DSM systems all have much better BER performance than other systems in non-correlated channel environment.

Keywords: Spatial modulation; space-time block coding; differential spatial modulation; spectral efficiency; bit error rate

1. Introduction

Spatial Modulation [1] (SM) is a recently developed low-complexity Multiple-Input Multiple-Output (MIMO) scheme that jointly uses antenna indices and a conventional constellation set to convey information. Compared with the conventional Single-Input Single-Output (SISO) system, SM system has a bigger Shannon Capacity, which is not able to mitigate in SISO system. Meanwhile, compared to the conventional MIMO system, SM system is free from Inter-Channel Interference (IAI) and Inter-Antenna Synchronization (IAS), which is achieved just by one transmit-antenna being activated for data transmission at any signaling time instance. On the other hand, SM system reduced the number of wireless communication channels which contains most of the cost. Furthermore, as conventional MIMO system needs degree of freedom which offered by a number of received antennas to distinguish the different independent transmit signal float. Usually, The number of receive antennas must not be less than the number of transmit antennas, which is not always can be realized in communication system obviously.

However, it has been shown that the complexity of the search detector at the receiver of a basic SM system grows linearly with the increasing of spectral efficiency. To circumvent the problem, people combines SM and space-time block coding (STBC)[2] to generate an improved communication system, which is called space-time block coded spatial modulation (STBC-SM). In this scheme, the transmitted information symbols are expanded not only to both of the space and time domains but also to the spatial (antenna) domain which corresponds to the on/off status of the transmit antennas available at the space domain, and therefore both core STBC and antenna indices carry information. Although the spectral efficiency has been greatly improved, a low-complexity maximum likelihood (ML) decoder is given for the new scheme which profits from the orthogonality of the core STBC in [3].

Since STBC-SM system has circumvent the problem of spectral efficiency, there is still a problem in channel state information. So far, the former two types of SM systems all have assumed that accurate channel state information is available at the receiver. Nevertheless, sometimes, it is costly and difficultly to obtain accurate channel state information especially in high-mobility situations. Therefore, differential signaling has been taken into SM systems and thus develop a Differential Spatial Modulation (DSM) scheme [4].This developed scheme keeps the key feature of SM that only one antenna is active at any symbol instance. Therefore, ICI is avoided and the requirement of IAS is relaxed. Then, the transmission matrix is differentially encoded and transmitted. At the receiver, Maximum Likelihood (ML) detection is applied.

The rest of the paper is organized as follows. In Section 2, briefly review of the SM system model has been given. In Section 3, STBC-SM and DSM has been briefly reviewed. In Section 4, performance analysis of the three scheme are given theoretically and demonstrated via simulations, leading to some interesting observations and useful conclusions. In Section 5, we conclude this paper.

2. Basic spatial modulation

Basic SM system having N_T transmit as well as N_R receive antennas and communicating over a quasi-static, frequency flat fading channel, can be modeled as:

$$\mathbf{y} = \mathbf{H}\mathbf{x}_{l,s} + \mathbf{N} \tag{1}$$

where $\mathbf{x}_{l,s}$ is modeled as the transmitted vector. Since one transmit-antenna being activated for data transmission at any signaling time instance, there is only one non-zero value in the vector, which is of the form:

$$\mathbf{x}_{l,s} = [\underbrace{0,\cdots,0}_{l-1},s,\underbrace{0,\cdots,0}_{N_T-l}]^T \tag{2}$$

where l is an arbitrary number during $[1, N_T]$, s is a complex symbol from the signal set S with |S| = M, and $\mathbf{y}$ is modeled as the received vector. The channel matrix $\mathbf{H}$ is the $N_R \times N_T$ fading matrix with the (i, j)-th entry, $h_{i,j}$, denoting the normalized complex fading gain from transmit antenna j to receive antenna i, $\mathbf{N}$ is the noise vector. The entries of the channel matrix and the noise vector are from $CN(0,1)$ and $CN(0, \sigma^2)$ such that $\sigma^2=\gamma/\rho$, respectively, where ρ is the average signal-to-noise ratio (SNR) at each receive antenna and γ is the average energy of the signal set S. Throughout this paper, all S are assumed to be a square- or a rectangular-QAM constellation. For basic SM system, we assumed that the receiver has already knew the channel matrix **H**. In such circumstance, the ML detector can be modeled as [5]:

$$\left(\hat{l},\hat{s}\right) = \arg\min_{l\in L, s\in S} \left\|\mathbf{y}-\mathbf{H}\mathbf{x}_{l,s}\right\|_2^2 \tag{3}$$

3. Space-time block coded spatial modulation

STBC-SM system is a scheme that has high spectral efficiency and simplified ML detection, both STBC symbols and the indices of the transmit antennas from which these symbols are transmitted, carry information. Take Alamouti's STBC for example. In Alamouti's STBC, two complex information symbols (x_1 and x_2) drawn from a M-QAM constellation are transmitted from two transmit antennas in two symbol intervals:

$$\mathbf{X} = \begin{bmatrix} x_1 & -x_2^* \\ x_2 & x_1^* \end{bmatrix} \tag{4}$$

where columns and rows correspond to the transmit antennas and the symbol intervals, respectively.

In STBC-SM system, two of the transmit-antennas are activated to transmit Alamouti's STBC symbols at any signaling time instance, and two signaling time instance constitute a transmit matrix, called the STBC-SM codeword. Since the activated combinations are chosen from N_T available transmit antennas for STBC transmission. This provides design flexibility. However, the total number of codeword combinations considered should be an integer power of 2. Usually, we divide all the defined codeword into several codebooks, in which. Thus, three steps are needed to design STBC-SM system:

(1) For a given N_T, we need to calculate the number of possible antenna combinations for the transmission of Alamouti's STBC and choose c=2^n combinations as the codewords, where c is the maximum number no bigger than the number of all the possible antenna combinations, n is an integer, respectively.

(2) Divide all the codewords into $a = \lfloor N_T/2 \rfloor$ codebooks, namely, each codebook has $m = \lceil c/a \rceil$ codewords, and the codewords in the same codebook do not interfere to each other.

(3) Sometimes, to get better BER performance, some rotation angles are given to each codebook. These rotation angles are aiming to make the distances between either two codebook maximum, which help a lot in ML detector at the receiver.

Since c antenna combinations, the resulting spectral efficiency of the STBC-SM scheme can be calculated as:

$$f = \frac{1}{2}\log_2 c + \log_2 M \left[\text{bit/s/Hz}\right] \tag{5}$$

Let s introduce the concept of STBC-SM via the following simple example of N_T=4. In this situation, there are six possible antenna combinations, so we need to choose c=4 of them first. Furthermore, we need to divide them into $a = \lfloor 4/2 \rfloor$ codebooks, and the codewords in the same codebook should not interfere to each other. Finally, we need to give the second codebook a rotation angle θ to make the distances between the two codebooks maximum:

$$\begin{aligned}\chi_1 &= \{\mathbf{X}_{11}, \mathbf{X}_{12}\} = \left\{ \begin{pmatrix} x_1 & x_2 & 0 & 0 \\ -x_2^* & x_1^* & 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 & x_1 & x_2 \\ 0 & 0 & -x_2^* & x_1^* \end{pmatrix} \right\} \\ \chi_2 &= \{\mathbf{X}_{21}, \mathbf{X}_{22}\} = \left\{ \begin{pmatrix} 0 & x_1 & x_2 & 0 \\ 0 & -x_2^* & x_1^* & 0 \end{pmatrix}, \begin{pmatrix} x_2 & 0 & 0 & x_1 \\ x_1^* & 0 & 0 & -x_2^* \end{pmatrix} \right\} e^{j\theta}\end{aligned} \tag{6}$$

where rows and columns correspond to the transmit antennas and the symbol intervals, respectively.

STBC-SM system having N_T transmit as well as N_R receive antennas and communicating over a quasi-static, frequency flat fading channel, can be modeled as:

$$\mathbf{Y} = \mathbf{X}_\chi \mathbf{H} + \mathbf{N} \tag{7}$$

and since we have assumed that the receiver has already knew the channel matrix **H**. In such circumstance, the ML detector can be modeled as:

$$\hat{\mathbf{X}}_\chi = \arg\min_{\mathbf{X}_\chi \in \chi} \left\| \mathbf{Y} - \mathbf{X}_\chi \mathbf{H} \right\|^2 \tag{8}$$

Because of c codewords the STBC-SM system have, cM^2 different transmission matrices can be constructed. The ML decoder must make an exhaustive search over all possible cM^2 transmission matrices, which is still having high complexity. However, this can be simplified due to the orthogonality of Alamouti's STBC as following equivalent channel model:

$$\mathbf{y} = \mathbf{H}_\chi \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \mathbf{n} \tag{9}$$

where $\mathbf{H}_\chi$ is the $2N_R \times 2$ equivalent channel matrix of the Alamouti coded SM scheme, which has c different realizations according to the STBC-SM codewords. Due to the orthogonality of Alamouti's STBC, the columns of $\mathbf{H}_\chi$ are orthogonal to each other for all cases and, consequently, no ICI occurs in our scheme as in the case of SM. For the former situation of N_T=4, the 4 $\mathbf{H}_\chi$ as follows:

$$\mathbf{H}_0 = \begin{bmatrix} h_{1,1} & h_{1,2} \\ h_{1,2}^* & -h_{1,1}^* \\ h_{2,1} & h_{2,2} \\ h_{2,2}^* & -h_{2,1}^* \\ \vdots & \vdots \\ h_{N_R,1} & h_{N_R,2} \\ h_{N_R,2}^* & -h_{N_R,1}^* \end{bmatrix} \mathbf{H}_1 = \begin{bmatrix} h_{1,3} & h_{1,4} \\ h_{1,4}^* & -h_{1,3}^* \\ h_{2,3} & h_{2,4} \\ h_{2,4}^* & -h_{2,3}^* \\ \vdots & \vdots \\ h_{N_R,3} & h_{N_R,4} \\ h_{N_R,4}^* & -h_{N_R,3}^* \end{bmatrix} \mathbf{H}_2 = \begin{bmatrix} h_{1,2}\varphi & h_{1,3}\varphi \\ h_{1,3}^*\varphi & -h_{1,2}^*\varphi^* \\ h_{2,2}\varphi & h_{2,3}\varphi \\ h_{2,3}^*\varphi & -h_{2,2}^*\varphi^* \\ \vdots & \vdots \\ h_{N_R,2}\varphi & h_{N_R,3}\varphi \\ h_{N_R,3}^*\varphi & -h_{N_R,2}^*\varphi^* \end{bmatrix} \mathbf{H}_3 = \begin{bmatrix} h_{1,4}\varphi & h_{1,1}\varphi \\ h_{1,1}^*\varphi & -h_{1,4}^*\varphi^* \\ h_{2,4}\varphi & h_{2,1}\varphi \\ h_{2,1}^*\varphi & -h_{2,4}^*\varphi^* \\ \vdots & \vdots \\ h_{N_R,4}\varphi & h_{N_R,1}\varphi \\ h_{N_R,1}^*\varphi & -h_{N_R,4}^*\varphi^* \end{bmatrix} \tag{10}$$

where $\varphi = e^{j\theta}$ is the rotation angle. For the cth combination, the simplified ML detector of x_1 and x_2 can be modeled as:

$$\begin{aligned}\hat{x}_{1,l} &= \arg\min_{x_1\in\gamma}\left\|y-h_{l,1}x_1\right\|^2\\ \hat{x}_{2,l} &= \arg\min_{x_2\in\gamma}\left\|y-h_{l,2}x_2\right\|^2\end{aligned}\tag{11}$$

and then the simplified ML detector of $m_{1,l}$ and $m_{2,l}$ can be modeled as:

$$\begin{aligned}\mathrm{m}_{1,l} &= \min_{x_1\in\gamma}\left\|y-h_{l,1}x_1\right\|^2\\ \mathrm{m}_{2,l} &= \min_{x_2\in\gamma}\left\|y-h_{l,2}x_2\right\|^2\end{aligned}\tag{12}$$

Since $m_{1,l}$ and $m_{2,l}$ are calculated by the ML decoder for the lth combination, their summation $m_l = m_{1,l} + m_{2,l}$, $0 \leqslant l \leqslant c-1$ gives the total ML metric for the l th combination. And the combination who has minimum m_l is right answer. Thus, the total number of ML metric calculations is reduced from cM^2 to $2cM$.

4. Differential spatial modulation

Former two types of SM systems all have assumed that accurate channel state information is available at the receiver. Nevertheless, sometimes, it is costly and difficultly to obtain accurate channel state information especially in high-mobility situations. Therefore, differential signaling has been taken into SM systems and thus develop a Differential Spatial Modulation (DSM) scheme. Similar to STBC-SM, DSM collect the modulation signal vectors over several adjacent time intervals to transmit complex information symbols drawn from a M-QAM constellation. Each interval only one transmit antenna was activated, however, the modulation signal matrix **X** satisfies the condition that each transmit antenna is activated only once. Take N_T=2 for example, the modulation signal matrix has only two types[6]:

$$\mathbf{G}=\left\{\begin{bmatrix}s_1,0\\0,s_2\end{bmatrix},\begin{bmatrix}0,s_2\\s_1,0\end{bmatrix}\middle| s_1,s_2\in\mathbf{\Lambda}\right\}\tag{13}$$

In a word, a modulation signal matrix S need $2M$+1 bits with M bits being mapped to the first symbol s_1, another M bits to the second symbol s_2, and the left 1 bit to the active order of the transmit antennas. Obviously, the spectral efficiency of the DSM scheme can be calculated as:

$$f = 0.5 + M\left[\mathrm{bit/s/Hz}\right]\tag{14}$$

At time intervals 0 and 1, the transmitter begins the first transmission by sending an arbitrary S(0) $\in$ **G**, which may be unknown to the receiver. Note that this transmitted signal matrix does not convey any information. Then all the transmit matrix are constructed by[7]:

$$\mathbf{S}(T+1)=\mathbf{S}(T)\cdot\mathbf{X}(T+1)\tag{15}$$

The system can be modeled as:

$$\mathbf{Y}=\mathbf{H}\cdot\mathbf{S}+\mathbf{N}\tag{16}$$

An example of the differential transmission process N_T=2 and complex information symbols drawn from a BPSK constellation are shown in table 1.

Table 1: The differential transmission process for 2-transmit-antenna DSM with BPSK

T	0	1	2	3
t	0,1	2,3	4,5	6,7
Input bits		000	101	110
$\mathbf{X}(T)$		$\begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$	$\begin{bmatrix} 0 & +1 \\ -1 & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & -1 \\ +1 & 0 \end{bmatrix}$
$\mathbf{S}(T)$	$\begin{bmatrix} +1 & 0 \\ 0 & +1 \end{bmatrix}$	$\begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$	$\begin{bmatrix} 0 & -1 \\ +1 & 0 \end{bmatrix}$	$\begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$

Note the two adjacent transmit matrics $\mathbf{Y}(T)$ and $\mathbf{Y}(T+1)$:

$$\mathbf{Y}(T) = \mathbf{H}(T)\mathbf{S}(T) + \mathbf{N}(T) \tag{17}$$

$$\mathbf{Y}(T+1) = \mathbf{H}(T+1)\mathbf{S}(T+1) + \mathbf{N}(T+1) \tag{18}$$

Substituting (15) and (17) into (18) and assuming that the fading channel remains constant over these two DSM blocks:

$$\mathbf{Y}(T+1) = \mathbf{Y}(T)\mathbf{X}(T+1) - \mathbf{N}(T)\,\mathrm{X}(T+1) + \mathbf{N}(T+1) \tag{19}$$

Therefore, the optimal ML detector can be derived as:

$$\hat{\mathbf{X}}(T+1) = \arg\min_{\forall \mathbf{X}} \left\| \mathbf{Y}(T+1) - \mathbf{Y}(T)\mathbf{X} \right\|_F^2 \tag{20}$$

5. Performance analysis

First, in our simulations we have assumed a frequency-flat block Rayleigh fading channel and the receiver is assumed to have perfect CSIR. Considering all the three scheme has the same N_R=1 and, the most important, the same spectral efficiency, which means N_T and M-QAM constellation of these scheme could be different.

In figure 1, we extend our simulation studies to spectral efficiency equals to 2 bits/s/Hz transmission schemes.

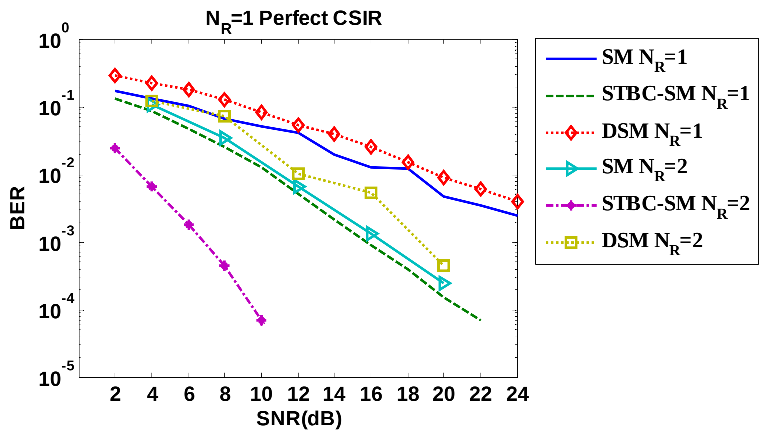

Figure 1: Performance of the SM, DSM and STBC-SM with The transmission rate 2bit/s/Hz of N_T=1,2, respectively

As we can conclude in fig 1, when has the same spectral efficiency, STBC-SM scheme has much better BER performance compared to basic SM scheme while the BER performance of DSM scheme over basic SM scheme can be less than 3dB. In addition, it can be observed from fig 2 that the BER performance all the three scheme increases with an increasing number of receive antennas.

However, sometimes, it is costly and difficultly to obtain accurate channel state information especially in high-mobility situations. In order to analysis the BER porformance of these scheme in time-invariant channel environment, sum-of-sinusoids statistical simulation models are used to generate uncorrelated fading waveforms for frequency-selective Rayleigh channels. These models employ random path gain, random initial phase, and conditional random Doppler frequency for all individual sinusoids. Let $X_k(t)$ be the th Rayleigh fader given by[10]:

$$X_k(t)=\sqrt{\frac{2}{M}}\left\{\sum_{n=1}^{M}\cos(\psi_{n,k})\cdot\cos[\omega_d t\cos(\frac{2\pi n-\pi+\theta_k}{4M})+\phi_k]\right.$$
$$\left.\sum_{n=1}^{M}\sin(\psi_{n,k})\cdot\cos[\omega_d t\cos(\frac{2\pi n-\pi+\theta_k}{4M})+\phi_k]\right\} \qquad (21)$$

where $\psi_{n,k},\theta_k,\phi_k$ and are mutually independent and uniformly distributed over $[-\pi,\pi)$ for all n and k. Furthermore, and $X_k(t)$ are $X_l(t)$ uncorrelated for all $k \neq l$. As it shows in figure.2, the change speed of Rayleigh channels can be adjuct by $w_d t$:

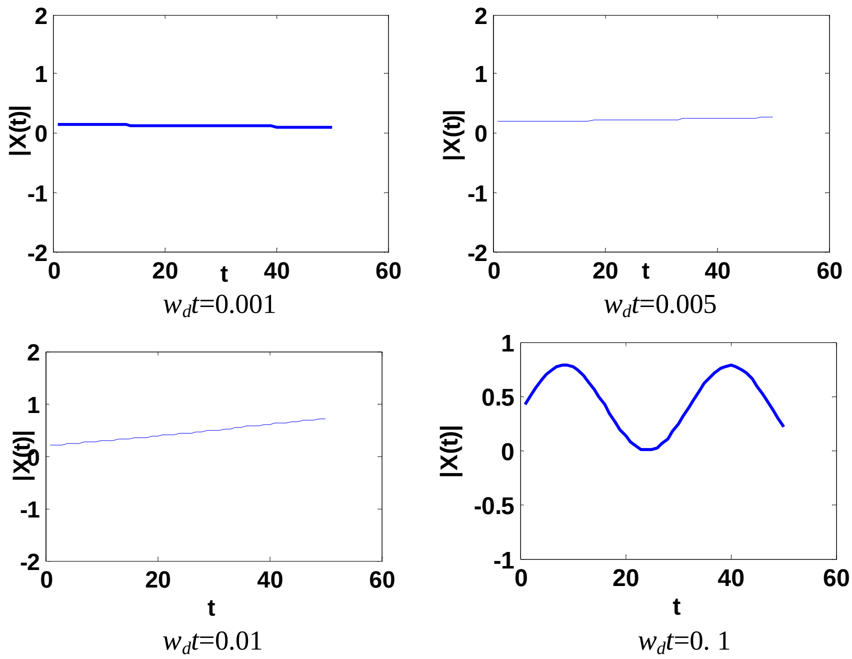

Figure 2: The change of Rayleigh channels when $w_d t$ changed

In figure 3, we extend our simulation studies to spectral efficiency equals to 2bits/s/Hz transmission schemes with time-varying channel environment. We fix the

SNR=22dB and change $w_d t$ from 0 to 1, which means the channel environment change from time-invariant to high-mobility situations.

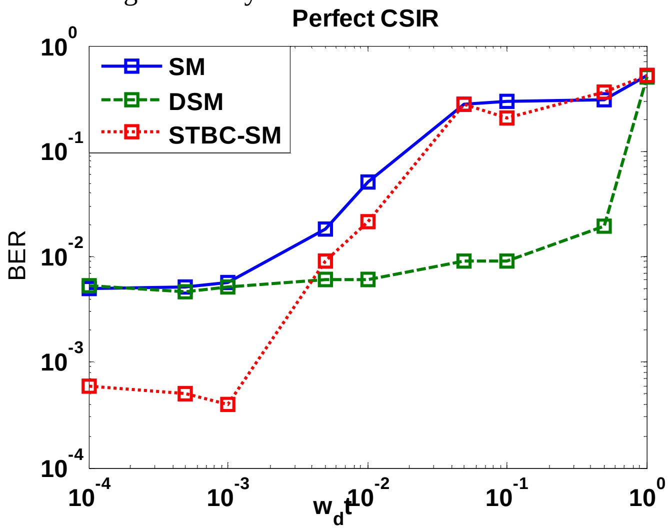

Figure 3: Spectral efficiency equals to 2bits/s/Hz transmission schemes with time-varying channel environment. SNR=22dB and change $w_d t$ from 0 to 1

It can seen in fig.3, when in time-invariant channel or channel vary with a very slow speed, basic SM and STBC-SM system still has better BER performance. Then, when the channel change a little faster, DSM scheme become the best one on BER performance. Nonetheless, when the channel change much more faster even high-mobility, all the three scheme become useless.

6. Conclusion

In this paper, basic spatial modulation, DSM STBC-SM are both simulated in time-invariant channel environment and in time-varying channel environment. The simulation results are compared to each other in terms of Bit Error Rate (BER) performance, leading to some interesting observations and useful conclusions. First, STBC-SM system has taken advantage of the benefits of SM and STBC both while avoiding their drawback, resulting in the best BER performance among the three communication systems. Moreover, DSM systems all have much better BER performance than other systems in non-correlated channel environment.

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