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# Intuitionistic (T, S)-fuzzy Magnified Translation Medial Ideals in BCI-algebras

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***Abstract.*** Fuzzy and intuitionistic fuzzy set theories span a wide range of application ranging from industrial process control to medical diagnosis and group decisions processes. For this reason, the generalization of the notions intuitionistic fuzzy medial subalgebras and intuitionistic fuzzy medial ideals in BCI-algebras by using t-norm T and s-norm S are given. Moreover some interesting results are investigated. Moreover, some algorithms for medial ideals and fuzzy medial ideals have been constructed.

***Keywords:*** medial BCI-algebras; intuitionistic fuzzy medial ideal; image ( pre-image ) of intuitionistic (T,S)-fuzzy medial ideal; intuitionistic (T,S)-fuzzy translations and multiplications

## 1. Introduction

The theory of BCK/BCI algebras introduced by Imai and Iséki [4,5] and has been studied deeply by several researchers. The concept of fuzzy subset and various operations on it were first introduced by Zadeh in [37]. Xi [36] introduced the concepts of fuzzy subalgebras and ideals in BCI-algebras and discussed some properties of them. The idea of intuitionistic fuzzy set was first published by Atanassov [1] as a generalization of the notion of fuzzy set. A similar treatment in the BCI-algebras and related algebraic structures were carried out by many researchers [2,6-9,26-35]. Meng and Jun [16] studied medial BCI-algebras. Mostafa et.al [18] introduced the notion of medial ideals in BCI-algebras. They stated the fuzzification of medial ideals and investigated its properties. On the other hand, triangular norm is a powerful tool in the theory research and application development of fuzzy sets [3,14,15,22-25]. Lee et al. [12] discussed fuzzy translations, (normalized, maximal) fuzzy extensions and fuzzy multiplications of fuzzy subalgebras in BCK/BCI-algebras and introduced the relations among fuzzy translations, (normalized, maximal) fuzzy extensions and fuzzy multiplications. In [20,21], concepts of (intuitionistic) fuzzy translation to (intuitionistic) fuzzy H-ideals in BCK/BCI-algebras are introduced. Moreover, the notion of fuzzy extensions and fuzzy multiplications of fuzzy H-ideals with several related properties are investigated. Also, the relationships between fuzzy translations, fuzzy extensions and fuzzy multiplications of fuzzy H-ideals are investigated. In the light of the developments made so far, in this paper the concepts of intuitionistic (T,S)- fuzzy magnified translation medial ideals in BCI-algebras is introduced and some interesting results are obtained.

**2. Preliminaries**

We review some definitions and properties that will be useful in our results.

**Definition 2.1.** [4,5]. An algebraic system $(X,*,0)$ of type (2, 0) is called a BCI-algebra if it satisfying the following conditions:

(BCI-1) $((x*y)*(x*z))*(z*y)=0$,
(BCI-2) $(x*(x*y))*y=0$,
(BCI-3) $x*x=0$,
(BCI-4) $x*y=0$ and $y*x=0$ imply $x=y$,

for all $x, y$ and $z \in X$. In a BCI-algebra X, we can define a partial ordering "$\leq$" by $x \leq y$ if and only if $x*y=0$.

In what follows, X will denote a BCI-algebra unless otherwise specified.

**Definition 2.2.** [16,19] A BCI-algebra $(X,*,0)$ of type (2, 0) is called a medial BCI-algebra if it satisfying the following condition: $(x*y)*(z*u)=(x*z)*(y*u)$, for all $x, y, z$ and $u \in X$.

**Lemma 2.3.** [16] An algebra (X, $*$, 0) of type (2, 0) is a medial BCI-algebra if and only if it satisfies the following conditions:

(i) $x*(y*z)=z*(y*x)$
(ii) $x*0=x$
(iii) $x*x=0$

**Lemma 2.4.** [16] In a medial BCI-algebra X, the following holds: $x*(x*y)=y$, for all $x, y \in X$.

**Lemma 2.5.** Let X be a medial BCI-algebra, then $0*(y*x)=x*y$, for all $x, y \in X$.

**Definition 2.6.** A non empty subset S of a medial BCI-algebra X is said to be medial sub-algebra of X, if $x*y \in S$, for all $x, y \in S$.

**Definition 2.7.** [10,17]. A non-empty subset I of a BCI-algebra X is said to be

(1 ) a BCI-ideal of X if it satisfies:

(I1) $0 \in I$,
(I2) $x*y \in I$ and $y \in I$ implies $x \in I$ for all $x, y \in X$.

(2) medial ideal of X if it satisfies

(M1) $0 \in M$,
(M2) $z*(y*x) \in M$ and $y*z \in M$ imply $x \in M$ for all $x, y$ and $z \in X$.

**Proposition 2.8.** [18] Any medial ideal of a BCI-algebra must be a BCI-ideal but the converse is not true.

**Proposition 2.9.** Any BCI- ideal of a medial BCI-algebra is a medial ideal.
**Proof:** Straightforward.

**Example 2.10.** Let X = {0,1,2,3,4,5} be a set with a binary operation * defined by the following table:

| * | 0 | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 4 | 4 |
| 1 | 1 | 0 | 1 | 0 | 4 | 4 |
| 2 | 2 | 2 | 0 | 0 | 4 | 4 |
| 3 | 3 | 2 | 1 | 0 | 4 | 4 |
| 4 | 4 | 4 | 4 | 4 | 0 | 0 |
| 5 | 5 | 4 | 5 | 4 | 1 | 0 |

We can prove that $(X,*,0)$ is a BCI-algebra and A = {0, 1, 2, 3} is a medial-ideal of X.

**Definition 2.11.** [13] A triangular norm (t-norm) is a function T: [0,1]x[0,1] $\rightarrow$ [0,1] that satisfies following conditions:

(T1) boundary condition :T(x, 1) = x,
(T2) commutativity condition: T(x, y) = T(y, x),
(T3) associativity condition :T(x, T(y, z)) = T(T(x, y), z),
(T4) monotonicity:T(x, y) ≤ T(x, z) ,whenever y ≤ z for all x, y, z $\in$ [0, 1].

A simple example of such defined t-norm is a function T (α, β) = min{α, β}.
In the general case T (α, β) ≤ min{α, β} and T (α, 0) = 0 for all α, β $\in$ [0, 1].

**Definition 2.12.** [11 ] Let X be a BCI-algebra. A fuzzy subset μ in X is called a fuzzy subalgebra of X with respect to a t-norm T (briefly, a T-fuzzy subalgbra of X) if $\mu(x) \geq T\{\mu(x*y), \mu(y)\}$, for all $x, y \in X$ .

**Definition 2.13.** [13] A triangular conorm (t-conorm S) is a mapping S: [0,1] x [0,1] $\rightarrow$ [0,1] ,that satisfies following conditions:

(S1) S(x, 0) = x,
(S2) S(x, y) = S(y, x),
(S3) S(x, S(y, z)) = S(S(x, y), z),
(S4) S(x, y) ≤ S(x, z) , whenever y ≤ z for all x, y, z $\in$ [0, 1].

A simple example of such definition s-norm S is a function S(x, y) = max{x, y}.
Every S- conorm S has a useful property: max{α, β}≤ S (α, β) for all α, β $\in$ [0, 1].

## 3. T, S -Fuzzy medial ideals

**Definition 3.1.** [11 ] Let *X* be a BCI-algebra.
(1) A fuzzy set $\mu$ in X is called T- fuzzy BCI- ideal of X if it satisfies:

(TI1) $\mu(0) \geq \mu(x)$,

(TI2) $\mu(x) \geq T\{\mu(x*y), \mu(y)\}$, for all $x, y$ and $z \in X$ .

(2) A fuzzy set $\lambda$ in X is called S- fuzzy BCI- ideal of X if it satisfies:

(SI1) $\lambda(0) \leq \lambda(x)$,

(SI2) $\lambda(x) \leq S\{\lambda(x*y), \lambda(y)\}$, for all $x, y$ and $z \in X$ .

**Definition 3.2.** Let X be a BCI-algebra.

(1) A fuzzy set $\mu$ in X is called T- fuzzy medial -ideal of X if it satisfies:

(FM1) $\mu(0) \geq \mu(x)$,

(FM2) $\mu(x) \geq T\{\mu(z*(y*x)), \mu(y*z)\}$, for all $x, y$ and $z \in X$ .

(2) A fuzzy set $\lambda$ in X is called S- fuzzy medial -ideal of X if it satisfies:

(FS1) $\lambda(0) \leq \lambda(x)$,

(FS2) $\lambda(x) \leq S\{\lambda(z*(y*x)), \lambda(y*z)\}$, for all $x, y$ and $z \in X$ .

**Example 3.3.** Let X = {0, 1, 2, 3, 4, 5} be a set with a binary operation * as Example 2.10. Let $T_m : [0,1] \times [0,1] \rightarrow [0,1]$ be a function defined by Tm (α, β) = max {α + β − 1, 0} for all α, β ∈ [0, 1] is a t-norm (cf. [10]). By routine calculations, we known that a fuzzy set μ in X defined by μ(1) = 0.3 and μ(0) = μ(2) = μ(3) = μ(4) = μ(5) =0.9 is a Tm-fuzzy BCI-ideal of X, which is a Tm-fuzzy medial-ideal because $\mu(x) \geq T\{\mu(z*(y*x)), \mu(y*z)\}$ and $S_m : [0,1] \times [0,1] \rightarrow [0,1]$ be a function defined by $S_m(\alpha, \beta) = \min\{1-(\alpha+\beta), 1\}$. Then By routine calculations, we known that a fuzzy set $\lambda$ in X defined by $\lambda$ (5) = 0.8 and $\lambda$ (0) = $\lambda$ (1) = $\lambda$ (2) = $\lambda$ (3) = $\lambda$ (4) = 0.3 is a $S_m$-fuzzy medial-ideal because $\lambda(x) \leq S_m\{\lambda(x*y), \lambda(y)\}$, for all $x, y$ and $z \in X$ .

**Lemma 3.4.** Any T-fuzzy medial- ideal of a BCI-algebra is T- fuzzy BCI- ideal of *X*.
**Proof:** Straightforward.

**Lemma 3.5.** Any S-fuzzy medial- ideal of a BCI-algebra is S- fuzzy BCI- ideal of *X*.
**Proof:** Straightforward.

**Example 3.6.** Let $X = \{0,1,2,3\}$ be a set with a binary operation $*$ define by the following table:

| $*$ | 0 | 1 | 2 | 3 |
|---|---|---|---|---|
| 0 | 0 | 1 | 2 | 3 |
| 1 | 1 | 0 | 3 | 2 |
| 2 | 2 | 3 | 0 | 1 |
| 3 | 3 | 2 | 1 | 0 |

We can prove that $(X,*,0)$ is a BCI-algebra. Define $\mu(x)$ and $\lambda(x)$ as follows:

$$\mu(x) = \begin{cases} 0.9 & if\ x = \{0,1\} \\ 0.6 & otherwise \end{cases} \quad \text{and} \quad \lambda(x) = \begin{cases} 0.2 & if\ x = \{0,1\} \\ 0.6 & if\ x = 2 \\ 0.7 & if x = 3 \end{cases}$$

Let $T_m : [0,1]\times[0,1] \to [0,1]$ be a function defined by $T_m(\alpha, \beta) = \max\{\alpha + \beta - 1, 0\}$ and $S_m : [0,1]\times[0,1] \to [0,1]$ be a function defined by $S_m(\alpha,\beta) = \min\{1-(\alpha+\beta),1\}$. For all, $\alpha,\beta \in [0,1]$ .It is easy to check that $\mu$ is $T_m$- fuzzy medial ideal and $\lambda$ is $S_m$-fuzzy medial -ideal of X.

**4. Intuitionistic (T,S)- fuzzy medial ideals**

An Intuitionistic fuzzy set (briefly IFS) A in a nonempty set X is an object having the form $A = \{(x, \mu_A(x), \lambda_A(x)) \mid x \in X\}$, where the function $\mu_A : X \to [0,1]$ and $\lambda_A : X \to [0,1]$ denote the degree of membership and degree of non membership, respectively and $0 \le \mu_A(x) + \lambda_A(x) \le 1$, for all $x \in X$ . An intuitionistic fuzzy set $A = \{(x, \mu_A(x), \lambda_A(x)) \mid x \in X\}$, in X can be identified to an order pair $(\mu_A, \lambda_A)$ in $I^X \times I^X$ . We shall use the symbol $A = (\mu_A, \lambda_A)$ for IFS $A = \{(x, \mu_A(x), \lambda_A(x)) \mid x \in X\}$.

**Definition 4.1.** **[**11] An IFS $A = (\mu_A, \lambda_A)$ in a BCI-algebra X is called intuitionistic (T,S)- fuzzy subalgebra of X if it satisfies the following :

(IFMS1) $\mu_A(x * y) \ge T\{\mu_A(x), \mu_A(y)\}$,

(IFMS2) $\lambda_A(x * y) \le S\{\lambda_A(x), \lambda_A(y)\}$,for all $x, y \in X$ .

**Example 4.2.** Let $X = \{0,1,2,3,4,5\}$ as in example 2.10, and $A = (\mu_A, \lambda_A)$ be an I F S in *X* defined by $\mu_A(1) = \mu_A(2) = \mu_A(3) = \mu_A(4) = \mu_A(5) = 0.3 < 0.7 = \mu_A(0)$ and $\lambda_A(1) = \lambda_A(2) = \lambda_A(3) = \lambda_A(4) = \lambda_A(5) = 0.5 > 0.2 = \lambda_A(0)$ . Let $T : [0,1]\times[0,1] \to [0,1]$ be a function defined by $T_m(\alpha,\beta) = \max\{\alpha+\beta-1,0\}$, and $S : [0,1]\times[0,1] \to [0,1]$ be a function defined by $S(\alpha,\beta) = \min\{1-(\alpha+\beta),1\}$.By routine calculations , we can prove that $A = (\mu_A, \lambda_A)$ is a intuitionistic (T,S)- fuzzy subalgebra of *X*.

**Lemma 4.3.** Every intuitionistic (T,S)- fuzzy subalgebra $A = (\mu_A, \lambda_A)$ of *X* satisfies the inequalities $\mu_A(0) \ge \mu_A(x)$ , and $\lambda_A(0) \le \lambda_A(x)$ for all $x \in X$ .

**Proof:** Straightforward.

**Definition 4.4.** [4] An IFS $A = (\mu_A, \lambda_A)$ in X is called intuitionistic (T,S)- fuzzy BCI-ideal of X if it satisfies the following inequalities:

(IFI1) $\mu_A(0) \ge \mu_A(x)$ and $\lambda_A(0) \le \lambda_A(x)$

(IFI2) $\mu_A(x) \ge T\{\mu_A(x * y), \mu_A(y)\}$, (IFI3) $\lambda_A(x) \le S\{\lambda_A(x*y), \lambda_A(y)\}$, for all $x, y \in X$.

**Definition 4.5.** An IFS $A = (\mu_A, \lambda_A)$ in $X$ is called intuitionistic (T,S)- fuzzy medial ideal of $X$ if it satisfies the following inequalities.

(IFM$_1$) $\mu_A(0) \ge \mu_A(x)$ and $\lambda_A(0) \le \lambda_A(x)$

(IFM$_2$) $\mu_A(x) \ge T\{\mu_A(z*(y*x)), \mu_A(y*z)\}$,

(IFM$_3$) $\lambda_A(x) \le S\{\lambda_A(z*(y*x)), \lambda_A(y*z)\}$, for all $x, y, z \in X$.

We now illustrate the above definitions by using some examples.

**Example 4.6.** Let $X = \{0,1,2,3\}$ be a set with a binary operation $*$ define by the following table:

| $*$ | 0 | 1 | 2 | 3 |
|---|---|---|---|---|
| 0 | 0 | 1 | 2 | 3 |
| 1 | 1 | 0 | 3 | 2 |
| 2 | 2 | 3 | 0 | 1 |
| 3 | 3 | 2 | 1 | 0 |

Define

| X | 0 | 1 | 2 | 3 |
|---|---|---|---|---|
| $\mu_A$ | 0.9 | 0.6 | 0.2 | 0.2 |
| $\lambda_B$ | 0.1 | 0.4 | 0.5 | 0.8 |

Let $T : [0,1] \times [0,1] \to [0,1]$ be a function defined by $T(\alpha, \beta) = \max\{\alpha + \beta - 1, 0\}$, and $S :\ [0,1] \times [0,1] \to [0,1]$ be a function defined by $S\ (\alpha, \beta) = \min\{1 - (\alpha + \beta), 1\}$. By routine calculations, we can prove that, $A = (\mu_A, \lambda_A)$ is intuitionistic (T,S)- fuzzy medial ideal (sub-algebra) of X.

**Lemma 4.7.** Let $A = (\mu_A, \lambda_A)$ be intuitionistic (T,S)- fuzzy medial ideal of *X*. If $x \le y$ in *X*, then $\mu_A(x) \ge \mu_A(y)$, $\lambda_A(x) \le \lambda_A(y)$, for all $x, y \in X$ .

**Proof:** Let $x, y \in X$ be such that $x \le y$, then $x * y = 0$. From (IFM$_2$), lemma2.5), we have

$\mu_A(x) \ge T\{\mu_A(0*(y*x)), \mu_A(y*0)\} = T\{\mu_A((x*y), \mu_A(y)\}$

$= T\{\mu_A(0), \mu_A(y)\} = \mu_A(y)$ .

Similarly, form (IFM$_3$), we have $\lambda_A(x) \le S\{\lambda_A(0*(y*x)), \lambda_A(y*0)\}$, hence,

$\lambda_A(x) \le S\{\lambda_A(x*y), \lambda_A(y)\} = S\{\lambda_A(0), \lambda_A(y)\} = \lambda_A(y)$ .

**Lemma 4.8.** Let $A=(\mu_A,\lambda_A)$ be intuitionistic (T,S)- fuzzy medial ideal of *X*, if the inequality $x*y\le z$ hold in *X*, then $\mu_A(x)\ge T\{\mu_A(y),\mu_A(z)\}$, $\lambda_A(x)\le S\{\lambda_A(y),\lambda_A(z)\}$, for all $x,y,z\in X$ .

**Proof:** Let $x,y,z\in X$ be such that $x*y\le z$ . Thus, put $z=0$ in (IFM2), (using Lemma 2.5 and Lemma 4.7), we get,

$$\mu_A(x)\ge T\{\mu_A(0*(y*x),\mu_A(y*0)\}=T\{\mu_A(x*y),\mu_A(y)\}\ge \overbrace{\max\{\mu_A(z),\mu_A(y)\}}^{\sin ce\quad \mu_A(x*y)\le\mu_A(z)}.$$

Similarly we can prove that, $\lambda_A(x)\le S\{\lambda_A(z)),\lambda_A(y)\}$ .

**Theorem 4.9.** Every intuitionistic (T,S)- fuzzy medial ideal of X is intuitionistic (T,S)- fuzzy subalgebra of X.

**Proof:** Let $A=(\mu_A,\lambda_A)$ be intuitionistic (T,S)- fuzzy medial ideal of *X*. Since $x*y\le x$, for all $x,y\in X$ , then $\mu_A(x*y)\ge\mu_A(x)$, $\lambda_A(x*y)\le\lambda_A(x)$ .Put z = 0 in (IFM2), (IFM3), we have $\mu_A(x*y)\ge\mu_A(x)\ge T\{\mu_A(0*(y*x)),\mu_A(y*0)\}=T\{\mu_A((x*y)),\mu_A(y)\}$ $\ge T\{\mu_A(x),\mu_A(y)\}$ Now $\lambda_A(x*y)\le\lambda_A(x)\le S\{\lambda_A(0*(y*x)),\lambda_A(y*0)\}$ = $S\{\lambda_A(x*y),\lambda_A(y)\}\le S\{\lambda_A(x),\lambda_A(y\}$ . Then $A=(\mu_A,\lambda_A)$ is intuitionistic (T,S)- fuzzy subalgebra of *X*.

The converse of theorem 4.10 may not be true. For example, the intuitionistic (T,S)- fuzzy subalgebra $A=(\mu_A,\lambda_A)$ in example 4.2 is not intuitionistic (T,S)- fuzzy medial ideal of *X* since $\mu_A(1)=0.2<0.5=T\{\mu_A(4*(4*1)),\mu_A(4*4)\}$ .

**Lemma 4.10.** Let $A=(\mu_A,\lambda_A)$ be intuitionistic (T,S)-fuzzy medial ideal (subalgebra) of X, such that $\mu_A(x)\ge T\{\mu_A(y),\mu_A(z)\}$, $\lambda_A(x)\le S\{\lambda_A(y),\lambda_A(z)\}$, and the inequality $x*y\le z$ are satisfied for all $x,y,z\in X$ . Then $A=(\mu_A,\lambda_A)$ is intuitionistic (T,S)- fuzzy medial ideal (subalgebra) of X.

**Proof:** Let $A=(\mu_A,\lambda_A)$ be intuitionistic (T,S)- fuzzy ideal (subalgebra) of X. Recall that $\mu_A(0)\ge\mu_A(x)$ and $\lambda_A(0)\le\lambda_A(x)$, for all $x\in X$ . Since, $x*(z*(y*x))=(y*x)*(z*x)\le y*z$ , it follows from the hypothesis that $\mu_A(x)\ge T\{\mu_A(z*(y*x)),\mu_A(y*z)\}$ and $\lambda_A(x)\le S\{\lambda_A(z*(y*x)),\lambda_A(y*z)\}$ . Hence $A=(\mu_A,\lambda_A)$ is intuitionistic (T,S)- fuzzy medial ideal of X.

**Theorem 4.11.** If $A=(\mu_A,\lambda_A)$ is an (T,S)- intuitionistic fuzzy medial ideal of X, then for any $x,a_1,a_2,.....a_n\in X$ ,$(...((x*a_1)*a_2)*...a_n=0$ , implies

$$\mu_A(x)\ge T\{\mu_A(a_1),\mu_A(a_2),\mu_A(a_3)......\mu_A(a_n)\},$$

$$\lambda_A(x)\le S\{\lambda_A(a_1),\lambda_A(a_2),.......\lambda_A(a_n)\}$$

**Proof:** Using induction on n and Definition , Lemmas 4.11 and 4.7, the proof is straightforward.

**Lemma 4.12.** An $A=(\mu_A,\lambda_A)$ is an intuitionistic (T,S)- fuzzy medial ideal of X if and only if the fuzzy sets $\mu_A$, $\tilde{\lambda}_A=1-\lambda_A$ are T- fuzzy medial ideals of X.

**Proof:** Let $A=(\mu_A,\lambda_A)$ is an intuitionistic (T,S)- fuzzy medial ideal of X. Clearly, $\mu_A$, is a T-fuzzy medial ideal of X. For every $x,y\in X$ , we have $\tilde{\lambda}_A(0)=1-\lambda_A(0)\geq 1-\lambda_A(x)=\tilde{\lambda}_A(x)$ and $\tilde{\lambda}_A(x)=1-\lambda(x)\geq 1-S\{\lambda_A(z*(y*x)),\lambda_A(y*z)\}=T\{ 1-\lambda_A(z*(y*x)),1-\lambda_A(y*z)\}=T\{\tilde{\lambda}_A(z*(y*x)),\tilde{\lambda}_A(y*z)\}$ .Hence $\tilde{\lambda}_A$ is a T-fuzzy medial ideal of X.

Conversely, assume that $\mu_A$ and $\tilde{\lambda}_A=1-\lambda_A$ are T-fuzzy medial ideals of X. For every $x,y\in X$ , we get $\mu_A(0)\geq\mu_A(x)$ , $1-\lambda_A(0)\geq\tilde{\lambda}_A(x)=1-\lambda_A(x)$, that is, $\lambda_A(0)\leq\lambda_A(x)$; $\mu_A(x)\geq T\{\mu_A(z*(y*x)),\mu_A(y*z)\}$, and $1-\lambda_A(x)=\tilde{\lambda}_A(x)\geq T\{\tilde{\lambda}_A(z*(y*x)),\tilde{\lambda}_A(y*z)\}=\mathrm{T}\{1\text{-}\lambda_A(z*(y*x)),1-\lambda_A(y*z)\}=$
1- $S\{\lambda_A(z*(y*x)),\lambda_A(y*z)\}$ that is, $\lambda_A(x)\leq S\{\lambda_A(z*(y*x)),\lambda_A(y*z)\}$ .Hence $A=(\mu_A,\lambda_A)$ is an intuitionistic (T,S)- fuzzy medial ideal of X.

**Theorem 4.13.** Let $A=(\mu_A,\lambda_A)$ be an IFS in X. Then $A=(\mu_A,\lambda_A)$ is an intuitionistic (T, S)-fuzzy medial ideal of X if and only if ΩA = ($\mu_A$, $\tilde{\mu}_A$) and ΣA = $(\lambda_A,\tilde{\lambda}_A)$ are (T,S)-fuzzy medial ideal of X.

**Proof:** If $A=(\mu_A,\lambda_A)$ is an intuitionistic (T,S)- fuzzy medial ideal of X, then $\mu_A$, $\tilde{\lambda}_A=1-\lambda_A$ are T- fuzzy medial ideals of X from Lemma 3.13, hence ΩA = ($\mu_A$, $\tilde{\mu}_A$) and ΣA = $(\lambda_A,\tilde{\lambda}_A)$ are intuitionistic (T,S)-fuzzy medial ideals of X.

Conversely, if _ΩA = ($\mu_A$, $\tilde{\mu}_A$) and ΣA = $(\lambda_A,\tilde{\lambda}_A)$ are intuitionistic (T,S)-fuzzy medial ideals of X, then the fuzzy sets $\mu_A$, $\tilde{\lambda}_A=1-\lambda_A$ are T- fuzzy medial ideals of X, hence $A=(\mu_A,\lambda_A)$ is an intuitionistic (T,S)- fuzzy medial ideals of X.

**Definition 4.14.** Let $A=(\mu_A,\lambda_A)$ be an intuitionistic fuzzy set of X, we define the following: For any $t\in[0,1]$ and nonempty fuzzy sets $\mu$, $\lambda$ in X, the set $L(\mu_A,t):=\{x\in X\mid\mu(x)\geq t\}$ is called t-level cut of $\mu_A$, and the set $U(\lambda_A,s):=\{x\in X\mid\lambda(x)\leq s\}$ is called s-level cut of $\lambda_A$.

**Theorem 4.15.** An IFS $A=(\mu_A,\lambda_A)$ is intuitionistic (T,S)- fuzzy medial ideal of *X* if and only if for all $s,t\in[0,1]$, the set $L(\mu_A,t)$ and $U(\lambda_A,s)$ are either empty or medial ideals of *X*.

**Proof:** Let $A=(\mu_A,\lambda_A)$ be intuitionistic (T,S)- fuzzy medial ideal of X and $L(\mu_A,t)$ $\neq\phi\neq U(\lambda_A,s)$. Since $\mu_A(0)\geq t$ and $\lambda_A(0)\leq s$, let $x,y,z\in X$ be such that $z*(y*x)\in L(\mu_A,t)$ and $y*z\in L(\mu_A,t)$, then $\mu_A(z*(y*x))\geq t$ and $\mu_A(y*z)\geq t$, it follows that $\mu_A(x)\geq T\{\mu_A(x*(y*z)),\mu_A(y*z)\}\geq t$, we get $x\in L(\mu_A,t)$. Hence $L(\mu_A,t)$ is a medial ideal of X. Now let $x,y,z\in X$ be such that $z*(y*x)\in U(\lambda_A,s)$ and $y*z\in U(\lambda_A,s)$, then $\lambda_A(z*(y*x))\leq s$ and $\lambda_A(y*z)\leq s$ which imply that $\lambda_A(x)\leq S\{\lambda_A(z*(y*x)),\lambda_A(y*z)\}\leq s$. Thus $x\in U(\lambda_A,s)$ and therefore $U(\lambda_A,s)$ is a medial ideal of X.

Conversely, assume that for each $s,t\in[0,1]$, the sets $L(\mu_A,t)$ and $U(\lambda_A,s)$ are either empty or medial ideal of X. For any $x\in X$, let $\mu_A(x)=t$ and $\lambda_A(x)=s$. Then $x\in L(\mu_A,t)\bigcap U(\lambda_A,s)$ and so $L(\mu_A,t)\neq\phi\neq U(\lambda_A,s)$. Since $L(\mu_A,t)$ and $U(\lambda_A,s)$ are medial ideals of X, therefore $0\in L(\mu_A,t)\bigcap U(\lambda_A,s)$. Hence, $\mu_A(0)\geq t=\mu_A(x)$ and $\lambda_A(0)\leq s=\lambda_A(x)$ for all $x\in X$. If there exist $x',y',z'\in X$ be such that $\mu_A(x')<T\{\mu_A(z'*(y'*x')),\mu_A(y'*z')\}$. Then by taking $t_0=\frac{1}{2}\{\mu_A(x')+T\{\mu_A(z'*(y'*x'),\mu_A(y'*z')\}\}$, we get $\mu_A(x')<t_0<T\{\mu_A(z'*(y'*x')),\mu_A(y'*z')\}$ and hence $x'\notin L(\mu_A,t_0)$, $z'*(y'*x')\in L(\mu_A,t_0)$ and $y'*z'\in L(\mu_A,t_0)$, i.e. $L(\mu_A,t_0)$ is not a medial ideal of X, which make a contradiction. Finally assume that there exist $a,b,c\in X$ such that $\lambda_A(a)>S\{\lambda_A(c*(b*a)),\lambda_A(b*c)\}$. Then by taking $s_0:=\frac{1}{2}\{\lambda_A(a)+S\{\lambda_A(c*(b*a),\lambda_A(b*c)\}\}$, we get $S\{\lambda_A(c*(b*a)),\lambda_A(b*c)\}<s_0<\lambda_A(a)$. Therefore, $(c*(b*a))\in U(\lambda_A,s_0)$ and $b*c\in U(\lambda_A,s_0)$, but $a\notin U(\lambda_A,s_0)$, which make a contradiction. This completes the proof.

## 5. The image (preimage) of intuitionistic (T,S)- fuzzy medial ideals

Let $(X,*,0)$ and $(Y,*',0')$ be BCI-algebras. A mapping $f:X\to Y$ is said to be a homomorphism if $f(x*y)=f(x)*'f(y)$ for all $x,y\in X$. Note that if $f:X\to Y$ is a homomorphism of BCI-algebras, then $f(0)=0'$. Let $f:X\to Y$ be a homomorphism of BCI-algebras. For any IFS $A=(\mu_A,\lambda_A)$ in *Y*, define an IFS $A^f=(\mu_A^f,\lambda_A^f)$ in *X* by $\mu_A^f(x):=\mu_A(f(x))$, and $\lambda_A^f(x):=\lambda_A(f(x))$ for all $x\in X$.

**Theorem 5.1.** Let $f:X\to Y$ be a homomorphism of BCI-algebras and let $A=(\mu_A,\lambda_A)$ be an IFS in Y. If $A$ is intuitionistic (T,S)-fuzzy medial, then $A^f=(\mu_A^f,\lambda_A^f)$ is intuitionistic (T,S)- fuzzy medial ideal of X.

**Proof:** For all $x, y, z \in X$ , we have $\mu_A^f(x) := \mu_A(f(x)) \le \mu_A(0) = \mu_A(f(0))$ = $\mu_A^f(0)$ , and $\lambda_A^f(x) := \lambda_A(f(x)) \ge \lambda_A(0)$ $= \lambda_A(f(0)) = \lambda_A^f(0)$ . Now $\mu_A^f(x) :=$ $\mu_A(f(x)) \ge T\{\mu_A(f(z) * (f(y) * f(x))), \mu_A(f(y) * f(z))\} = T\{\mu_A(f(z) * f(y * x$ $)), \mu_A(f(y * z))\} = T\{\mu_A(f(z * (y * x)), \mu_A(f(y * z))\} = T\{\mu_A^f(z * (y * x)),\ \mu_A^f(y$ $* z)\}$. Similarly, $\lambda_A^f(x) \le S\{\lambda_A^f(z * (y * x)), \lambda_A^f(y * z)\}$. Hence $A^f = (\mu_A^f, \lambda_A^f)$ is an intuitionistic (T,S)-fuzzy medial ideal in X.

**Theorem 5.2.** Let $f : X \to Y$ be an epimorphism of BCI-algebras and let $A = (\mu_A, \lambda_A)$ be an IFS in Y. If $A^f = (\mu_A^f, \lambda_A^f)$ is intuitionistic (T,S)- fuzzy medial ideal of X, then $A = (\mu_A, \lambda_A)$ is intuitionistic (T,S)- fuzzy medial ideal in Y.

**Proof:** For any $a \in Y$ , there exists $x \in X$ such that $f(x) = a$ .Then

$$\mu_A(a) = \mu_A(f(x)) = \mu_A^f(x) \le \mu_A^f(0) = \mu_A(f(0)) = \mu_A(0),$$
$$\lambda_A(a) = \lambda_A(f(x)) = \lambda_A^f(x) \ge \lambda_A^f(0) = \lambda_A(f(0)) = \lambda_A(0).$$

Let $a, b, c \in Y$ , there exists $x, y, z \in X$ such that $f(x) = a, f(y) = b, f(z) = c$ . It follows that $\mu_A(a) = \mu_A(f(x)) = \mu_A^f(x) \ge T\{\mu_A^f(z * (y * x)), \mu_A^f(y * z)\}$ = $T\{\mu_A$ $= (f(z * (y * x)), \mu_A(f(y * z))\}$ = $T\{\mu_A(f(z) * f(y * x)), \mu_A(f(y) * f(z))\}$ $= T\{\mu_A$ $(f(z) * (f(y) * f(x))), \mu_A(f(y) * f(z))\} = T\{\mu_A(c * (b * a)), \mu_A(b * c)\}$.
Similarly, $\lambda_A(a) \le S\{\lambda_A(c * (b * a)), \lambda_A(b * c)\}$. This completes the proof.

**6. Cartesian product of intuitionistic (T,S)- fuzzy medial ideals**

Let $\mu$ and $\lambda$ be two fuzzy sets in the set *X*. the product $\lambda \times \mu : X \times X \to [0,1]$ is defined by $(\lambda \times \mu)(x, y) = T\{\lambda(x), \mu(y)\}$, for all $x, y \in X$ . Let $A = (X, \mu_A, \lambda_A)$ and $B = (X, \mu_B, \lambda_B)$ be two IFS of *X*, the Cartesian product $A \times B = (X \times X,$ $\mu_A \times \mu_B, \lambda_A \times \lambda_B)$ is defined by $(\mu_A \times \mu_B)(x, y) = T\{\mu_A(x), \mu_B(y)\}$ and $(\lambda_A \times \lambda_B)$ $(x, y) = S\{\lambda_A(x), \lambda_B(y)\}$ , where $\mu_A \times \mu_B : X \times X \to [0,1]$ and $\mu_A \times \mu_B : X \times X \to$ $[0,1]$ for all $x, y \in X$ .

**Remark 6.1.** Let *X* and *Y* be BCI-algebras, we define* on $X \times Y$ by,for every $(x, y), (u, v) \in X \times Y$ , $(x, y) * (u, v) = (x * u, y * v)$ . Clearly $(X \times Y; *, (0,0))$ is BCI-algebra.

**Proposition 6.2.** Let $A = (X, \lambda_A, \mu_A)$, $B = (X, \lambda_B, \mu_B)$ be intuitionistic (T,S)- fuzzy medial ideals of *X*, then $A \times B$ is intuitionistic (T,S)- fuzzy medial ideal of $X \times X$ .

**Proof:** For all $x, y \in X$ , $(\mu_A \times \mu_B)(0,0) = T\{\mu_A(0), \mu_B(0)\} \ge$ $T\{\mu_A(x), \mu_B(y)\} =$ $(\mu_A \times \mu_B)(x, y)$ and $(\lambda_A \times \lambda_B)(0,0) = S\{\lambda_A(0), \lambda_B(0)\} \le S\{\lambda_A(x), \lambda_B(y)\}$ $= (\lambda_A \times$ $\lambda_B)(x, y)$. Now, let $(x_1, x_2), (y_1, y_2), (z_1, z_2) \in X \times X$ , then

$T\{(\mu_A \times \mu_B)((z_1, z_2) * ((y_1, y_2) * (x_1, x_2))), (\mu_A \times \mu_B)((y_1, y_2) * (z_1, z_2))\}$
$= T\{(\mu_A \times \mu_B)((z_1, z_2) * (y_1 * x_1, y_2 * x_2)), (\mu_A \times \mu_B)(y_1 * z_1, y_2 * z_2)\}$
$= T\{(\mu_A \times \mu_B)(z_1 * (y_1 * x_1), z_2 * (y_2 * x_2)), (\mu_A \times \mu_B)(y_1 * z_1, y_2 * z_2)\}$
$= T\{T\{\mu_A(z_1 * (y_1 * x_1)), \mu_B(z_2 * (y_2 * x_2))\}, T\{\mu_A(y_1 * z_1), \mu_B(y_2 * z_2)\}\}$
$= T\{T\{\mu_A(z_1 * (y_1 * x_1)), \mu_A(y_1 * z_1)\}, T\{\mu_B(z_2 * (y_2 * x_2)), \mu_B(y_2 * z_2)\}$
$\leq T\{\mu_A(x_1), \mu_B(x_2) = (\mu_A \times \mu_B)(x_1, x_2)$.
Similarly we can prove that, $S\{(\lambda_A \times \lambda_B)((z_1, z_2) * ((y_1, y_2) * (x_1, x_2))), (\lambda_A \times \lambda_B)$ $((y_1, y_2) * (z_1, z_2))\} \geq (\lambda_A \times \lambda_B)(x_1, x_2)$.

**Example 6.3.** Let $X = \{0, 1, 2, 3\}$ be a set with a binary operation $*$ as example 4.6 Then $X \times X = \begin{Bmatrix} (0,0), (0,1), (0,2), (0,3), (1,0), (1,1), (1,2), (1,3), \\ (2,0), (2,1), (2,2), (2,3), (3,0), (3,1), (3,2), (3,3) \end{Bmatrix}$, we define* on $X \times X$ by for every $(x, y), (u, v) \in X \times X$, $(x, y) * (u, v) = (x * u, y * v)$. By routine calculations $(X \times X; *, (0,0))$ is BCI-algebra.

Let $T_m : [0,1] \times [0,1] \to [0,1]$ be a functions defined by $T_m(\alpha, \beta) = \max\{\alpha + \beta - 1, 0\}$ and $S_m : [0,1] \times [0,1] \to [0,1]$ by $S_m(\alpha, \beta) = \min\{1 - (\alpha + \beta), 1\}$ for all $\alpha, \beta \in [0, 1]$. Let $A \times B = (X \times X, \mu_A \times \mu_B, \lambda_A \times \lambda_{\mathbf{B}})$ intuitionistic fuzzy medial ideals of $X$ *define* by $\mu_A(x, y) = \begin{cases} 0.9 \, if & x \ or \ y \in \{0,1\} \\ 0.2 \, if & x \ or \ y \in \{2,3\} \end{cases}$ , $\lambda_A(x, y) = \begin{cases} 0.6 \ \ if & x \ or \ y = 3 \\ 0.2 \ \ if & x \ or \ y \in \{2,3\} \end{cases}$

By routine calculations, we can prove that $A \times B = (X \times X, \mu_A \times \mu_B, \lambda_A \times \lambda_{\mathbf{B}})$, is intuitionistic (T,S)- fuzzy medial ideals of $X \times X$.

**Definition 6.4.** Let $A = (X, \lambda_A, \mu_A)$ and $B = (X, \lambda_B, \mu_B)$ be IFS of a BCI-algebra *X*. for $s, t \in [0,1]$ the set $U(\mu_A \times \mu_B, s) := \{(x, y) \in X \times X \mid (\mu_A \times \mu_B)(x, y) \geq s\}$ is called upper s-level of $(\mu_A \times \mu_B)(x, y)$ and the set $L(\lambda_A \times \lambda_B, t) = \{(x, y) \in X \times X \mid (\lambda_A \times \lambda_B)(x, y) \leq t\}$ is called lower t-level of $(\lambda_A \times \lambda_B)(x, y)$.

**Theorem 6.5.** The intuitionistic fuzzy sets $A = (X, \lambda_A, \mu_A)$ and $B = (X, \lambda_B, \mu_B)$ are intuitionistic (T,S)- fuzzy medial ideals of *X* if the non-empty set upper *s*-level cut $U(\mu_A \times \mu_B, s)$ and the non-empty lower *t*-level cut $L(\lambda_A \times \lambda_B, t)$ are medial ideals of $X \times X$ for all $s, t \in [0,1]$.

**Proof:** Let $A = (X, \lambda_A, \mu_A)$ and $B = (X, \lambda_B, \mu_B)$ be intuitionistic (T,S)- fuzzy medial ideals of *X*, therefore for any $(x, y) \in X \times X$, we have $(\mu_A \times \mu_B)$ (0,0) $= T\{\mu_A(0), \mu_B(0)\} \geq T\{\mu_A(x), \mu_B(y)\} = (\mu_A \times \mu_B)(x, y)$.

Let $(x_1, x_2), (y_1, y_2), (z_1, z_2) \in X \times X$ and $s \in [0,1]$, be such that $((z_1, z_2) *$

$((y_1, y_2) * (x_1, x_2))) \in U(\mu_A \times \mu_B, s)$ and $(y_1, y_2) * (z_1, z_2) \in U(\mu_A \times \mu_B, s)$ . Now $(\mu_A \times \mu_B)(x_1, x_2) \geq T\{(\mu_A \times \mu_B)((z_1, z_2) * ((y_1, y_2) * (x_1, x_2))), (\mu_A \times \mu_B)((y_1, y_2) * (z_1, z_2))\} = T\{(\mu_A \times \mu_B)((z_1, z_2) * (y_1 * x_1, y_2 * x_2)), (\mu_A \times \mu_B)(y_1 * z_1, y_2 * z_2)\}$ $= T\{(\mu_A \times \mu_B)(z_1 * (y_1 * x_1), z_2 * (y_2 * x_2)), (\mu_A \times \mu_B)(y_1 * z_1, y_2 * z_2)\} \geq T\{s, s\} = s$ , Therefore $(x_1, x_2) \in U((\mu_A \times \mu_B)(x, y), s)$ is a medial ideal of $X \times X$ . Similarly we can prove that $L((\lambda_A \times \lambda_B)(x, y), t)$ is a medial ideal of $X \times X$ .This completes the proof.

**7. Intuitionistic (T,S)- fuzzy magnified translation medial ideals**

Let $A = (\mu_A, \lambda_A)$ be intuitionistic (T,S)- fuzzy subset of a set $X$, $\alpha \in [0, 1 - \sup\{\mu(x), x \in X\}]$, $\beta \in (0,1]$. An object having the form $A_\beta^\alpha = ((\mu_A)_\beta^\alpha, (\lambda_A)_\beta^\alpha)$ is called magnified translation of A if $(\mu_A)_\beta^\alpha(x) = \beta\mu_A(x) + \alpha$ and $(\lambda_A)_\beta^\alpha(x) = \beta\lambda_A(x) + \alpha$ be such that $, \alpha \in [0, 1 - \sup\{\mu(x), \forall x \in X\}$ , $\beta \in (0, 1 - 2\alpha]$ .In particular if $\beta = 1$ , then $A_1^\alpha = ((\mu_A)_1^\alpha, (\lambda_A)_1^\alpha)$ is called intuitionistic (T,S)- fuzzy translation of A. If $\alpha = 0$ , then $A_\beta^0 = ((\mu_A)_\beta^0, (\lambda_A)_\beta^0)$ is called intuitionistic (T,S)- fuzzy multiplication of A.

**Example 7.1.** Consider the BCI-algebra $X = \{0,1,2,3\}$ in example 4.7. Define a fuzzy Subsets $\mu_A, \lambda_A$ of X by

| X | 0 | 1 | 2 | 3 |
|---|---|---|---|---|
| $\mu_A$ | 0.8 | 0.5 | 0.4 | 0.1 |
| $\lambda_B$ | 0.2 | 0.2 | 0.6 | 0.9 |

Since α ϵ [ 0, 1 - sup{μ(x), xϵ X}], β ϵ (0, 1-2 α] , then α ϵ [ 0, 1 - 0.8]= [0, 0.2]. If we take α =0.1 , therefore $\beta \in (0, 1 - 2\alpha] = (0, 0.2]$. Hence, we can take $\alpha = 0.1$ , $\beta = 0.2$ and therefore we get the following table :

| X | 0 | 1 | 2 | 3 |
|---|---|---|---|---|
| $\mu_A$ | 0.8 | 0.5 | 0.4 | 0.1 |
| $\lambda_B$ | 0.2 | 0.2 | 0.6 | 0.9 |
| $(\mu_A)_{0.2}^{0.1}(x)$ | 0.26 | 0.20 | 0.16 | 0.12 |
| $(\lambda_A)_{0.2}^{0.1}(x)$ | 0.14 | 0.14 | 0.22 | 0.28 |

Let $T : [0,1] \times [0,1] \to [0,1]$ be a function defined by $T(\alpha, \beta) = \max\{\alpha + \beta - 1, 0\}$, and $S : [0,1] \times [0,1] \to [0,1]$ be a function defined by $S(\alpha, \beta) = \min\{1 - (\alpha + \beta), 1\}$ .By routine calculations, it is easy to show that $A_{0.2}^{0.1} = ((\mu_A)_{0.2}^{0.1}, (\lambda_A)_{0.2}^{0.1})$, is an intuitionistic (T,S) -fuzzy magnified translation of medial ideal on X.

**Theorem 7.2.** The intuitionistic fuzzy magnified translation $A_\beta^\alpha = ((\mu_A)_\beta^\alpha, (\lambda_A)_\beta^\alpha)$ of $A = (\mu_A, \lambda_A)$ is intuitionistic (T,S)- fuzzy medial ideal of X if and only if $A$ is intuitionistic (T,S)- fuzzy medial ideal of X.

**Proof:** Let $A = (\mu_A, \lambda_A)$ be intuitionistic (T,S)- fuzzy medial ideal of X. Then $A$ is a non-empty intuitionistic fuzzy subset of X, and hence $A_\beta^\alpha$ is also non-empty. Now for $x \in X$ we have $(\mu_A)_\beta^\alpha(0) = \beta\mu_A(0) + \alpha \geq \beta\mu_A(x) + \alpha = (\mu_A)_\beta^\alpha(x)$,

$(\lambda_A)_\beta^\alpha(0) = \beta\lambda_A(0) + \alpha \leq \beta\lambda_A(x) + \alpha = (\lambda_A)_\beta^\alpha(x)$.

and $(\mu_A)_\beta^\alpha(x) = \beta\mu_A(x) + \alpha \geq \beta(T\{\mu_A(z*(y*x), \mu_A(y*z)\}) + \alpha =$
$T\{\beta\mu_A(z*(y*x) + \alpha, \beta\mu_A(y*z) + \alpha\} = T\{(\mu_A)_\beta^\alpha(z*(y*x)), (\mu_A)_\beta^\alpha(y*z)\}$

and $(\lambda_A)_\beta^\alpha(x) = \beta\lambda_A(x) + \alpha \leq \beta(S\{\lambda_A(z*(y*x), \lambda_A(y*z)\}) + \alpha =$
$S\{\beta\lambda_A(z*(y*x) + \alpha, \beta\lambda_A(y*z) + \alpha\} = S\{(\lambda_A)_\beta^\alpha(z*(y*x)), (\lambda_A)_\beta^\alpha(y*z)\}$.

Hence $A_\beta^\alpha$ is intuitionistic (T,S) - fuzzy magnified translation medial ideal of X.

Conversely, let $A_{\beta,}^\alpha = ((\mu_A)_\beta^\alpha, (\lambda_A)_\beta^\alpha)$ be intuitionistic (T,S) - fuzzy magnified translation medial ideal of X. Then $(\mu_A)_\beta^\alpha(0) \geq (\mu_A)_\beta^\alpha(x)$ . i.e $\beta\mu_A(0) + \alpha \geq \beta\mu_A(x) + \alpha$ , therefore $\mu_A(0) \geq \mu_A(x)$ and $(\lambda_A)_\beta^\alpha(0) = \beta\lambda_A(0) + \alpha \leq \beta\lambda_A(x) + \alpha = (\lambda_A)_\beta^\alpha(x)$, i.e $\beta\lambda_A(0) + \alpha \leq \beta\lambda_A(x) + \alpha$, therefore $\lambda_A(0) \leq \lambda_A(x)$.

Now for all $x, y, z \in X$ , we have

$$\begin{aligned}\beta\mu_A(x) + \alpha = (\mu_A)_\beta^\alpha(x) &\geq T\{(\mu_A)_\beta^\alpha(z*(y*x)), (\mu_A)_\beta^\alpha(y*z)\} \\ &= T\{\beta\mu_A(z*(y*x) + \alpha, \beta\mu_A(y*z) + \alpha\} \\ &= \beta(T\{\mu_A(z*(y*x), \mu_A(y*z)\}) + \alpha ,\end{aligned}$$

therefore $\mu_A(x) \geq T\{\mu_A(z*(y*x), \mu_A(y*z)\}$ and

$$\begin{aligned}\beta\lambda_A(x) + \alpha = (\lambda_A)_\beta^\alpha(x) &\leq S\{(\lambda_A)_\beta^\alpha(z*(y*x)), (\lambda_A)_\beta^\alpha(y*z)\} \\ &= S\{\beta\lambda_A(z*(y*x) + \alpha, \beta\lambda_A(y*z) + \alpha\} \\ &= \beta(S\{\lambda_A(z*(y*x), \lambda_A(y*z)\}) + \alpha\end{aligned}$$

i.e. $\lambda_A(x) \leq S\{\lambda_A(z*(y*x), \lambda_A(y*z)\}$ . Hence $A = (\mu_A, \lambda_A)$ is intuitionistic (T,S)- fuzzy medial ideal of X.

**Lemma 7.3.** If $A_\beta^\alpha = ((\mu_A)_\beta^\alpha, (\lambda_A)_\beta^\alpha)$ is intuitionistic (T,S) - fuzzy magnified translation medial ideal and $x \leq y$ in *X*, then $(\mu_A)_\beta^\alpha(x) \geq (\mu_A)_\beta^\alpha(y)$, $(\lambda_A)_\beta^\alpha(x) \leq (\lambda_A)_\beta^\alpha(y)$ . That is $(\mu_A)_\beta^\alpha$ is order reserving and $(\lambda_A)_\beta^\alpha$ is order preserving.

**Proof:** Let $x, y \in X$ be such that $x \leq y$, by lemma 4.8, we have $\mu_A(x) \geq \mu_A(y)$ , and

$$(\mu_A)_\beta^\alpha(x) = \beta\mu_A(x) + \alpha \geq \beta\mu_A(y) + \alpha = (\mu_A)_\beta^\alpha(y)$$

Similarly, $(\lambda_A)^\alpha_\beta(x) = \beta\lambda_A(x)+\alpha \le \beta\lambda_A(y)+\alpha = (\lambda_A)^\alpha_\beta(y)$.

**Lemma 7.4.** If $A^\alpha_\beta = ((\mu_A)^\alpha_\beta, (\lambda_A)^\alpha_\beta)$ is intuitionistic (T,S)- fuzzy magnified translation medial ideal and the inequality $x*y \le z$ hold in X, then $(\mu_A)^\alpha_\beta(x) \ge T\{(\mu_A)^\alpha_\beta(y), (\mu_A)^\alpha_\beta(z)\}$, $(\lambda_A)^\alpha_\beta(x) \le S\{(\lambda_A)^\alpha_\beta(y), (\lambda_A)^\alpha_\beta(z)\}$.

**Proof:** Let $x, y, z \in X$ be such that $x*y \le z$. Thus, by Lemma 4.9, we have

$$(\mu_A)^\alpha_\beta(x) = \beta\mu_A(x)+\alpha \ge \beta(T\{\mu_A(y), \mu_A(z)\})+\alpha$$
$$= T\{\beta\mu_A(y)+\alpha, \beta\mu_A(z)+\alpha\} = T\{(\mu_A)^\alpha_\beta(y), (\mu_A)^\alpha_\beta(z)\}.$$

Similarly, we can prove that, $(\lambda_A)^\alpha_\beta(x) \le S\{(\lambda_A)^\alpha_\beta(y), (\lambda_A)^\alpha_\beta(z)\}$.

**Definition 7.5.** Let $f : X \to Y$ be a homomorphism of BCI-algebras, for any (T,S)-intuitionistic fuzzy magnified translation $A^\alpha_\beta = ((\mu_A)^\alpha_\beta, (\lambda_A)^\alpha_\beta)$ of $A$ in Y. We define (T,S)- intuitionistic fuzzy magnified translation $(A^\alpha_\beta)^f = ((\mu^f_A)^\alpha_\beta, (\lambda^f_A)^\alpha_\beta)$ in $X$ by $(\mu^f_A)^\alpha_\beta(x) = (\mu_A)^\alpha_\beta(f(x))$ and $(\lambda^f_A)^\alpha_\beta(x) = (\lambda_A)^\alpha_\beta(f(x))$, for all $x \in X$.

**Theorem 7.6.** Let $f : X \to Y$ be a homomorphism of BCI-algebras. If $A^\alpha_\beta = ((\mu_A)^\alpha_\beta, (\lambda_A)^\alpha_\beta)$ is (T,S)- intuitionistic fuzzy magnified translation medial ideal, then $(A^\alpha_\beta)^f = ((\mu^f_A)^\alpha_\beta, (\lambda^f_A)^\alpha_\beta)$ is (T,S)- intuitionistic fuzzy magnified translation medial ideal of *X*.

**Proof:** For all *x, y, z*$\in X$, we have

$$(\mu^f_A)^\alpha_\beta(x) := (\mu_A)^\alpha_\beta(f(x)) \le (\mu_A)^\alpha_\beta(0) = (\mu_A)^\alpha_\beta(f(0)) = (\mu^f_A)^\alpha_\beta(0),$$

and $(\lambda^f_A)^\alpha_\beta(x) := (\lambda_A)^\alpha_\beta(f(x)) \ge (\lambda_A)^\alpha_\beta(0) = (\lambda_A)^\alpha_\beta(f(0)) = (\lambda^f_A)^\alpha_\beta(0)$.

Now

$$(\mu^f_A)^\alpha_\beta(x) = (\mu_A)^\alpha_\beta(f(x)) \ge T\{(\mu_A)^\alpha_\beta(f(z)*(f(y)*f(x))), (\mu_A)^\alpha_\beta(f(y)*f(z))\}$$
$$= T\{(\mu_A)^\alpha_\beta(f(z)*(f(y*x))), (\mu_A)^\alpha_\beta(f(y*z))\}$$
$$= T\{(\mu_A)^\alpha_\beta(f(z*(y*x))), (\mu_A)^\alpha_\beta(f(y*z))\}$$
$$= T\{(\mu^f_A)^\alpha_\beta(z*(y*x)), (\mu^f_A)^\alpha_\beta(y*z)\},$$

and

$$(\lambda^f_A)^\alpha_\beta(x) = (\lambda_A)^\alpha_\beta(f(x)) \le S\{(\lambda_A)^\alpha_\beta(f(z)*(f(y)*f(x))), (\lambda_A)^\alpha_\beta(f(y)*f(z))\}$$
$$= S\{(\lambda_A)^\alpha_\beta(f(z)*(f(y*x))), (\lambda_A)^\alpha_\beta(f(y*z))\}$$
$$= S\{(\lambda_A)^\alpha_\beta(f(z*(y*x))), (\lambda_A)^\alpha_\beta(f(y*z))\}$$
$$= S\{(\lambda^f_A)^\alpha_\beta(z*(y*x)), (\lambda^f_A)^\alpha_\beta(y*z)\}.$$

Then $(A^\alpha_\beta)^f$ is (T,S)- intuitionistic fuzzy magnified translation medial ideal of *X*.

**Theorem 7.**7. Let $f: X \rightarrow Y$ be an epimorphism of BCI-algebras and $A_\beta^\alpha = ((\mu_A)_\beta^\alpha,$ $(\lambda_A)_\beta^\alpha)$ a (T,S)- intuitionistic fuzzy magnified translation in Y. If $(A_\beta^\alpha)^f = ((\mu_A^f)_\beta^\alpha,$ $(\lambda_A^f)_\beta^\alpha)$ is (T,S)- intuitionistic fuzzy magnified translation medial ideal of X, then $A_\beta^\alpha$ $= ((\mu_A)_\beta^\alpha, (\lambda_A)_\beta^\alpha)$ is (T,S)- intuitionistic fuzzy magnified translation medial ideal in Y.

**Proof:** For any $a \in Y$, there exists $x \in X$ such that $f(x) = a$. Then $(\mu_A)_\beta^\alpha(a) = (\mu_A)_\beta^\alpha$ $(f(x)) = (\mu_A^f)_\beta^\alpha(x) \le (\mu_A^f)_\beta^\alpha(0) = (\mu_A)_\beta^\alpha(f(0)) = (\mu_A)_\beta^\alpha(0)$, and $(\lambda_A)_\beta^\alpha(a) = (\lambda_A)_\beta^\alpha$ $(f(x)) = (\lambda_A^f)_\beta^\alpha(x) \ge (\lambda_A^f)_\beta^\alpha(0) = (\lambda_A)_\beta^\alpha(f(0)) = (\lambda_A)_\beta^\alpha(0)$. Now, let $a, b, c \in Y$, and $f(x) = a,\ f(y) = b,\ f(z) = c$, for some $x, y, z \in X$. It follows that

$$
\begin{aligned}
(\mu_A)_\beta^\alpha(a) = (\mu_A)_\beta^\alpha(f(x)) = (\mu_A^f)_\beta^\alpha(x) \\
&\ge T\{(\mu_A^f)_\beta^\alpha(z*(y*x)), (\mu_A^f)_\beta^\alpha(y*z)\} \\
&= T\{(\mu_A)_\beta^\alpha(f(z*(y*x))), (\mu_A)_\beta^\alpha(f(y*z))\} \\
&= T\{(\mu_A)_\beta^\alpha(f(z)*(f(y*x))), (\mu_A)_\beta^\alpha(f(y*z))\} \\
&= T\{(\mu_A)_\beta^\alpha(f(z)*(f(y)*f(x))), (\mu_A)_\beta^\alpha(f(y)*f(z))\} \\
&= T\{(\mu_A)_\beta^\alpha(c*(b*a)), (\mu_A)_\beta^\alpha(b*c)\}, \text{and}
\end{aligned}
$$

$$
\begin{aligned}
(\lambda_A)_\beta^\alpha(a) = (\lambda_A)_\beta^\alpha(f(x)) = (\lambda_A^f)_\beta^\alpha(x) \\
&\le S\{(\lambda_A^f)_\beta^\alpha(z*(y*x)), (\lambda_A^f)_\beta^\alpha(y*z)\} \\
&= S\{(\lambda_A)_\beta^\alpha(f(z*(y*x))), (\lambda_A)_\beta^\alpha(f(y*z))\} \\
&= S\{(\lambda_A)_\beta^\alpha(f(z)*(f(y*x))), (\lambda_A)_\beta^\alpha(f(y*z))\} \\
&= S\{(\lambda_A)_\beta^\alpha(f(z)*(f(y)*f(x))), (\lambda_A)_\beta^\alpha(f(y)*f(z))\} \\
&= S\{(\lambda_A)_\beta^\alpha(c*(b*a)), (\lambda_A)_\beta^\alpha(b*c)\}
\end{aligned}
$$

This completes the proof.

## 8. Conclusion

In the present paper, we have introduced the concept of (T,S)-intuitionistic fuzzy magnified translation medial ideal in BCI –algebras and investigated some of their useful properties. We believe that these results are very useful in developing algebraic structures also these definitions and main results can be similarly extended to some other algebraic structure such as PS -algebras, Q-algebras, SU-algebras, IS- algebras, $\beta$-algebras, PU-algebras and semirings. It is our hope that this work would other foundations for further study of the theory of BCI-algebras. In our future study of fuzzy structure of BCI-algebras, may be the following topics should be considered:

1) To establish the interval valued intuitionistic (T,S)-fuzzy medial subalgebras (ideals).
2) Bipolar of intuitionistic (T,S)-fuzzy medial subalgebras (ideals).

3) To consider the structure of $(\tilde{\tau})$-interval–valued intuitionistic (T,S)-fuzzy medial subalgebras (ideals) of BCI-algebras.
4) Soft sets with applications of intuitionistic (T,S)-fuzzy medial subalgebras (ideals) of BCI-algebras.

***Algorithm for BC I-algebras***

Input ( $X$ : set, $*$ : binary operation)
Output (“ $X$ is a BCI -algebra or not”)
Begin
If $X = \phi$ then go to (1.);
End If
If $0 \notin X$ then go to (1.);
End If
Stop: =false;
$i := 1$;
While $i \le |X|$ and not (Stop) do
If $x_i * x_i \ne 0$ then
Stop: = true;
End If
$j := 1$
While $j \le |X|$ and not (Stop) do
If $(x_i * (x_i * y_j)) * y_j \ne 0,$ then
Stop: = true;
End If
End If
$k := 1$
While $k \le |X|$ and not (Stop) do
If $((x_i * y_j) * (x_i * z_k)) * (z_k * y_i) \ne 0,$ then
Stop: = true;
End If
End While
End While
End While
If Stop then
Output (“ $X$ is not a BCI-algebra”)
Else
Output (“ $X$ is a BCI -algebra”)
End If.

***Algorithm for fuzzy subsets***

Input ( $X$ : BCI-algebra, $\mu : X \to [0,1]$ );
Output (" $A$ is a fuzzy subset of $X$ or not")
Begin
Stop: =false;
$i := 1$;
While $i \le |X|$ and not (Stop) do
If ( $\mu(x_i) < 0$ ) or ( $\mu(x_i) > 1$ ) then
Stop: = true;
End If
End While
If Stop then
Output (" $\mu$ is a fuzzy subset of $X$ ")
Else
Output (" $\mu$ is not a fuzzy subset of $X$ ")
End If
End.

***Algorithm for medial -ideals***

Input ( $X$ : BCI-algebra, $I$ : subset of $X$ );
Output (" $I$ is an medial -ideals of $X$ or not");
Begin
If $I = \phi$ then go to (1.);
End If
If $0 \notin I$ then go to (1.);
End If
Stop: =false;
$i := 1$;
While $i \le |X|$ and not (Stop) do
$j := 1$
While $j \le |X|$ and not (Stop) do
$k := 1$
While $k \le |X|$ and not (Stop) do
If $z_k * (y_j * x_i) \in I$ and $y_j * z_k \in I$ then
If $x_i \notin I$ then
Stop: = true;
End If
End If
End While
End While
End While
If Stop then

Output ("$I$ is is an medial -ideals of $X$")
Else
(1.) Output ("$I$ is not is an medial -ideals of $X$")
End If
End .

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