Journal of Mathematics and Informatics
Vol. 31, 2026, 5-18
ISSN: 2349-0632 (P), 2349-0640 (online)
Published 14 September 2026
www.researchmathsci.org
DOI: http://doi.org/10.22457/jmi.v31a02275

Journal of
Mathematics and Informatics

Tripolar Fuzzy HyperGraph and Tripolar Fuzzy SuperHyperGraph

Takaaki Fujita

Independent Researcher, Tokyo, Japan.
Email: takaaki.fujita060@gmail.com

Received 30 August 2026; accepted 13 September 2026

Abstract. Graph-based models provide a fundamental mathematical framework for representing relational systems through vertices and their connections. Hypergraphs extend ordinary graphs by allowing a single hyperedge to connect multiple vertices, whereas SuperHyperGraphs employ iterated power-set constructions to represent nested and multilevel structures. In parallel, tripolar fuzzy sets describe uncertain information through positive, neutral, and negative (counter-property) components. In this paper, we formulate Tripolar Fuzzy HyperGraphs and Tripolar Fuzzy SuperHyperGraphs in a mathematically explicit manner. We define the admissible tripolar domain, the associated order and meet, and compatibility conditions for vertices, hyperedges, supervertices, and superedges. We prove that the canonical edge assignment is always admissible, characterize it as the greatest compatible assignment in the tripolar order, establish monotonicity under hyperedge or incidence-set inclusion, and show the expected reductions to Tripolar Fuzzy Graphs and Tripolar Fuzzy HyperGraphs. These results provide a rigorous basis for higher-order and hierarchical relational models carrying three-component fuzzy information.

Keywords: SuperHyperGraph, HyperGraph, Tripolar Fuzzy HyperGraph, Tripolar Fuzzy SuperHyperGraph

AMS Mathematics Subject Classification (2010): 03E72, 05C65, 05C69

1. Introduction

1.1. HyperGraphs and SuperHyperGraphs

Graphs are fundamental mathematical tools for describing relational systems, with vertices representing objects and edges expressing pairwise connections between them [1]. Nevertheless, many real-world interactions cannot be adequately characterized by binary relationships alone. Collaborative research teams, protein assemblies, and transactions involving multiple parties are typical examples in which a single relation may involve several entities simultaneously. Hypergraphs provide a natural framework for such situations by replacing conventional edges with hyperedges that can join arbitrary nonempty collections of vertices [2,3].

In many complex systems, however, modeling multiway interaction is only part of the problem. The underlying organization may also contain several levels of aggregation, such as subcommunities embedded in larger communities, components grouped into modules, or teams organized within broader institutional units. One way to formalize this nested structure is through repeated application of the power-set construction. This idea yields finite SuperHyperGraphs, where set-valued structures created at one level may serve as higher-level supervertices at subsequent levels [4-6]. Consequently, SuperHyperGraphs can represent both higher-order incidence and hierarchical organization within a single mathematical framework. Related hierarchical modeling approaches have also appeared in studies of complex networks and applied sciences [7,8, 52].

Fuzzy-set theory can incorporate uncertainty into graph-based models. Fuzzy Graphs [9-11,42] associate graded information with vertices, edges, or both, allowing vague or partially defined relationships to be represented mathematically. This approach has subsequently been generalized to higher-order settings, including Fuzzy HyperGraphs [12-14] and Fuzzy SuperHyperGraphs [15-17]. Further uncertainty-based extensions have also been investigated, notably Neutrosophic HyperGraphs [18,19] and Neutrosophic SuperHyperGraphs [20,21], which provide additional mechanisms for representing incomplete and indeterminate information.

1.2. Tripolar fuzzy sets

A Fuzzy Set assigns each element a membership degree in $[0,1]$, allowing partial membership rather than only binary membership [26, 57, 58]. Related uncertainty frameworks include Intuitionistic Fuzzy Sets [42,43], Picture Fuzzy Sets [44,45], Hesitant Fuzzy Sets [46,47], Neutrosophic Sets [27-29] and Plithogenic Sets [30,31]. A Bipolar Fuzzy Set represents information from two opposite perspectives, typically through positive and negative membership components [32,33,43]. Related concepts include Bipolar Neutrosophic Sets [34,35], m-Polar Fuzzy Sets [36,37], and other multipolar extensions. A Tripolar Fuzzy Set is a three-pole fuzzy model; in the Rao-type formulation used in this paper, its components represent positive, neutral, and negative/counter-property degrees [38-41]. This formulation is particularly suitable for constructing compatibility relations on graph-like structures because the positive component is ordered in the usual direction while the neutral and counter-property components use reversed inequalities. Furthermore, related concepts such as Tripolar Intuitionistic Fuzzy Sets [48] and Tripolar Neutrosophic Sets [49] are also known.

1.3. Research gap and contributions

This paper does not address the absence of fuzzy graph models, hypergraphs, multipolar sets, or SuperHyperGraphs individually. Rather, the existing frameworks reviewed above address different parts of the modeling problem: ordinary fuzzy graph models primarily provide one graded component; bipolar models provide two opposing components; hypergraphs provide multiway incidence; and SuperHyperGraphs provide nested, multilevel objects. A mathematically explicit construction is therefore needed when all three requirements occur simultaneously: (i) higher-order incidence, (ii) hierarchical supervertex structure, and (iii) three-component fuzzy information with a compatibility rule that respects the tripolar order.

To address this gap, the paper makes four main contributions. First, it gives complete definitions of the tripolar admissible domain, Tripolar Fuzzy Graphs, Tripolar Fuzzy HyperGraphs, and Tripolar Fuzzy n-SuperHyperGraphs, including all domains, ranges, admissibility conditions, and componentwise compatibility inequalities. Second, it proves that the canonical assignment obtained from the tripolar meet of incident vertices or supervertices is always admissible. Third, it shows that this canonical assignment is the greatest compatible edge assignment in the tripolar order and is monotone with respect to inclusion of hyperedges or superedge incidence sets. Fourth, it establishes reduction and restriction properties that relate the proposed models to ordinary Tripolar Fuzzy Graphs and Tripolar Fuzzy HyperGraphs. These results clarify both the mathematical meaning and the structural role of the proposed constructions.

2. Preliminaries

This section introduces the fundamental concepts and notation required below. Unless otherwise specified, all sets and graph-like structures considered in this paper are finite.

2.1. SuperHyperGraphs

Graph theory offers a fundamental mathematical framework for describing relational structures, where objects are represented by vertices and pairwise connections by edges [1]. Hypergraphs extend this setting by allowing a single hyperedge to link any finite collection of vertices, thereby providing a natural representation of higher-order or multi-entity interactions [3,22,23,24]. SuperHyperGraphs generalize this hierarchy further by constructing supervertices and superedges through iterated power-set operations on an underlying base set. Such structures have recently been considered in various theoretical and applied settings, including molecular modeling, complex-network representation, and signal-processing applications [7, 50, 51]. In what follows, the hierarchical level n is assumed to be a fixed nonnegative integer.

Definition 1 (Base set). Let V_0 be a fixed finite nonempty set. It is called the *base (ground) set* from which higher-level supervertices are generated.

Definition 2 (Power set). For a set X, its power set is

$$\mathcal{P}(X) = \{A : A \subseteq X\},$$

and its nonempty power set is denoted by $P_+(X)$:

$P_+(X) = \{A \subseteq X : A \neq \emptyset\}$.

Definition 3 (Iterated power set) [25]. For $k \in \mathbb{N}_0$, define

$$\mathcal{P}^0(X) = X, \qquad \mathcal{P}^{k+1}(X) = \mathcal{P}\left(\mathcal{P}^k(X)\right).$$

Thus, elements of $\mathcal{P}^k(V_0)$ may represent objects nested through k successive set-forming levels.

Definition 4 (Hypergraph) [4]. A finite hypergraph is a pair

$$H = (V, E),$$

where V is a finite nonempty vertex set and E is a finite family of nonempty subsets of V; equivalently,

$E \subseteq P_+(V)$.

Definition 5 (n-SuperHyperGraph) [4]. Fix a finite base set V_0 and $n \in \mathbb{N}_0$. A finite n-SuperHyperGraph over V_0 is a triple

$$\mathcal{S}^{(n)} = (V_n, E, \partial),$$

where

$$V_n \subseteq \mathcal{P}^n(V_0)$$

is a finite set of n-supervertices, E is a finite set of superedge identifiers, and the incidence map ∂ satisfies

∂: E ⟶ $P_+(V_n)$.

Thus, for every superedge identifier, $\partial(e)$ is a nonempty subset of V_n. For each $e \in E$, the nonempty set $\partial(e) \subseteq V_n$ is called the *superedge incidence set*. This notation separates the hierarchical construction of the supervertices ($V_n \subseteq \mathcal{P}^n(V_0)$) from the incidence structure (∂).

2.2. Tripolar fuzzy set and graph

We use the Rao-type tripolar formulation cited in [38-41]. The following admissible domain makes the ranges and constraint explicit:

$$\mathbb{T} = \{(p, q, r) \in [0,1] \times [0,1] \times [-1,0]: p + q \leq 1\}.$$

Definition 6 (Tripolar Fuzzy Set) (cf. [38-41]). Let X be a set. A *Tripolar Fuzzy Set* (TFS) A on X is specified by three membership functions

$$\rho_A: X \to [0,1], \qquad \nu_A: X \to [0,1], \qquad \delta_A: X \to [-1,0],$$

such that, for every $x \in X$,

$$0 \leq \rho_A(x) + \nu_A(x) \leq 1.$$

Equivalently,

$$A(x) = \big(\rho_A(x), \nu_A(x), \delta_A(x)\big) \in \mathbb{T} \qquad (x \in X).$$

Here, $\rho_A(x)$ denotes the positive membership degree, $\nu_A(x)$ the neutral/indeterminate/irrelevant degree, and $\delta_A(x)$ the negative or counter-property degree. We may write

$$A = \{\langle x, \rho_A(x), \nu_A(x), \delta_A(x)\rangle : x \in X\}.$$

For two admissible values

$$a = (p_a, n_a, d_a), \qquad b = (p_b, n_b, d_b),$$

define the tripolar order by

$$a \preccurlyeq_T b \quad \Leftrightarrow \quad p_a \leq p_b, \qquad n_a \geq n_b, \qquad d_a \geq d_b.$$

The corresponding tripolar meet is

$$a \wedge_T b = (\min\{p_a, p_b\}, \max\{n_a, n_b\}, \max\{d_a, d_b\}).$$

For any nonempty finite $Y \subseteq X$, we use the notation

$$\bigwedge_{y \in Y}^{T} A(y) = \left(\min_{y \in Y} \rho_A(y), \max_{y \in Y} \nu_A(y), \max_{y \in Y} \delta_A(y)\right).$$

Definition 7 (Tripolar Fuzzy Graph) [55]. Let

$$G = (V, E), \qquad E \subseteq \binom{V}{2},$$

be a finite simple graph. Let $A_V = (\rho_V, \nu_V, \delta_V)$ be a TFS on V and $A_E = (\rho_E, \nu_E, \delta_E)$ a TFS on E. The structure

$$\mathcal{G} = (G, A_V, A_E)$$

is called a *Tripolar Fuzzy Graph* (TFG) if, for every edge $e = \{u, v\} \in E$,
$$A_E(e) \preccurlyeq_T A_V(u) \wedge_T A_V(v).$$
Equivalently, for every $\{u, v\} \in E$,
$$\rho_E(\{u, v\}) \leq \min\{\rho_V(u), \rho_V(v)\},$$
$$\nu_E(\{u, v\}) \geq \max\{\nu_V(u), \nu_V(v)\},$$
and
$$\delta_E(\{u, v\}) \geq \max\{\delta_V(u), \delta_V(v)\}.$$
The first inequality is the standard fuzzy-graph compatibility bound for positive membership. The second and third inequalities arise from the reversed order of the neutral and counter-property components in $\preccurlyeq_T$. Thus, the edge value cannot exceed the tripolar meet of its two incident vertex values.

3. Results: tripolar fuzzy hypergraphs

A hypergraph allows one edge to contain any nonempty finite collection of vertices. The tripolar extension below therefore replaces the binary vertex meet in Definition 7 with the finite meet over all vertices incident with a hyperedge.

Definition 8 (Tripolar Fuzzy HyperGraph). Let
$$H = (V, E)$$
be a finite hypergraph. Let $A_V = (\rho_V, \nu_V, \delta_V)$ be a TFS on V and $A_E = (\rho_E, \nu_E, \delta_E)$ a TFS on E; explicitly,
$$\rho_V, \nu_V : V \to [0,1], \qquad \delta_V : V \to [-1,0],$$
$$\rho_E, \nu_E : E \to [0,1], \qquad \delta_E : E \to [-1,0],$$
with
$$\rho_V(v) + \nu_V(v) \leq 1 \quad (v \in V), \qquad \rho_E(e) + \nu_E(e) \leq 1 \quad (e \in E).$$
The structure
$$\mathcal{H} = (H, A_V, A_E)$$
is called a *Tripolar Fuzzy HyperGraph* (TFHG) if, for every $e \in E$,
$$A_E(e) \preccurlyeq_T \bigwedge_{v \in e}^{T} A_V(v).$$
Equivalently, for every $e \in E$,
$$\rho_E(e) \leq \min_{v \in e} \rho_V(v), \qquad \nu_E(e) \geq \max_{v \in e} \nu_V(v), \qquad \delta_E(e) \geq \max_{v \in e} \delta_V(v).$$
The positive hyperedge membership is therefore bounded by the least positive membership among its incident vertices, whereas the neutral and counter-property components are bounded according to the reversed components of the tripolar order.

Theorem 1 (Existence and well-definedness of the canonical assignment). Let $H = (V, E)$ be a finite hypergraph and let $A_V = (\rho_V, \nu_V, \delta_V)$ be a TFS on V. For each $e \in E$, define
$$M_e := \bigwedge_{v \in e}^{T} A_V(v) = (r_e, n_e, d_e),$$
where
$$r_e := \min_{v \in e} \rho_V(v), \qquad n_e := \max_{v \in e} \nu_V(v), \qquad d_e := \max_{v \in e} \delta_V(v).$$
Then $M_e \in \mathbb{T}$ for every $e \in E$. Consequently, the canonical assignment
$$A_E^{\mathrm{can}}(e) := M_e$$

defines a TFS on E and $(H, A_V, A_E^{\mathrm{can}})$ is a TFHG.
Proof. Because every hyperedge e is finite and nonempty, the three extrema defining r_e, n_e, d_e exist. Since $\rho_V(v), \nu_V(v) \in [0,1]$ and $\delta_V(v) \in [-1,0]$ for all $v \in V$, we immediately have

$$r_e \in [0,1], \qquad n_e \in [0,1], \qquad d_e \in [-1,0].$$

Choose $v_0 \in e$ such that $\nu_V(v_0) = n_e$. By the definition of r_e,

$$r_e \leq \rho_V(v_0).$$

Hence, using the admissibility of $A_V(v_0)$,

$$0 \leq r_e + n_e \leq \rho_V(v_0) + \nu_V(v_0) \leq 1.$$

Therefore $(r_e, n_e, d_e) \in \mathbb{T}$, so A_E^{can} is a well-defined TFS on E. Moreover,

$$A_E^{\mathrm{can}}(e) = \bigwedge_{v \in e}^{T} A_V(v),$$

so the TFHG compatibility condition holds with equality for every $e \in E$. ■
Theorem 1 shows that the proposed class is nonempty for every finite underlying hypergraph and every admissible vertex TFS; no additional consistency assumption is required to obtain compatible hyperedge memberships. The theorem also produces a distinguished, saturated edge assignment from vertex data alone.

Proposition 1 (Maximality of the canonical edge assignment). Under the assumptions of Theorem 1, let A_E be any TFS on E such that (H, A_V, A_E) is a TFHG. Then, for every $e \in E$,

$$A_E(e) \preccurlyeq_T A_E^{\mathrm{can}}(e).$$

Thus, $A_E^{\mathrm{can}}(e)$ is the greatest edge value permitted by the vertex memberships with respect to $\preccurlyeq_T$.
Proof: The TFHG compatibility condition gives

$$A_E(e) \preccurlyeq_T \bigwedge_{v \in e}^{T} A_V(v) = A_E^{\mathrm{can}}(e),$$

for every $e \in E$. ■

Proposition 2 (Monotonicity under hyperedge inclusion). Let $e, f \in E$ satisfy $e \subseteq f$. Then the canonical values satisfy

$$A_E^{\mathrm{can}}(f) \preccurlyeq_T A_E^{\mathrm{can}}(e).$$

Proof: Since $e \subseteq f$,

$$\min_{v \in f} \rho_V(v) \leq \min_{v \in e} \rho_V(v),$$

while

$$\max_{v \in f} \nu_V(v) \geq \max_{v \in e} \nu_V(v), \qquad \max_{v \in f} \delta_V(v) \geq \max_{v \in e} \delta_V(v).$$

These are exactly the three componentwise conditions for $A_E^{\mathrm{can}}(f) \preccurlyeq_T A_E^{\mathrm{can}}(e)$. ■

Proposition 3 (Reduction to Tripolar Fuzzy Graphs). If every hyperedge of $H = (V, E)$ has cardinality two, then Definition 8 reduces exactly to Definition 7. In particular, a TFHG on a 2-uniform simple hypergraph is a TFG after identifying each two-element hyperedge with the corresponding graph edge.
Proof: For $e = \{u, v\}$,

$$\bigwedge_{x\in e}^{T} A_V(x) = A_V(u) \wedge_T A_V(v),$$

so the compatibility inequalities in Definitions 7 and 8 are identical. ■

Proposition 4 (Restriction property). Let $\mathcal{H} = (H, A_V, A_E)$ be a TFHG with $H = (V, E)$. For any $V' \subseteq V$, put

$$E' := \{e \in E : e \subseteq V'\}.$$

Let A'_V and A'_E denote the restrictions of A_V to V' and of A_E to E', respectively. Then the restricted structure

$$\mathcal{H}' = \left((V', E'), A'_V, A'_E\right)$$

is a TFHG.

Proof: For each $e \in E'$, the original compatibility relation already gives

$$A_E(e) \preccurlyeq_T \bigwedge_{v\in e}^{T} A_V(v).$$

Restriction does not change any membership value on e, so the same inequality holds in $\mathcal{H}'$. ■

4. Results: tripolar fuzzy superhypergraphs

For an n-SuperHyperGraph, tripolar fuzzy values are assigned directly to the n-supervertices and superedge identifiers. The hierarchical information is encoded by $V_n \subseteq \mathcal{P}^n(V_0)$, while the interaction among those supervertices is encoded by the incidence map ∂. No flattening of a supervertex to elements of V_0 is required for the definition.

Definition 9 (Tripolar Fuzzy n-SuperHyperGraph). Let

$$\mathcal{S}^{(n)} = (V_n, E, \partial)$$

be a finite n-SuperHyperGraph over V_0. Let $A_V = (\rho_V, \nu_V, \delta_V)$ be a TFS on V_n and $A_E = (\rho_E, \nu_E, \delta_E)$ a TFS on E; explicitly,

$$\rho_V, \nu_V : V_n \to [0,1], \qquad \delta_V : V_n \to [-1,0],$$
$$\rho_E, \nu_E : E \to [0,1], \qquad \delta_E : E \to [-1,0],$$

with

$$\rho_V(x) + \nu_V(x) \le 1 \quad (x \in V_n), \qquad \rho_E(e) + \nu_E(e) \le 1 \quad (e \in E).$$

The structure

$$\mathfrak{S}^{(n)} = \left(\mathcal{S}^{(n)}, A_V, A_E\right)$$

is called a *Tripolar Fuzzy n-SuperHyperGraph* (TFSHG) if, for every $e \in E$,

$$A_E(e) \preccurlyeq_T \bigwedge_{x\in\partial(e)}^{T} A_V(x).$$

Equivalently,

$$\rho_E(e) \le \min_{x\in\partial(e)} \rho_V(x), \qquad \nu_E(e) \ge \max_{x\in\partial(e)} \nu_V(x), \qquad \delta_E(e) \ge \max_{x\in\partial(e)} \delta_V(x).$$

Thus, the hierarchical level determines what counts as a supervertex, whereas the tripolar compatibility condition is applied to the supervertices in each incidence set $\partial(e)$. The three fuzzy components interact with the hierarchy only through these level-n incidence sets, which keeps the construction independent of any particular flattening convention.

Theorem 2 (Existence and well-definedness of the canonical superedge assignment). Let $\mathcal{S}^{(n)} = (V_n, E, \partial)$ be a finite n-SuperHyperGraph and let A_V be a TFS on V_n. For every $e \in E$, define

$$M_e^{(n)} := \bigwedge_{x\in\partial(e)}^{T} A_V(x) = \left(r_e^{(n)}, n_e^{(n)}, d_e^{(n)}\right),$$

where

$$r_e^{(n)} := \min_{x\in\partial(e)} \rho_V(x), \quad n_e^{(n)} := \max_{x\in\partial(e)} \nu_V(x), \quad d_e^{(n)} := \max_{x\in\partial(e)} \delta_V(x).$$

Then $M_e^{(n)} \in \mathbb{T}$ for every $e \in E$. Hence

$$A_E^{\mathrm{can}}(e) := M_e^{(n)}$$

defines a TFS on E and produces a TFSHG.

Proof: Because $\partial(e)$ is finite and nonempty, the defining extrema exist and satisfy

$$r_e^{(n)}, n_e^{(n)} \in [0,1], \qquad d_e^{(n)} \in [-1,0].$$

Choose $x_0 \in \partial(e)$ with $\nu_V(x_0) = n_e^{(n)}$. Since $r_e^{(n)} \le \rho_V(x_0)$ and $A_V(x_0)$ is admissible,

$$0 \le r_e^{(n)} + n_e^{(n)} \le \rho_V(x_0) + \nu_V(x_0) \le 1.$$

Thus $M_e^{(n)} \in \mathbb{T}$. By construction,

$$A_E^{\mathrm{can}}(e) = \bigwedge_{x\in\partial(e)}^{T} A_V(x),$$

so the compatibility condition is satisfied with equality. ■

As in Theorem 1, Theorem 2 guarantees existence for arbitrary admissible supervertex data and supplies a canonical saturated superedge assignment.

Proposition 5 (Maximality of the canonical superedge assignment). Let A_E be any superedge TFS such that $\left(\mathcal{S}^{(n)}, A_V, A_E\right)$ is a TFSHG. Then

$$A_E(e) \preccurlyeq_T A_E^{\mathrm{can}}(e) \qquad (e \in E).$$

Proof: This is the defining compatibility inequality together with the definition of $A_E^{\mathrm{can}}(e)$. ■

Proposition 6 (Monotonicity under incidence inclusion). If $e_1, e_2 \in E$ satisfy

$$\partial(e_1) \subseteq \partial(e_2),$$

then

$$A_E^{\mathrm{can}}(e_2) \preccurlyeq_T A_E^{\mathrm{can}}(e_1).$$

Proof: Enlarging the incidence set can only decrease the minimum positive degree and can only increase the maximum neutral and counter-property degrees. These three monotonicities are precisely the componentwise definition of $\preccurlyeq_T$. ■

Proposition 7 (Reduction to Tripolar Fuzzy HyperGraphs). Suppose $n = 0$, $V_n = V_0$, and the superedge identifiers are themselves nonempty subsets of V_0 with

$$\partial(e) = e \qquad (e \in E).$$

Then Definition 9 reduces exactly to Definition 8. Hence a TFSHG at level 0 with ordinary hyperedges is a TFHG.

Proof: Under the stated assumptions, each incidence set $\partial(e)$ is exactly the hyperedge e, so

$$\bigwedge_{x\in\partial(e)}^{T} A_V(x) = \bigwedge_{v\in e}^{T} A_V(v).$$

Therefore, the two compatibility conditions coincide componentwise. ■

5. Conclusion

This paper developed Tripolar Fuzzy HyperGraphs and Tripolar Fuzzy SuperHyperGraphs as higher-order and hierarchical graph structures equipped with positive, neutral, and negative/counter-property fuzzy information. The definitions were stated using an explicit admissible tripolar domain and a tripolar partial order, which makes the edge and superedge compatibility conditions precise. For both TFHGs and TFSHGs, the canonical assignment obtained by taking the tripolar meet of the incident vertex or supervertex values was proved to be admissible. Moreover, this assignment was characterized as the greatest compatible edge value in the tripolar order, and its monotonicity under hyperedge or incidence-set inclusion was established. Reduction results show that the framework is consistent with Tripolar Fuzzy Graphs in the 2-uniform case and with Tripolar Fuzzy HyperGraphs at SuperHyperGraph level 0. We also obtained a restriction property for TFHGs.

These results provide a basic structural foundation beyond mere existence. Future research may investigate unions, intersections, products, homomorphisms and isomorphisms, connectivity, degree concepts, paths and cycles, matrix or tensor representations, and algorithmic problems for TFHGs and TFSHGs. Extensions based on Neutrosophic Sets [53,54] and Plithogenic Sets [56] are also natural directions. In addition, application-oriented studies may examine whether the three tripolar components can better represent higher-order and multilevel systems in decision support, information networks, and other domains where positive, neutral, and counter-property information coexist.

Acknowledgments. The author sincerely thanks the reviewers and readers whose comments and suggestions helped improve the clarity and mathematical presentation of this work.

Conflicts of Interest. The author declares no conflicts of interest related to this research or its publication.

Author Contributions: The author solely conceptualized the study, conducted the research, prepared the manuscript, and reviewed and approved the final version for publication.

Use of Generative AI and AI-Assisted Tools

Generative AI and AI-assisted tools were used only for language and grammar checking and did not change the author's mathematical responsibility.

REFERENCES

1. R. Diestel, *Graph Theory*, 6th ed., Springer, 2024.
2. Y. Feng, H. You, Z. Zhang, R. Ji and Y. Gao, Hypergraph neural networks, *Proceedings of the AAAI Conference on Artificial Intelligence*, 33 (2019) 3558–3565.

3. Y. Gao, Z. Zhang, H. Lin, X. Zhao, S. Du and C. Zou, Hypergraph learning: Methods and practices, *IEEE Transactions on Pattern Analysis and Machine Intelligence*, 44(5) (2022) 2548–2566.
4. F. Smarandache, *Extension of HyperGraph to n-SuperHyperGraph and to Plithogenic n-SuperHyperGraph, and Extension of HyperAlgebra to n-ary (Classical-/Neutro-/Anti-) HyperAlgebra*, Infinite Study, 2020.
5. T. Fujita, Metahypergraphs, metasuperhypergraphs, and iterated metagraphs: Modeling graphs of graphs, hypergraphs of hypergraphs, superhypergraphs of superhypergraphs, and beyond, *Current Research in Interdisciplinary Studies*, 4(5) (2025) 1–23.
6. T. Fujita and F. Smarandache, *HyperGraph and SuperHyperGraph Theory with Applications*, Neutrosophic Science International Association Publishing House, 2026.
7. S. M. Berrocal Villegas, W. Montalvo Fritas, C. R. Berrocal Villegas, M. Y. F. Flores Fuentes Rivera, R. Espejo Rivera, L. D. Bautista Puma and D. M. Macazana Fernández, Using plithogenic n-SuperHyperGraphs to assess the degree of relationship between information skills and digital competencies, *Neutrosophic Sets and Systems*, 84 (2025) 513–524.
8. N. Hodelin Amable, E. E. Vergel De Salazar, M. G. Martinez Isaac, O. C. Olavarria Sanchez and J. M. Solis Palma, Representation of motivational dynamics in school environments through plithogenic n-SuperHyperGraphs with family participation, *Neutrosophic Sets and Systems*, 92 (2025) 570–583.
9. J. Xu et al., Fuzzy graph convolutional network for hyperspectral image classification, *Engineering Applications of Artificial Intelligence*, 127 (2024) Article 107280.
10. J. Dinar et al., Wiener index for an intuitionistic fuzzy graph and its application in water pipeline network, *Ain Shams Engineering Journal*, 14(1) (2023) Article 101826.
11. R. Lal et al., Enhancing security in electromagnetic radiation therapy using fuzzy graph theory, *Scientific Reports*, 15 (2025) Article 13139.
12. S. Jiang, M. Kang, X. Xu, J. Xiao and L. Lu, A multi-scale evolutionary framework for fuzzy hypergraph community detection, *IEEE Transactions on Fuzzy Systems* (2026) doi: 10.1109/TFUZZ.2026.3706939.
13. T. Yin et al., The fuzzy hypergraph neural network model based on sparse k-nearest neighborhood granule, *Applied Soft Computing*, 170 (2025) Article 112721.
14. X.-A. Bi et al., FHG-GAN: Fuzzy hypergraph generative adversarial network with large foundation models for Alzheimer's disease risk prediction, *IEEE Transactions on Fuzzy Systems*, 33(8) (2025) 2599–2613.
15. T. Fujita, Recursive intuitionistic fuzzy SuperHyperGraphs, *Journal of Analytical Uncertainty*, 1(2) (2026) 52–64, doi: 10.63924/jau.v1i2.273.

16. T. Fujita, Modeling hierarchical and higher-order uncertainty in IT systems management using strong intuitionistic fuzzy hypergraphs and superhypergraphs, *Journal of Engineering Management and Systems Science*, 1(1) (2026) 1–25, doi: 10.67334/jemsci11202621.
17. T. Fujita and A. Mehmood, Fuzzy tolerance hypergraphs and fuzzy tolerance superhypergraphs, *Neutrosophic Computing and Machine Learning*, 41 (2025) 46–70, doi: 10.5281/zenodo.17254700.
18. M.P. Shareef, B. R. Jose and J. Mathew, LLM-semantic enrichment of neutrosophic fuzzy hypergraphs for indeterminacy-driven reinforcement learning and explainable path-based recommendations, *IEEE Access*, 14 (2026) 112891–112911, doi: 10.1109/ACCESS.2026.3715708.
19. T. Fujita, Hesitant neutrosophic network models for operations research: Graphs, hypergraphs, and SuperHyperGraphs, *Advances in Neutrosophic Mathematics and Informatics*, 1(2) (2026) 25–38.
20. M. Ghods, Z. Rostami and F. Smarandache, Introduction to neutrosophic restricted superhypergraphs and neutrosophic restricted superhypertrees and several of their properties, *Neutrosophic Sets and Systems*, 50(1) (2022) Article 28.
21. T. Fujita and F. Smarandache, Fuzzy and neutrosophic directed SuperHyperGraphs: Definitions, properties, and illustrative real-world applications in decision-making, *IAENG International Journal of Applied Mathematics*, 56(5) (2026).
22. T. Dillan, S. M. Isa, A. S. Girsang et al., Temporal-spatial reasoning over hypergraph knowledge structures for multimodal retrieval-augmented generation, *Scientific Reports* (2026) doi: 10.1038/s41598-026-63525-9.
23. Y. Belkadi et al., Multi-view hypergraph networks for imbalanced multi-class intrusion detection, *Machine Learning with Applications*, 25 (2026) Article 100964, doi: 10.1016/j.mlwa.2026.100964.
24. Z. Long et al., HyperPIM: A redundancy-aware heterogeneous PIM accelerator for hypergraph neural networks, *IEEE Transactions on Computer-Aided Design of Integrated Circuits and Systems* (2026).
25. T. Fujita, Fractional powersets and SuperHyperStructures: Toward a framework for fractional set theory and discrete hierarchical systems, *Journal of Fractional Calculus and Nonlinear Systems*, 6(2) (2025) 104–146, doi: 10.48185/jfcns.v6i2.1777.
26. W. Pedrycz and F. Gomide, *An Introduction to Fuzzy Sets: Analysis and Design*, MIT Press, 1998.
27. H. Wang et al., *Interval Neutrosophic Sets and Logic: Theory and Applications in Computing*, Vol. 5, Infinite Study, 2005.
28. M. Ali and F. Smarandache, Complex neutrosophic set, *Neural Computing and Applications*, 28(7) (2017) 1817–1834.

29. R. Mallick and S. Pramanik, *Pentapartitioned Neutrosophic Set and Its Properties*, Vol. 36, Infinite Study, 2020.
30. F. Smarandache, *Plithogenic Set, an Extension of Crisp, Fuzzy, Intuitionistic Fuzzy, and Neutrosophic Sets—Revisited*, Infinite Study, 2018.
31. M. Y. Leyva-Vázquez and F. Smarandache, *Plithogenic Tensor Neutrosophic Logic: A Formal Foundation for Declared-Loss Evaluation in Large Language Models*, Zenodo, 2026, doi: 10.5281/zenodo.20030718.
32. L. Ebanezar and M. Vanniarajan, Dominating energy in cubic bipolar fuzzy graphs, *Journal of Science and Arts*, 26(2) (2026) 435–448.
33. U. ur Rehman et al., Selection of optimal combined heat and power systems for industrial energy needs by using bipolar fuzzy rough MCDM based on Schweizer-Sklar operators, *International Journal of Computational Intelligence Systems* (2026).
34. I. Deli, M. Ali and F. Smarandache, Bipolar neutrosophic sets and their application based on multi-criteria decision making problems, in *2015 International Conference on Advanced Mechatronic Systems*,, IEEE, 2015.
35. S. Pramanik et al., Cross entropy measures of bipolar and interval bipolar neutrosophic sets and their application for multi-attribute decision-making, *Axioms.* 7(2) (2018) Article 21.
36. K.R. Maity and M. Pal, A novel approach on m-polar fuzzy set with Dombi power aggregation operators to solve multi-attribute decision-making problems, *TWMS Journal of Applied and Engineering Mathematics*, 15(9) (2025) 2313–2330.
37. H. A. Alharbi and K. M. Alsager, Fermatean m-polar fuzzy soft rough sets with application to medical diagnosis, *AIMS Mathematics*, 10(6) (2025) 14314–14346.
38. N. Wattanasiripong, N. Tiprachot and S. Lekkoksung, On tripolar complex fuzzy sets and their application in ordered semigroups, *International Journal of Analysis and Applications*, 23 (2025) Article 139.
39. A. Phukhaengsi, P. Khamrot and T. Gaketem, Tripolar fuzzy [symbol unavailable in source]-quasi-ideals of ordered semigroups, *International Journal of Analysis and Applications*, 24 (2026) Article 156,
doi: 10.28924/2291-8639-24-2026-156.
40. Minhaj, A. Khan, Muneeza, A. Gumaei, S. M. Alzanin and S. Ashraf, Advancing AI-powered assistive technologies with tripolar fuzzy integration: Enhancing decision-making for individuals with disabilities, *International Journal of Fuzzy Systems*, 28(3) (2026) 972–981, doi: 10.1007/s40815-025-01997-z.
41. M. M. K. Rao and B. Venkateswarlu, Tripolar fuzzy interior ideals of a [symbol unavailable in source]-semiring, *Asia Pacific Journal of Mathematics*, 5(2) (2018) 192–207.

42. P.A. Ejegwa, New operations on intuitionistic fuzzy multisets, *Journal of Mathematics and Informatics*, 3 (2015) 17–23.
43. P. Sengar et al., Common coupled fixed-point theorems in intuitionistic fuzzy metric spaces, *Journal of Mathematics and Informatics*, 6 (2016) 41–51.
44. T.O. Sangodapo, Some results on picture fuzzy multirelations, *Journal of Mathematics and Informatics*, 28 (2025) 45–58.
45. T.O. Sangodapo, Homomorphism of picture fuzzy subgroup of a group, *Journal of Mathematics and Informatics*, 30 (2026) 7–16.
46. A.F. Talee, M. Y. Abassi and S. Ali Khan, Hesitant fuzzy ideals in semigroups with a frontier, *Arya Bhatta Journal of Mathematics and Informatics*, 9(1) (2017) 163–170.
47. A.F. Talee et al., Hesitant fuzzy sets approach to ideal theory in ordered Γ-semigroups, *Italian Journal of Pure and Applied Mathematics*, 43 (2020) 73–85.
48. T. Fujita, A mathematical study of tripolar intuitionistic fuzzy graphs, *International Journal of Computer Application*, 16(4) (2026) 16.
49. T. Fujita, A mathematical study of tripolar neutrosophic graphs, *International Journal of Research in Engineering & Science*, 16(4) (2026) 13.
50. V. Wanjala and C. Amenya, Feature-Augmented Drug SuperHyperGraphs: Neural Architectures for Hierarchical Pharmaceutical Knowledge Representation, *Neutrosophic Knowledge*, 12 (2026).
51. T. Fujita, A. K. Das, S. Das, S. P. Mondal and V. Duran, Note on Drug Hypergraph and Drug SuperHyperGraph in Pharmaceutical Sciences, *Neutrosophic Knowledge*, 12 (2026).
52. A.K. Das, T. Fujita and K. Bhattacharjee, Topological properties of neutrosophic hyperedge-weighted superhypergraphs and their applications in social networks, in *SuperHyperTopologies and SuperHyperStructures with their Applications*, 2025, 279.
53. X. Wu and X. Chen, New Quality Productive Forces-Empowered Neutrosophic Group Decision-Making Approach for Engineering Cost Risk Assessment: Evidence from Farmland Water Conservancy Projects, *Frontiers in Physics*, 14 (2026) Article 1964509.
54. V.A. Khan et al., Some new type of approaches to deferred Riesz statistical convergence in non-Archimedean neutrosophic sequences, *Advanced Studies: Euro-Tbilisi Mathematical Journal*, 19(2) (2026) 213–233.
55. T. Fujita, On tripolar fuzzy offgraph, *International Journal of Computer Application*, 16(4) (2026) 13.
56. T. Fujita and F. Smarandache, *A Comprehensive and Updated Survey of Fuzzy, Neutrosophic, and Uncertain Sets*, Infinite Study, 2026.
57. Y. Zhang and Z. Gong, An approximate technique for solving fully fuzzy complex linear systems, *Journal of Mathematics and Informatics*, 28 (2025) 1–19.

58. C.Z. Jia, L.Q. Li and X. R. Li, Enriched fuzzy σ-algebra and fuzzy rough approximation operators, *Journal of Mathematics and Informatics*, 25 (2023) 59–69.