Journal of Mathematics and Informatics
Vol. 4, 2015, 37-50
ISSN: 2349-0632 (P), 2349-0640 (online)
Published 26 December 2015
www.researchmathsci.org

Journal of
Mathematics and Informatics

Model Operator $F_{\alpha,\beta}$ on Doubt Intuitionistic Fuzzy H-ideals in BCK/BCI-algebras

***S.R.Barbhuiya*[a] *and Tapan Senapati*[b]**

[a]Department of Mathematics, Srikishan Sarda College,
Hailakandi-788151, Assam, India
E-mail: saidurbarbhuiya@gmail.com
[b]Department of Applied Mathematics with Oceanology and Computer Programming, Vidyasagar University, Midnapore 721102 India
E-mail: math.tapan@gmail.com

Received 5 December 2015; accepted 22 December 2015

Abstract. In this paper, we study the effect of model operators on doubt intuitionistic fuzzy H-ideals in BCK/BCI-algebras and the effect of model operators on doubt intuitionistic fuzzy H-ideals in BCK/BCI -algebras under homomorphism and obtained some interesting properties.

Keywords: BCK/BCI -algebra, Intuitionistic fuzzy ideal, Intuitionistic fuzzy H -ideal, Doubt intuitionistic fuzzy H -ideal, Model Operator, (α,β) -Model Operator.

Mathematics Subject Classification (2010): 06F35, 03E72, 03G25, 94D05

1. Introduction

The concept of fuzzy sets was first initiated by Zadeh([36]) in 1965. Since then these ideas have been applied to other algebraic structures such as group, semigroup, ring, vector spaces etc. Imai and Iseki [12] introduced BCK-algebras as a generalization of notion of the concept of set theoretic difference and propositional calculus and in the same year Iseki [13] introduced the notion of BCI-algebra which is a generalization of BCK-algebra. Senapati together with colleagues [7, 21-35] had done lot of works on BCK/BCI -algebras and related algebraic systems. Jun [14] introduced the notion of doubt(anti) fuzzy ideals in BCK/BCI -algebras. In 1999, Khalid and Ahmad [16] introduced fuzzy H -ideals in BCI -algebras. Further, in [2, 3, 4, 18] several authors discussed more results on BCK/BCI -algebras. In 2003, Zhan and Tan [35] introduced doubt fuzzy H -ideals in BCK -algebras. The concept of intuitionistic fuzzy subset(IFS) was introduced by Atanassov [1] in 1983, which is a generalization of the notion of fuzzy sets [34]. In 2010, Satyanarayan et al. ([17]) introduced intuitionistic fuzzy H -ideals in BCK-algebras respectively and also several interesting properties of these concepts were studied. Doubt intuitionistic fuzzy H -ideals in BCK/BCI -algebras were introduced in [6] by Bej and Pal. Senapati et al. [18, 19, 20] introduce the concepts of (intuitionistic) fuzzy translation to (intuitionistic) fuzzy subalgebras, ideals and H-ideals in BCK/BCI-algebras.

The intuitionistic fuzzy model operators $\Box$ and $\Diamond$ introduced by Atanassov in 1983. The extension on both the operators $\Box$ and $\Diamond$ is the new operator D_α which represents both of them. Further the extension of all the operators is the operator $F_{\alpha,\beta}$ called (α,β) -model operator which were introduced by Atanassov. The effect of all the model operator on IFSs is again an IFSs. The model operators play an important rule in the study of IFSs. Here in this paper, we study the effect of model operators in particular (α,β) -model operator on doubt intuitionistic fuzzy H-ideals in BCK/BCI-algebras.

2. Preliminaries

Definition 2.1. ([14, 15]) An algebra $(X,*,0)$ of type (2, 0) is called a BCK -algebra if it satisfies the following axioms:

(i) $((x*y)*(x*z))*(z*y)=0$

(ii) $(x*(x*y))*y=0$

(iii) $x*x=0$

(iv) $0*x=0$

(v) $x*y=0$ and $y*x=0 \Rightarrow x=y$ for all $x,y,z\in X$.

We can define a partial ordering " $\le$ " on X by $x\le y$ iff $x*y=0$.

Definition 2.2. ([14, 15]) A BCK-algebra X is said to be commutative if it satisfies the identity $x\wedge y=y\wedge x$ where $x\wedge y=y*(y*x)\ \forall x,y\in X$. In a commutative BCK-algebra, it is known that $x\wedge y$ is the greatest lower bound of x and y.

In a BCK-algebra X, the following hold:

(i) $x*0=x$

(ii) $(x*y)*z=(x*z)*y$

(iii) $x*y\le x$

(iv) $(x*y)*z\le(x*z)*(y*z)$

(v) $x\le y$ implies $x*z\le y*z$ and $z*y\le z*x$.

A BCK-algebra X is said to be associative [6] if it satisfies the identity $(x*y)*z=x*(y*z)\ \forall x,y,z\in X$. A non empty subset S of a BCK-algebra X is called a subalgebra X if $x*y\in X$, for all x,y $\in$ S . A non empty subset I of a BCK-algebra X is called an ideal [15] of X if (i) $0\in I$ (ii) $x*y\in I$ and $y\in I\Rightarrow x\in I$ for all $x,y\in X$. A nonempty subset I of a BCK -algebra X is said to be a H-ideal [16, 35] of X if it satisfies (i) and (iii) $x*(y*z)\in I$ and $y\in I\Rightarrow x*z\in I$ for all $x,y,z\in X$.

A fuzzy subset μ of a BCK-algebra X is called a

(**A**) fuzzy subalgebra of X if $\mu(x*y)\ge min\{\mu(x),\mu(y)\}$, for all x, y $\in$ X.

(B) doubt fuzzy subalgebra of X if $\mu(x*y)\le min\{\mu(x),\mu(y)\}$, for all x, y $\in$ X.

(**C**) fuzzy ideal [15] of X if it satisfies the following axioms:

(i) $\mu(0) \geq \mu(x)$

(ii) $\mu(x) \geq min\{\mu(x * y), \mu(y)\}$ for all $x, y \in X$.

(**D**) doubt fuzzy ideal [15] of X if it satisfies the following axioms:

(i) $\mu(0) \leq \mu(x)$

(ii) $\mu(x) \leq max\{\mu(x * y), \mu(y)\}$ for all $x, y \in X$.

(**E**) fuzzy H-ideal [16, 35] of X if it satisfies the following axioms:

(i) $\mu(0) \geq \mu(x)$

(ii) $\mu(x * z) \geq min\{\mu(x * (y * z)), \mu(y)\}$ for all $x, y, z \in X$.

(**F**) doubt fuzzy H-ideal of X if it satisfies the following axioms:

(i) $\mu(0) \leq \mu(x)$

(ii) $\mu(x * z) \leq max\{\mu(x * (y * z)), \mu(y)\}$ for all $x, y, z \in X$.

Definition 2.3. ([1]) An intuitionistic fuzzy set (IFS) A of a non empty set X is an object of the form $A = \{< x, \mu_A(x), \nu_A(x) > | x \in X\}$, where $\mu_A : X \to [0,1]$ and $\nu_A : X \to [0,1]$ with the condition $0 \leq \mu_A(x) + \nu_A(x) \leq 1, \forall x \in X$. The numbers $\mu_A(x)$ and $\nu_A(x)$ denote respectively the degree of membership and the degree of non-membership of the element x in set A. For the sake of simplicity, we shall use the symbol $A = (\mu_A, \nu_A)$ for the intuitionistic fuzzy set $A = \{< x, \mu_A(x), \nu_A(x) > | x \in X\}$. The function $\pi_A(x) = 1 - \mu_A(x) - \nu_A(x)$ for all $x \in X$. is called the degree of uncertainty of $x \in A$. The class of IFSs on a universe X is denoted by $IFS(X)$.

Definition 2.4. ([1]) If $A = \{< x, \mu_A(x), \nu_A(x) > | x \in X\}$ and $B = \{< x, \mu_B(x), \nu_B(x) > | x \in X\}$ are any two IFSs of a set X , then $A \subseteq B$ if and only if for all $x \in X$, $\mu_A(x) \leq \mu_B(x)$ and $\nu_A(x) \geq \nu_B(x)$, $A = B$ if and only if for all $x \in X, \mu_A(x) = \mu_B(x)$ and $\nu_A(x) = \nu_B(x)$ $A \cap B = \{< x, (\mu_A \cap \mu_B)(x), \quad (\nu_A \cup \nu_B)(x) > | x \in X\}$, where $(\mu_A \cap \mu_B)(x) = \min\{\mu_A(x), \mu_B(x)\}$ and $(\nu_A \cup \nu_B)(x) = \max\{\nu_A(x), \nu_B(x)\}$, $A \cup B = \{< x, (\mu_A \cup \mu_B)(x), (\nu_A \cap \nu_B)(x) > | x \in X\}$,where $(\mu_A \cup \mu_B)(x) = \max \ \{\mu_A(x), \ \mu_B(x)\}$ and $(\nu_A \cap \nu_B)(x) = \min\{\nu_A(x), \nu_B(x)\}$.

Definition 2.5. ([1]) If $A = \{< x, \mu_A(x), \nu_A(x) > | x \in X\}$ and $B = \{< x, \mu_B(x), \ \nu_B(x) > | x \in X\}$ are any two IFSs of a set X , then their cartesian product is defined by $A \times B = \{< (x, y), (\mu_A \times \mu_B)(x, y), (\nu_A \times \nu_B)(x, y) > | x, y \in X\}$ where $(\mu_A \times \mu_B)(x, y) = \min\{\mu_A(x), \mu_B(y)\}$ & $(\nu_A \times \nu_B)(x, y) = \max\{\nu_A(x), \nu_B(y)\}$.

Definition 2.6. ([14]) If $A = \{< x, \mu_A(x), \nu_A(x) > | x \in X\}$ and $B = \{< x, \mu_B(x), \nu_B(x) > | x \in X\}$ are any two IFSs of a set X , then their doubt cartesian product is defined by

$A\times B=\{<(x,y),(\mu_A\times\mu_B)(x,y),(\nu_A\times\nu_B)(x,y)>|\,x,y\in X\}$ where $(\mu_A\times\mu_B)(x,y)=\max\{\mu_A(x),\mu_B(y)\}$ & $(\nu_A\times\nu_B)(x,y)=\min\{\nu_A(x),\nu_B(y)\}$.

Definition 2.7. ([1]) For any IFS $A=\{<x,\mu_A(x),\nu_A(x)>|\,x\in X\}$ of X and $\alpha\in[01]$, the operators $\Box: IFS(X)\to IFS(X)$, $\Diamond: IFS(X)\to IFS(X)$, $D_\alpha: IFS(X)\to IFS(X)$ are defined as

(i) $\Box(A)=\{<x,\mu_A(x),1-\mu_A(x)>|\,x\in X\}$ is called necessity operator

(ii) $\Diamond(A)=\{<x,1-\nu_A(x),\nu_A(x)>|\,x\in X\}$ is called possibility operator

(iii) $D_\alpha(A)=\{<x,\mu_A(x)+\alpha\pi_A(x),\nu_A(x)+(1-\alpha)\pi_A(x)>|\,x\in X\}$ is called α-model operator.

Clearly $\Box(A)\subseteq A\subseteq\Diamond(A)$ and the equality hold, when A is a fuzzy set also $D_0(A)=\Box(A)$ and $D_1(A)=\Diamond(A)$. Therefore the α-Model operator $D_\alpha(A)$ is an extension of necessity operator $\Box(A)$ and possibility operator $\Diamond(A)$.

Definition 2.8. ([1]) For any IFS $A=\{<x,\mu_A(x),\nu_A(x)>|\,x\in X\}$ of X and for any $\alpha,\beta\in[01]$ such that $\alpha+\beta\le 1$, the (α,β)-model operator $F_{\alpha,\beta}: IFS(X)\to IFS(X)$ is defined as $F_{\alpha,\beta}(A)=\{<x,\mu_A(x)+\alpha\pi_A(x),\nu_A(x)+\beta\pi_A(x)>|\,x\in X\}$, where $\pi_A(x)=1-\mu_A(x)-\nu_A(x)$ for all $x\in X$. Therefore we can write $F_{\alpha,\beta}(A)$ as $F_{\alpha,\beta}(A)(x)=(\mu_{F_{\alpha,\beta}(A)}(x),\nu_{F_{\alpha,\beta}(A)}(x))$ where $\mu_{F_{\alpha,\beta}}(x)=\mu_A(x)+\alpha\pi_A(x)$ and $\nu_{F_{\alpha,\beta}(A)}(x))=\nu_A(x)+\beta\pi_A(x)$. Clearly, $F_{0,1}(A)=\Box(A), F_{1,0}(A)=\Diamond(A)$ and $F_{\alpha,1-\alpha}(A)=D_\alpha(A)$

Definition 2.9. Let X and Y be two non empty sets and $f: X\to Y$ be a mapping. Let A and B be IFS's of X and Y respectively . Then the image of A under the map f is denoted by f(A) and is defined by $f(A)(y)=(\mu_{f(A)}(y),\nu_{f(A)}(y))$, where

$$\mu_{f(A)}(y)=\begin{cases}\vee\{\mu_A(x): x\in f^{-1}(y)\}\\ 0 \quad otherwise\end{cases}\nu_{f(A)}(y)=\begin{cases}\wedge\{\nu_A(x): x\in f^{-1}(y)\}\\ 1 \quad otherwise\end{cases}$$ also pre image of B under f is denoted by $f^{-1}(B)$ and is defined as

$$f^{-1}(B)(x)=(\mu_{f^{-1}(B)}(x),\nu_{f^{-1}(B)}(x))=(\mu_B(f(x)),\nu_B(f(x)));\forall x\in X$$

Remark 2.16. $\mu_A(x)\le\mu_{f(A)}(f(x))$ and $\nu_A(x)\ge\nu_{f(A)}(f(x))$ $\forall x\in X$ however equality hold when the map f is bijective.

An IFS $A=(\mu_A,\nu_A)$ in X is called

(**A**)an intuitionistic fuzzy subalgebra of X, if it satisfies the following axioms:

(i) $\mu_A(x*y) \geq \min\{\mu_A(x), \mu_A(y)\}$,

(ii) $\nu_A(x*y) \leq \max\{\nu_A(x), \nu_A(y)\}\ \forall\ x, y \in X$.

(B) an intuitionistic fuzzy ideal of X, if it satisfies the following axioms:

(i) $\mu_A(0) \geq \mu_A(x)$

(ii) $\nu_A(0) \leq \nu_A(x)$

(iii) $\mu_A(x) \geq \min\{\mu_A(x*y), \mu_A(y)\}$,

(iv) $\nu_A(x) \leq \max\{\nu_A(x*y), \nu_A(y)\}\ \forall\ x, y \in X$.

(C) an intuitionistic fuzzy H-ideal of X, if it satisfies the following axioms:

(i) $\mu_A(0) \geq \mu_A(x)$

(ii) $\nu_A(0) \leq \nu_A(x)$

(iii) $\mu_A(x*z) \geq \min\{\mu_A(x*(y*z)), \mu_A(y)\}$,

(iv) $\nu_A(x*z) \leq \max\{\nu_A(x*(y*z)), \nu_A(y)\}\ \forall\ x, y, z \in X$.

(D) a doubt intuitionistic fuzzy subalgebra of X, if it satisfies the following axioms:

(i) $\mu_A(x*y) \leq \max\{\mu_A(x), \mu_A(y)\}$,

(ii) $\nu_A(x*y) \geq \min\{\nu_A(x), \nu_A(y)\}\ \forall\ x, y \in X$.

(**E**) a doubt intuitionistic fuzzy ideal of X, if it satisfies the following axioms:

(i) $\mu_A(0) \leq \mu_A(x)$

(ii) $\nu_A(0) \geq \nu_A(x)$

(iii) $\mu_A(x) \leq \max\{\mu_A(x*y), \mu_A(y)\}$,

(iv) $\nu_A(x) \geq \min\{\nu_A(x*y), \nu_A(y)\}\ \forall\ x, y \in X$.

(**F**) a doubt intuitionistic fuzzy H-ideal of X, if it satisfies the following axioms:

(i) $\mu_A(0) \leq \mu_A(x)$

(ii) $\nu_A(0) \geq \nu_A(x)$

(iii) $\mu_A(x*z) \leq \max\{\mu_A(x*(y*z)), \mu_A(y)\}$,

(iv) $\nu_A(x*z) \geq \min\{\nu_A(x*(y*z)), \nu_A(y)\}\ \forall\ x, y, z \in X$.

3. Model operator $F_{\alpha,\beta}$ on doubt intuitionistic fuzzy H-ideals in BCK/BCI-algebras

In this section, we study the effect of model operator on doubt intuitionistic fuzzy H-ideals in BCK/BCI -algebras.

Theorem 3.1. If A is a doubt intuitionistic fuzzy (DIF) H-ideal of X, then $F_{\alpha,\beta}(A)$ is also a DIF H-ideal of X.

Proof: Let $x \in X$, then $F_{\alpha,\beta}(x) = (\mu_{F_{\alpha,\beta}(A)}(x), \nu_{F_{\alpha,\beta}(A)}(x))$. where $\mu_{F_{\alpha,\beta}(A)}(x) = \mu_A(x) + \alpha\pi_A(x)$ and $\nu_{F_{\alpha,\beta}(A)}(x) = \nu_A(x) + \beta\pi_A(x)$. Now

(i) $\mu_{F_{\alpha,\beta}(A)}(0) = \mu_A(0) + \alpha\pi_A(0) = \mu_A(0) + \alpha(1 - \mu_A(0) - \nu_A(0))$

$= \alpha + (1-\alpha)\mu_A(0) - \alpha\nu_A(0) \leq \alpha + (1-\alpha)\mu_A(x) - \alpha\nu_A(x)$

$= \mu_A(x) + \alpha(1 - \mu_A(x) - \nu_A(x)) = \mu_A(x) + \alpha\pi_A(x) = \mu_{F_{\alpha,\beta}(A)}(x)$

$\therefore \mu_{F_{\alpha,\beta}(A)}(0) \leq \mu_{F_{\alpha,\beta}(A)}(x)$

(ii)

$\nu_{F_{\alpha,\beta}(A)}(0) = \nu_A(0) + \beta\pi_A(0) = \nu_A(0) + \beta(1 - \mu_A(0) - \nu_A(0))$

$= \beta + (1-\beta)\nu_A(0) - \beta\mu_A(0) \geq \beta + (1-\beta)\nu_A(x) - \beta\mu_A(x)$

$= \nu_A(x) + \beta(1 - \nu_A(x) - \mu_A(x)) = \nu_A(x) + \beta\pi_A(x) = \mu_{F_{\alpha,\beta}(A)}(x)$

$\therefore \nu_{F_{\alpha,\beta}(A)}(0) \geq \nu_{F_{\alpha,\beta}(A)}(x)$

(iii)

$\mu_{F_{\alpha,\beta}(A)}(x*z) = \mu_A(x*z) + \alpha\pi_A(x*z) = \mu_A(x*z) + \alpha(1 - \mu_A(x*z) - \nu_A(x*z))$

$= \alpha + (1-\alpha)\mu_A(x*z) - \alpha\nu_A(x*z)$

$\leq \alpha + (1-\alpha)max(\mu_A(x*(y*z)), \mu_A(y)) - \alpha min(\nu_A(x*(y*z)), \nu_A(y))$

$= \alpha\{1 - min(\nu_A(x*(y*z)), \nu_A(y))\} + (1-\alpha)max(\mu_A(x*(y*z)), \mu_A(y))$

$= \alpha\, max(1 - \nu_A(x*(y*z)), 1 - \nu_A(y))\} + (1-\alpha)max(\mu_A(x*(y*z)), \mu_A(y))$

$= max\{\alpha(1 - \nu_A(x*(y*z))) + (1-\alpha)\mu_A(x*(y*z)), \alpha(1 - \nu_A(y)) + (1-\alpha)\mu_A(y)\}$

$= max\{\mu_A(x*(y*z)) + \alpha(1 - \mu_A(x*(y*z)) - \nu_A(x*(y*z))), \mu_A(y) + \alpha(1 - \mu_A(y) - \nu_A(y))\}$

$= max\{\mu_{F_{\alpha,\beta}(A)}(x*(y*z)), \mu_{F_{\alpha,\beta}(A)}(y)\}$.

$\therefore \mu_{F_{\alpha,\beta}(A)}(x*z) \leq max\{\mu_{F_{\alpha,\beta}(A)}(x*(y*z)), \mu_{F_{\alpha,\beta}(A)}(y)\}$.

Similarly we can prove $\nu_{F_{\alpha,\beta}(A)}(x*z) \geq min\{\nu_{F_{\alpha,\beta}(A)}(x*(y*z)), \nu_{F_{\alpha,\beta}(A)}(y)\}$.

Hence $F_{\alpha,\beta}(A)$ is a DIF H-ideal of X.

Remark 3.2. The converse of above Theorem need not be true as shown in Example below.

Example 3.3. Consider a BCK-algebra $X = \{0,1,2,3,4\}$ with the following cayley table:

Table 1: Illustration of converse of Theorem 3.1.

*	0	1	2	3	4
0	0	0	0	0	0
1	1	0	1	1	1
2	2	2	0	2	2
3	3	3	3	0	3
4	4	4	4	4	0

The IF subset $A = \{< x, \mu_A(x), \nu_A(x) >| x \in X\}$ given by $\mu_A(0) = 0.34$, $\mu_A(1) = 0.3$, $\mu_A(2) = 0.5$, $\mu_A(3) = 0.5$, $\mu_A(4) = 0.6$ and $\nu_A(0) = 0.66$, $\nu_A(1) = 0.6$, $\nu_A(2) = 0.5$, $\nu_A(3) = 0.5$, $\nu_A(4) = 0.4$ is not a DIF H -ideal of X . Since $\mu_A(0) \not\leq \mu_A(1)$. Now take $\alpha = 0.7, \beta = 0.2, \alpha + \beta \leq 1$, then $F_{\alpha,\beta}(A) = \{< x, \mu_{F_{\alpha,\beta}(A)}(x), \nu_{F_{\alpha,\beta}(A)}(x) >| x \in X\}$ is $\mu_{F_{0.7,0.2}(A)}(0) = 0.34$, $\mu_{F_{0.7,0.2}(A)}(1) = 0.37$, $\mu_{F_{0.7,0.2}(A)}(2) = 0.5$, $\mu_{F_{0.7,0.2}(A)}(3) = 0.5$, $\mu_{F_{0.7,0.2}(A)}(4) = 0.6$ and $\nu_{F_{0.7,0.2}(A)}(0) = 0.66$, $\nu_{F_{0.7,0.2}(A)}(1) = 0.62$, $\nu_{F_{0.7,0.2}(A)}(2) = 0.5$, $\nu_{F_{0.7,0.2}(A)}(3) = 0.5$, $\nu_{F_{0.7,0.2}(A)}(4) = 0.4$. It can easily verified that $F_{0.7,0.2}(A)$ is a DIF H -ideal of X.

Corollary 3.4. If A is a DIF H-ideal of X, then

(i) $\square(A)$ is also a DIF H-ideal of X.

(ii) $\Diamond(A)$ is also a DIF H-ideal of X.

(iii) $D_\alpha(A)$ is also a DIF H-ideal of X.

Theorem 3.5. If $A = (\mu_A, \nu_A)$ be a doubt intuitionistic fuzzy H-ideal of an associative BCK/BCI-algebra X. Then if the inequility $x * a \leq b$ holds in X, then

(i) $\mu_{F_{\alpha,\beta}(A)}(x * a) \leq \mu_{F_{\alpha,\beta}(A)}(b)$

(ii) $\nu_{F_{\alpha,\beta}(A)}(x * a) \geq \nu_{F_{\alpha,\beta}(A)}(b)$

Proof: Let $x, a, b \in X$ be such that $x * a \leq b$ then $(x * a) * b = 0$ and since A is a doubt intuitionistic fuzzy H-ideal of X, so

(i) $\mu_{F_{\alpha,\beta}(A)}(x * a) \leq max\{\mu_{F_{\alpha,\beta}(A)}((x * (b * a)), \mu_{F_{\alpha,\beta}(A)}(b)\}$

$= max\{\mu_{F_{\alpha,\beta}(A)}((x * b) * a), \mu_{F_{\alpha,\beta}(A)}(b)\}$ [Since X is associative]

$= max\{\mu_{F_{\alpha,\beta}(A)}((x * a) * b), \mu_{F_{\alpha,\beta}(A)}(b)\}$

$= max\{\mu_{F_{\alpha,\beta}(A)}(0), \mu_{F_{\alpha,\beta}(A)}(b)\}$

$= \mu_{F_{\alpha,\beta}(A)}(b)$ [Since $\mu_{F_{\alpha,\beta}(A)}(0) \leq \mu_{F_{\alpha,\beta}(A)}(b)$]

Therefore $\mu_{F_{\alpha,\beta}(A)}(x * a) \leq \mu_{F_{\alpha,\beta}(A)}(b)$

(ii)

$\nu_{F_{\alpha,\beta}(A)}(x * a) \geq min\{\nu_{F_{\alpha,\beta}(A)}((x * (b * a)), \nu_{F_{\alpha,\beta}(A)}(b)\}$

$= min\{\nu_{F_{\alpha,\beta}(A)}((x * b) * a), \nu_{F_{\alpha,\beta}(A)}(b)\}$ [Since X is associative]

$= min\{\nu_{F_{\alpha,\beta}(A)}((x * a) * b), \nu_{F_{\alpha,\beta}(A)}(b)\}$

$= min\{\nu_{F_{\alpha,\beta}(A)}(0), \nu_{F_{\alpha,\beta}(A)}(b)\}$

$= \nu_{F_{\alpha,\beta}(A)}(b)$ [Since $\nu_{F_{\alpha,\beta}(A)}(0) \geq \nu_{F_{\alpha,\beta}(A)}(b)$].

Therefore $\nu_{F_{\alpha,\beta}(A)}(x * a) \geq \nu_{F_{\alpha,\beta}(A)}(b)$.

Theorem 3.6. If $A = (\mu_A, \nu_A)$ be a doubt intuitionistic fuzzy H-ideal of a BCK/BCI-algebra X. Then

(i) $\mu_{F_{\alpha,\beta}(A)}(0*(0*x)) \leq \mu_{F_{\alpha,\beta}(A)}(x)$

(ii) $\nu_{F_{\alpha,\beta}(A)}(0*(0*x)) \geq \nu_{F_{\alpha,\beta}(A)}(x)$ for all $x \in X$.

Proof: (i) $\mu_{F_{\alpha,\beta}(A)}(0*(0*x)) \leq max\{\mu_{F_{\alpha,\beta}(A)}(0*(x*(0*x))), \mu_{F_{\alpha,\beta}(A)}(x)\}$

$= max\{\mu_{F_{\alpha,\beta}(A)}(0*(x*0)), \mu_{F_{\alpha,\beta}(A)}(x)\}$

$= max\{\mu_{F_{\alpha,\beta}(A)}(0*x), \mu_{F_{\alpha,\beta}(A)}(x)\}$

$= max\{\mu_{F_{\alpha,\beta}(A)}(0), \mu_{F_{\alpha,\beta}(A)}(x)\}$

$= \mu_{F_{\alpha,\beta}(A)}(x)$ forall $x \in X$.

Therefore $\mu_{F_{\alpha,\beta}(A)}(0*(0*x)) \leq \mu_{F_{\alpha,\beta}(A)}(x)$ for all $x \in X$.

(ii) Proof is similar to (i)

Theorem 3.7. *If A and B are two DIF H-ideals of BCK/BCI-algebra X, then*

(i) $F_{\alpha,\beta}(A \cup B)$ is also a DIF H-ideal of BCK/BCI-algebra X.

(ii) $F_{\alpha,\beta}(A \times B)$ is also a DIF H-ideal of BCK/BCI-algebra X.

Proof: (i) We have $F_{\alpha,\beta}(A \cup B)(x) = \{< x, \mu_{F_{\alpha,\beta}(A\cup B)}(x), \nu_{F_{\alpha,\beta}(A\cap B)}(x) > | x \in X\}$,

where $\mu_{(A\cup B)}(x) = (\mu_A \cup \mu_B)(x) = \max\{\mu_A(x), \mu_B(x)\}$ and

$\nu_{(A\cap B)}(x) = (\nu_A \cap \nu_B)(x) = \min\{\nu_A(x), \nu_B(x)\}$,

Let $x, y \in X$. Since both A, B are DIF H-ideals of BCK/BCI-algebra X, therefore

$(i)\ \mu_A(0) \leq \mu_A(x)$ & $\mu_B(0) \leq \mu_B(x)$

$(ii)\ \nu_A(0) \geq \nu_A(x)$ & $\nu_B(0) \geq \nu_B(x)$

$(iii) \mu_A(x*z) \leq max\{\mu_A(x*(y*z), \mu_A(y)\}$ & $\mu_B(x*y) \leq max\{\mu_B(x*(y*z), \mu_B(y)\}$

$(iv)\ \nu_A(x*z) \geq min\{\nu_A(x*(y*z))\ \nu_A(y)\}$, & $\nu_B(x*z) \geq min\{\nu_B(x*(y*z)), \nu_B(y)\}$

Now,

$\mu_{F_{\alpha,\beta}(A\cup B)}(0) = \mu_{(A\cup B)}(0) + \alpha\pi_{(A\cup B)}(0) = \mu_{(A\cup B)}(0) + \alpha\{1 - \mu_{(A\cup B)}(0) - \nu_{(A\cap B)}(0)\}$

$= \alpha + (1-\alpha)\mu_{(A\cup B)}(0) - \alpha\nu_{(A\cap B)}(0)$

$= \alpha + (1-\alpha)max(\mu_A(0), \mu_B(0)) - \alpha min(\nu_A(0), \nu_B(0))$

$\leq \alpha + (1-\alpha)max(\mu_A(x), \mu_B(x)) - \alpha min(\nu_A(x), \nu_B(x))$

$= \alpha(1 - min(\nu_A(x), \nu_B(x))) + (1-\alpha)max(\mu_A(x), \mu_B(x))$

$= max(\mu_A(x), \mu_B(x)) + \alpha\{(1 - max(\mu_A(x), \mu_B(x)) - min(\nu_A(x), \nu_A(x))\}$

$= \mu_{(A\cup B)}(x) + \alpha\{1 - \mu_{(A\cup B)}(x) - \nu_{(A\cap B)}(x)\} = \mu_{(A\cup B)}(x) + \alpha\pi_{(A\cup B)}(x)$

$= \mu_{F_{\alpha,\beta}(A\cup B)}(x)$

Therefore, $\mu_{F_{\alpha,\beta}(A\cup B)}(0) \leq \mu_{F_{\alpha,\beta}(A\cup B)}(x)$.

Similarly we can prove $\nu_{F_{\alpha,\beta}(A\cup B)}(0) \geq \nu_{F_{\alpha,\beta}(A\cup B)}(x)$.

Again,

$\mu_{F_{\alpha,\beta}(A\cup B)}(x*z) = \mu_{(A\cup B)}(x*z) + \alpha\pi_{(A\cup B)}(x*z)$

$= \mu_{(A\cup B)}(x*z) + \alpha\{1 - \mu_{(A\cup B)}(x*z) - \nu_{(A\cap B)}(x*z)\}$

$= \alpha + (1-\alpha)\mu_{(A\cup B)}(x*z) - \alpha\nu_{(A\cap B)}(x*z)$

$= \alpha + (1-\alpha)max\{\mu_A(x*z), \mu_A(x*z)\} - \alpha min\{\nu_A(x*z), \nu_A(x*z)\}$

$\leq (1-\alpha)max\{max\{\mu_A(x*(y*z)), \mu_A(y)\}, max\{\mu_B(x*(y*z)), \mu_B(y)\}\}$
$\quad +\alpha - \alpha min\{min\{\nu_A(x*(y*z)), \nu_A(y)\}, min\{\nu_B(x*(y*z)), \nu_B(y)\}\}$

$= (1-\alpha)max\{max\{\mu_A(x*(y*z)), \mu_B(x*(y*z))\}, max\{\mu_A(y), \mu_B(y)\}\} +$
$\quad \alpha - \alpha min\{min\{\nu_A(x*(y*z)), \nu_B(x*(y*z))\}, min\{\nu_A(y), \nu_B(y)\}\}$

$= (1-\alpha)max\{\mu_{(A\cup B)}(x*(y*z)), \mu_{(A\cup B)}(y)\} + \alpha -$
$\alpha min\{\nu_{(A\cap B)}(x*(y*z)), \nu_{(A\cap B)}(y)\}$

$= (1-\alpha)max\{\mu_{(A\cup B)}(x*(y*z)), \mu_{(A\cup B)}(y)\} +$
$\alpha max\{1 - \nu_{(A\cap B)}(x*(y*z)), 1 - \nu_{(A\cap B)}(y)\}$

$\leq max\{(1-\alpha)\mu_{(A\cup B)}(x*(y*z)) + \alpha(1 - \nu_{(A\cap B)}(x*(y*z))), (1-\alpha)\mu_{(A\cup B)}(y)$
$\quad +\alpha(1 - \nu_{(A\cap B)}(y))\}$

$\leq max\{\mu_{(A\cup B)}(x*(y*z)) + \alpha(1 - \mu_{(A\cup B)}(x*(y*z)) - \nu_{(A\cap B)}(x*(y*z))), \mu_{(A\cup B)}(y)$

$\leq max\{\mu_{(A\cup B)}(x*(y*z)) + \alpha\pi_{(A\cup B)}(x*(y*z))), \mu_{(A\cup B)}(y) + \alpha\pi_{(A\cup B)}(y))\}$

$\leq max\{\mu_{F_{\alpha,\beta}(A\cup B)}(x*(y*z)), \mu_{F_{\alpha,\beta}(A\cup B)}(y)\}$.

Therefore $\mu_{F_{\alpha,\beta}(A\cup B)}(x*z) \leq max\{\mu_{F_{\alpha,\beta}(A\cup B)}(x*(y*z)), \mu_{F_{\alpha,\beta}(A\cup B)}(y)\}$.

Similarly we can prove that

$$\nu_{F_{\alpha,\beta}(A\cap B)}(x*z) \geq min\{\nu_{F_{\alpha,\beta}(A\cap B)}(x*(y*z)), \nu_{F_{\alpha,\beta}(A\cap B)}(y)\}.$$

(ii) Similar to proof of (i)

Theorem 3.8. *An intuitionistic fuzzy set* $A=(\mu_A,\nu_A)$ *is a DIF H-ideal of a BCK/BCI-algebra X if f the fuzzy sets* $\mu_{F_{\alpha,\beta}(A)}$ *and* $\mu_{F_{\alpha,\beta}(A)}$ *are doubt fuzzy H-ideals of X.*

Theorem 3.9. *If* $\{A_i : i=1,2,...,n\}$ *be n DIF H-ideals of X, then*

(i) $F_{\alpha,\beta}(\cap_{i=1}^{n} A_i : i=1,2,...,n\}$ is also a DIF H-ideal of X.

(ii) $F_{\alpha,\beta}(\times_{i=1}^{n} A_i : i=1,2,...,n\}$ is also a DIF H-ideal of X.

Theorem 3.10. *If* $A=(\mu_A,\nu_A)$ *be a DIF H-ideal of a BCK/BCI-algebra* X *. Then the sets*

$X_{\mu_{F_{\alpha,\beta}}} = \{x\in X \mid \mu_{F_{\alpha,\beta}(A)}(x)=\mu_{F_{\alpha,\beta}(A)}(0)\}$

$X_{\nu_{F_{\alpha,\beta}}} = \{x\in X \mid \nu_{F_{\alpha,\beta}(A)}(x)=\nu_{F_{\alpha,\beta}(A)}(0)\}$

are H-ideals of X.

Proof: Let $A=(\mu_A,\nu_A)$ be a doubt intuitionistic fuzzy H-ideal of a BCK/BCI-algebra X. Clearly $0\in X_{\mu_{F_{\alpha,\beta}}}$ and $X_{\nu_{F_{\alpha,\beta}}}$. Let $x,y,z\in X$ such that $x*(y*z), y\in \mu_{F_{\alpha,\beta}(A)}$, then $\mu_{F_{\alpha,\beta}(A)}(x*(y*z))=\mu_{F_{\alpha,\beta}(A)}(y)=\mu_{F_{\alpha,\beta}(A)}(0).$ Now

$$\mu_{F_{\alpha,\beta}(A)}(x*z)\le max\{\mu_{F_{\alpha,\beta}(A)}(x*(y*z)),\mu_{F_{\alpha,\beta}(A)}(y)\}$$
$$= max\{\mu_{F_{\alpha,\beta}(A)}(0),\mu_{F_{\alpha,\beta}(A)}(0)\} = \mu_{F_{\alpha,\beta}(A)}(0)$$
$$\Rightarrow \mu_{F_{\alpha,\beta}(A)}(x*z)\le \mu_{F_{\alpha,\beta}(A)}(0).$$

Also $\mu_{F_{\alpha,\beta}(A)}(0)\le \mu_{F_{\alpha,\beta}(A)}(x*z)$ by Theorem 3.8 and Definition 2.22.

Therefore $\mu_{F_{\alpha,\beta}(A)}(x*z)=\mu_{F_{\alpha,\beta}(A)}(0) \Rightarrow x*z\in X_{\mu_{F_{\alpha,\beta}}}$

Similarly we can show that $x*z\in X_{\nu_{F_{\alpha,\beta}}}$

Hence $X_{\mu_{F_{\alpha,\beta}}}$ and $X_{\nu_{F_{\alpha,\beta}}}$ are H-ideals of X.

Proposition 3.11. *If A and B be two IFS sets of X and Y respectively and* $f: X\to Y$ *be a mapping, then*

(i) $f^{-1}(F_{\alpha,\beta}(B)) = F_{\alpha,\beta}(f^{-1}(B))$

(ii) $f(F_{\alpha,\beta}(A))\subseteq F_{\alpha,\beta}(f(A))$

4. Effect of model operators on doubt intuitionistic fuzzy H -ideals under homomorphism

Definition 4.1. *Let* X *and* Y *be two DIF H-ideals, then a mapping* $f: X\to Y$ *is said to be homomorphism if* $f(x*y)=f(x)*f(y), \forall x,y\in X$.

Theorem 4.2. *Let $f: X \to Y$ be a homomorphism of DIF H-ideals. If $F_{\alpha,\beta}(A)$ is a DIF H-ideal of Y, then $f^{-1}(F_{\alpha,\beta}(A))$ is also a DIF H-ideal of X.*

Proof: Since $f^{-1}(F_{\alpha,\beta}(A)) = F_{\alpha,\beta}(f^{-1}(A))$. It is enough to show that $F_{\alpha,\beta}(f^{-1}(A))$ is a DIF H-ideal of X. Let $x, y, z \in X$, then $\mu_{F_{\alpha,\beta}(f^{-1}(A))}(0) = \mu_{F_{\alpha,\beta}(A)}(f(0))$

$$\leq \mu_{F_{\alpha,\beta}(A)}(f(x)) = \mu_{F_{\alpha,\beta}(f^{-1}(A))}(x)$$

$$\Rightarrow \mu_{F_{\alpha,\beta}(f^{-1}(A))}(0) \leq \mu_{F_{\alpha,\beta}(f^{-1}(A))}(x)$$

and $\mu_{F_{\alpha,\beta}(f^{-1}(A))}(x*z) = \mu_{F_{\alpha,\beta}(A)} f(x*z) = \mu_{F_{\alpha,\beta}(A)}(f(x)*f(z))$

$$\leq max\{\mu_{F_{\alpha,\beta}(A)}(f(x)*(f(y))*f(z)), \mu_{F_{\alpha,\beta}(A)}(f(y))\}$$

$$\leq max\{\mu_{F_{\alpha,\beta}(A)}(f(x*(y*z))), \mu_{F_{\alpha,\beta}(A)}(f(y))\}$$

$$= max\{\mu_{F_{\alpha,\beta}(f^{-1}(A))}(x*(y*z)), \mu_{F_{\alpha,\beta}(f^{-1}(A))}(y)\}$$

$$\mu_{F_{\alpha,\beta}(f^{-1}(A))}(x*z) \leq max\{\mu_{F_{\alpha,\beta}(f^{-1}(A))}(x*(y*z)), \mu_{F_{\alpha,\beta}(f^{-1}(A))}(y)\}$$

Similarly we can show $\nu_{F_{\alpha,\beta}(f^{-1}(A))}(0) \geq \nu_{F_{\alpha,\beta}(f^{-1}(A))}(x)$ and

$$\nu_{F_{\alpha,\beta}(f^{-1}(A))}(x*z) \geq min\{\nu_{F_{\alpha,\beta}(f^{-1}(A))}(x*(y*z)), \nu_{F_{\alpha,\beta}(f^{-1}(A))}(y)\}.$$

Hence $f^{-1}(F_{\alpha,\beta}(A))$ is also a DIF H-ideal of X.

Corollary 4.3. *Let $f: X \to Y$ be a homomorphism of BCK-algebras.*

(i) If $\square(A)$ is a DIF H-ideal of Y, then $f^{-1}(\square(A))$ is also a DIF H-ideal of X.

(ii) If $\Diamond(A)$ is a DIF H-ideal of Y, then $f^{-1}(\Diamond(A))$ is also a DIF H-ideal of X.

Theorem 4.4. *Let $f: X \to Y$ be an onto homomorphism of DIF H-ideals. If $F_{\alpha,\beta}(A)$ is a DIF H-ideal of X, then $f(F_{\alpha,\beta}(A))$ is also a DIF H-ideal of Y.*

Proof: Let $y_1, y_2, y_3 \in Y$. Since f is onto, therefore there exists $x_1, x_2, x_3 \in X$ such that $f(x_1) = y_1$, $f(x_2) = y_2$, $f(x_3) = y_3$

$$f(F_{\alpha,\beta}(A))(y_1 * y_3) = (\mu_{f(F_{\alpha,\beta}(A))}(y_1 * y_3), \nu_{f(F_{\alpha,\beta}(A))}(y_1 * y_3))$$

Now $\mu_{f(F_{\alpha,\beta}(A))}(y_1 * y_3) = \mu_{F_{\alpha,\beta}(A)}(x_1 * x_3)$ where $y_1 * y_3 = f(x_1) * f(x_3) = f(x_1 * x_3)$

$$\mu_{f(F_{\alpha,\beta}(A))} f(0) = \mu_{F_{\alpha,\beta}(A)}(0) = \mu_A(0) + \alpha\pi_A(0) = \mu_A(0) + \alpha[1 - \mu_A(0) - \nu_A(0)]$$

$$= \alpha + (1-\alpha)\mu_A(0) - \alpha\nu_A(0) \leq \alpha + (1-\alpha)\mu_A(x) - \alpha\nu_A(x)$$

$$= \mu_A(x) + \alpha[1 - \mu_A(x) - \nu_A(x)] = \mu_A(x) + \alpha\pi_A(x)$$

$$= \mu_{F_{\alpha,\beta}(A)}(x) = \mu_{f(F_{\alpha,\beta}(A))} f(x)$$

$$\therefore \mu_{f(F_{\alpha,\beta}(A))} f(0) \le \mu_{f(F_{\alpha,\beta}(A))} f(x).$$

$$\mu_{f(F_{\alpha,\beta}(A))}(y_1 * y_3) = \mu_{F_{\alpha,\beta}(A)}(x_1 * x_3) = \mu_{F_{\alpha,\beta}(A)}(x_1 * x_3)$$
$$= \mu_A(x_1 * x_3) + \alpha\pi_A(x_1 * x_3) = \mu_A(x_1 * x_3) + \alpha[1 - \mu_A(x_1 * x_3) - \nu_A(x_1 * x_3)]$$
$$= \alpha + (1-\alpha)\mu_A(x_1 * x_3) - \alpha\nu_A(x_1 * x_3)$$
$$\le \alpha + (1-\alpha) max\{\mu_A(x_1 * (x_2 * x_3)), \mu_A(x_2)\} - \alpha min\{\nu_A(x_1 * (x_2 * x_3)), \nu_A(x_2)\}$$
$$= \alpha\{1 - min\{\nu_A(x_1 * (x_2 * x_3)), \nu_A(x_2)\}\} + (1-\alpha) max\{\mu_A(x_1 * (x_2 * x_3)), \mu_A(x_2)\}$$
$$= \alpha max\{1 - \nu_A(x_1 * (x_2 * x_3)), 1 - \nu_A(x_2)\} + (1-\alpha) max\{\mu_A(x_1 * (x_2 * x_3)), \mu_A(x_2)\}$$
$$= max\{(1-\alpha)\mu_A(x_1 * (x_2 * x_3)) + \alpha(1 - \nu_A(x_1 * (x_2 * x_3))), (1-\alpha)\mu_A(x_2) + \alpha(1 - \nu_A(x_2))\}$$
$$= max\{\mu_A(x_1 * (x_2 * x_3)) + \alpha(1 - \nu_A(x_1 * (x_2 * x_3)) - \mu_A(x_1 * (x_2 * x_3))), \mu_A(x_2) + \alpha(1 - \nu_A(x_2) - \mu_A(x_2))\}$$
$$= max\{\mu_A(x_1 * (x_2 * x_3)) + \alpha\pi_A(x_1 * (x_2 * x_3)), \mu_A(x_2) + \alpha\pi_A(x_2)\}$$
$$= max\{\mu_{F_{\alpha,\beta}(A)}(x_1 * (x_2 * x_3)), \mu_{F_{\alpha,\beta}(A)}(x_2)\}$$
$$= max\{\mu_{f(F_{\alpha,\beta}(A))}(f(x_1 * (x_2 * x_3))), \mu_{f(F_{\alpha,\beta}(A))}(f(x_2))\}$$
$$= max\{\mu_{f(F_{\alpha,\beta}(A))}(y_1 * (y_2 * y_3)), \mu_{f(F_{\alpha,\beta}(A))}(y_2)\}$$
$$\therefore \mu_{f(F_{\alpha,\beta}(A))}(y_1 * y_3) \le max\{\mu_{f(F_{\alpha,\beta}(A))}(y_1 * (y_2 * y_3)), \mu_{f(F_{\alpha,\beta}(A))}(y_2)\}.$$

Similarly we can show that $\nu_{f(F_{\alpha,\beta}(A))} f(0) \ge \nu_{f(F_{\alpha,\beta}(A))} f(x)$ and

$$\nu_{f(F_{\alpha,\beta}(A))}(y_1 * y_3) \ge min\{\nu_{f(F_{\alpha,\beta}(A))}(y_1 * (y_2 * y_3)), \nu_{f(F_{\alpha,\beta}(A))}(y_2)\}.$$

Hence $f(F_{\alpha,\beta}(A))$ is a DIF H-ideal of Y.

Corollary 4.5. *Let $f : X \to Y$ be a homomorphism of BCK-algebras.*

(i) If $\Box(A)$ is a DIF H-ideal of X, then $f(\Box(A))$ is also a DIF H-ideal of Y.

(ii) If $\Diamond(A)$ is a DIF H-ideal of X, then $f(\Diamond(A))$ is also a DIF H-ideal of Y.

REFERENCES

1. K.T. Atanassov, Intuitionistic Fuzzy Sets: Theory and Applications, Studies in fuzziness and soft computing, 35, Physica-Verlag, Heidelberg/New York 1999.
2. S. R. Barbhuiya, Intuitionistic Q-fuzzy Ideals of BG-Algebra, *International Journal of Computer Applications*, 94(16),(2014), 8-14.
3. S. R. Barbhuiya, Doubt fuzzy ideals of BF-algebra, *IOSR Journal of Mathematics (IOSR-JM)*, 10(2), Ver. VII (Mar-April), (2014), 65-70.
4. S. R. Barbhuiya, $(\in, \in \vee q)$ -Intuitionistic Fuzzy Ideals of BG-algebra, *Fuzzy Information and Engineering*, 7(1), (2015), 31-48.

5. S. R. Barbhuiya, Model Operator $\square_{\alpha,\beta,\gamma,\delta,\varepsilon,\zeta}$ on intuitionistic fuzzy BG -algebras, *Annals of Fuzzy Mathematics and Informatics,*(2015) (In press, available online).
6. T.Bej, M. Pal, Doubt intuitionistic fuzzy H-ideals in BCK/BCI-algebras, *Annals of Fuzzy Mathematics and Informatics*, 8(4),(2014), 593-605.
7. M. Bhowmik, T. Senapati and M. Pal, Intuitionistic L -fuzzy ideals of BG -algebras, *Afrika Matematika*, **25**(3), (2014) 577-590.
8. C. Jana, T. Senapati and M. Pal, Derivation, f-derivation and generalized derivation of KUS-algebras, *Cogent Mathematics*, 2 (2015) http://dx.doi.org/10.1080/23311835.2015.1064602.
9. C. Jana, T. Senapati, M. Bhowmik and M. Pal, On intuitionistic fuzzy G -subalgebras of G -algebras, *Fuzzy Information and Engineering* 7 (2015) 195-209.
10. C. Jana, M. Pal, T. Senapati and M. Bhowmik, Atanassov's intutionistic L -fuzzy G -subalgebras of G -algebras, *The Journal of Fuzzy Mathematics* 23 (2015) 325-340.
11. C. Jana and T. Senapati, Cubic G-subalgebras of G-algebras, *Annals of Pure and Applied Mathematics*, 10(1) (2015), 105-115.
12. Y. Imai, and K. Iseki, On Axiom systems of Propositional calculi XIV, *Proc. Japan Academy,* 42, (1966), 19-22.
13. K. Iseki, An algebra related with a propositional calculus, *Proc. Japan Acad.* 42 (1966),26-29.
14. Y. B. Jun, Doubt fuzzy BCK/BCI algebras, *Soochow Journal of Mathematics,* 20(3), (1991), 351-358.
15. Y. B. Jun and K. H. Kim, Intuitionistic fuzzy ideals of BCK-algebras, *Int. J. Math. Math.Sci.,* 24, (2000), 839-849.
16. H. M. Khalid and B. Ahmad, Fuzzy H-ideals in BCI-algebras, *Fuzzy Sets and Systems,* 101,(1999), 153-158.
17. B. Satyanarayan, U. B. Madhavi and R. D. Prasad, On intuitionistic fuzzy H-ideals in BCK-algebras, *Int. J. Algebra,* 4(13-16), (2010), 743-749.
18. T. Senapati, M. Bhowmik, M. Pal and B. Davvaz, Fuzzy translations of fuzzy H -ideals in BCK/BCI -algebras, *Journal of the Indonesian Mathematical Society*, 21 (2015) 45-58.
19. T. Senapati, M. Bhowmik, M. Pal, Atanassov's intuitionistic fuzzy translations of intuitionistic fuzzy H -ideals in BCK/BCI -algebras, *Notes on Intuitionistic Fuzzy Sets*, 19(1) (2013) 32-47.
20. T. Senapati, M. Bhowmik, M. Pal and B. Davvaz, Atanassovâ intuitionistic fuzzy translations of intuitionistic fuzzy subalgebras and ideals in BCK/BCI - algebras, *Eurasian Mathematical Journal*, 6(1) (2015), 96-114.
21. T. Senapati, C.S. Kim, M. Bhowmik, M. Pal, Cubic subalgebras and cubic closed ideals of B -algebras, *Fuzzy Information and Engineering* 7 (2015) 129-149.
22. T. Senapati, M. Bhowmik, M. Pal, Fuzzy dot subalgebras and fuzzy dot ideals of B -algebras, Journal of Uncertain Systems 8 (2014) 22-30.
23. T. Senapati, M. Bhowmik, M. Pal, Fuzzy B -subalgebras of B -algebra with respect to t -norm, *Journal of Fuzzy Set Valued Analysis* 2012 (2012) 11-pages, doi: 10.5899/2012/jfsva-00111.

24. T. Senapati, M. Bhowmik, M. Pal, Triangular norm based fuzzy BG-algebras, *Afrika Matematika* (2015), DOI 10.1007/s13370-015-0330-y.
25. T. Senapati, M. Bhowmik and M. Pal, Fuzzy dot structure of BG-algebras, *Fuzzy Information and Engineering*, **6**(3) (2014), 315-329.
26. T. Senapati, Bipolar fuzzy structure of BG-subalgebras, *The Journal of Fuzzy Mathematics* 23 (2015) 209-220.
27. T. Senapati, M. Bhowmik, M. Pal, Interval-valued intuitionistic fuzzy BG-subalgebras, *The Journal of Fuzzy Mathematics* 20 (2012) 707-720.
28. T. Senapati, M. Bhowmik, M. Pal, Interval-valued intuitionistic fuzzy closed ideals BG-algebras and their products, *International Journal of Fuzzy Logic Systems* 2 (2012) 27-44.
29. T. Senapati, M. Bhowmik, M. Pal, Intuitionistic fuzzifications of ideals in BG-algebras, Mathematica Aeterna 2 (2012) 761-778.
30. T. Senapati, T-fuzzy KU-subalgebras of KU-algebras, *Annals of Fuzzy Mathematics and Informatics,* 10(2) (2015), 261-270.
31. T. Senapati and K.P. Shum, Atanassov's intuitionistic fuzzy bi-normed KU-ideals of a KU-algebra, available as online first article in *Journal of Intelligent and Fuzzy Systems*, DOI: 10.3233/IFS-151841
32. T. Senapati, M. Bhowmik and M. Pal, Fuzzy dot subalgebras and fuzzy dot ideals of B-algebras, *Journal of Uncertain Systems*, **8**(1), (2014) 22-30.
33. T. Senapati, C. Jana, M. Bhowmik and M. Pal, L-fuzzy G-subalgebras of G-algebras, *J. of the Egyptian Mathematical Society*, **23** (2015), 219-223.
34. T. Senapati, M. Bhowmik and M. Pal, Fuzzy closed ideals of B-algebras with interval-valued membership function, *Intern. J. Fuzzy Mathematical Archive*, 1(10) (2011), 79-91.
35. T. Senapati, M. Bhowmik and M. Pal, Fuzzy closed ideals of B-algebras, *Int. J. of computer Science, Engineering and Technology*, 1(10) (2011), 669-673.
36. L. A. Zadeh, Fuzzy sets, *Information and Control,* 8, (1965), 338-353.
37. J. Zhan and Z. Tan, Characterization of doubt fuzzy H-ideals in BCK-algebras, *Soochow J.Math.*, 29(3), (2003), 293-298.