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Generalized (Γ,ϒ)-derivations on Subtraction Algebras

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Abstract. In this paper, the notion of generalized (Γ,ϒ)-derivation determined by Γ-derivation d is introduced on complicated subtraction (c-subtraction) algebra X and investigated some related properties in details. The c-subtraction algebra is a very important topic of subtraction algebra. If Γ is an increasing function such that Γ(x) ≤ Γ(y) for all x϶X, then some equivalent condition are studied on the basis of generalized (Γ,ϒ)-derivation determined by Γ-derivation d on X.

Keywords: subtraction algebra, *c*-subtraction algebra, derivation, Generalized (Γ,ϒ)-derivation, Generalized isotone (Γ,ϒ)-derivation.

1. Introduction

The study of *BCK*/*BCI*-algebra was initiated by Imai and Iśeki [5, 6] in 1966 as a generalization of the concept of set-theoretic difference and propositional calculus. Jana et al. [7-12] and, Bej and Pal [3] and Senapati et al. [24-43] has done lot of works on *BCK*/*BCI*-algebra and *B*/*BG*/*G*-algebras which is related to these algebras.

Schein [23] considered systems of the form (Φ, *o*, \), where Φ is a set of functions closed under the composition "*o*" of functions (and hence (Φ,*o*) is a functions of semi-group), and set theoretic subtraction "\" (and hence (Φ,\) is a subtraction algebra in the sense of [1]). He proved that every subtraction semigroup is isomorphic to a difference semigroup of invertible functions. Zelinka discussed a problem proposed by Schein concerning the structure of multiplication in a subtraction semigroup. He solved the problem for subtraction algebras of a special type, called atomic subtraction algebras. Jun et al. [13] introduced the notion of ideals, prime ideals and irreducible ideals, *N*-ideals and order systems, ideals and right fixed maps on subtraction algebras and investigated their characterizations. Again, Jun et al. introduced the notion of complicated subtraction algebras and investigated some of its properties. Ceven and Öztürk [4] introduced the notion of additional concepts on subtraction algebras, so called subalgebra, bounded subtraction algebra, union of subtraction algebras and investigated some of its related properties. Lee and Kim [19] gave a investigation of multipliers in subtraction algebras.

Derivations is a very important research area in the theory of algebraic structure in mathematics. It has been applied to the classical Galois theory and the theory of invariants. An extensive and analytic theory has been developed for derivations in different algebraic structures and also plays important role in functional analysis, algebraic geometry, algebras and linear differential equations. The notion of derivation is also applied in many mathematical areas such as near-rings, rings, Banach algebras and lattices [21]. Thereafter, so many generalized derivation have been done in different algebraic structures such as, Ali and Chaudhry [2] introduced generalized (α, β)-derivation on semiprime rings, Jung and Park [14] studied generalized (α, β)-derivations and commutativity on prime rings. Öztürk and Ceven [20] introduced the notion of derivation on subtraction algebras. Lee et al. [18] and Kim [15] introduced notion of *f*-derivation on subtraction algebras. Motivated by the above works and best of our knowledge there is no work available on generalized (Γ, ϒ)-derivation *D* determined by Γ-derivation *d* on *c*-subtraction algebras. For this reason we developed theories for (Γ, ϒ)-derivation *D* determined by Γ-derivation *d* on *c*-subtraction algebras.

In this paper, the notion of generalized (Γ, ϒ)-derivations determined by a Γ-derivation of *c*-subtraction algebras is introduced and characterized some of its related properties by this notion.

2. Subtraction algebra

In this section, we recall some basic concepts which are necessary to present the paper. By a subtraction algebra we mean an algebra $(X,-)$ with a single binary operation "–" that satisfies the following identities, for any $x, y, z \in X$,

(S1) $x-(y-x)=x$

(S2) $x-(x-y)=y-(y-x)$

(S3) $(x-y)-z=(x-z)-y$.

The last identity permits us to omit parenthesis in expressions of the form $(x-y)-z$. The subtraction determines an order relation on X: $a \leq b$ implies $a-b=0$, where $0=a-a$ is an element that does not depend on the choice of $a \in X$. The ordered set $(X, \leq)$ is a semi-Boolean algebra in the sense of [1], that is, it is a meet semi-lattice with zero (0) in which every interval $[0,a]$ is a Boolean algebra with respect to the induced order.

Here $a \wedge b=a-(a-b)$, the complement of an element $b \in [0,a]$ is $a-b$.

In a subtraction algebra, for any $x,y,z \in X$, the following are true [15,16]

(p1) $(x-y)-y=x-y$

(p2) $x-0=x$ and $0-x=0$

(p3) $(x-y)-x=0$

(p4) $x-(x-y)\leq y$

(p5) $(x-y)-(y-x)=x-y$

(p6) $x-(x-(x-y))=x-y$

(p7) $(x-y)-(z-y)\leq x-z$

(p8) $x\leq y$ if and only if $x=y-w$ for some $w \in X$

(p9) $x\leq y$ implies $x-z\leq y-z$ and $z-y\leq z-x$ for all $z \in X$

(p10) $x,y\leq z$ implies $x-y=x\wedge(z-y)$

(p11) $(x\wedge y)-(x\wedge z)\leq x\wedge(y-z)$

(p12) $(x-y)-z=(x-z)-(y-z)$.

A non-empty subset S of a subtraction algebra X is called a *subalgebra* of X if $x-y \in S$ for all $x,y \in X$. A mapping f from a subtraction algebra X to a subtraction algebra Y is called a *homomorphism* of X if $f(x-y)=f(x)-f(y)$ for all $x,y \in X$. A homomorphism f from a subtraction algebra X to itself is called an *endomorphism* of X. If $x\leq y$ implies $f(x)\leq f(y)$, then f is called an *isotone* mapping.

Definition 2.1. [9] *A nonempty subset S of a subtraction algebra X is called an ideal of X if it satisfies*

(a) $0 \in S$

(b) for all $x \in X$, $y \in S$ *and* $x - y \in S$ *implies* $x \in S$.

For an ideal *S* of a subtraction algebra *X*, we have that $x \leq y$ and $y \in S$ imply that $x \in S$ for any $x,y \in X$ [16].

Lemma 2.2. [18] *Let X be a subtraction algebra. Then*

(a) $x \wedge y = y \wedge x$ *for any* $x,y \in X$

(b) $x - y \leq x$ *for any* $x,y \in X$.

Definition 2.3. [12] *Let X be a subtraction algebra. For any* $a,b \in X$, *let* $G(a,b)=\{x|x-a \leq b\}$. *Then X is said to be complicated subtraction algebra (c-subtraction algebra) if for any* $a,b \leq X$, *the set G(a,b) has the greatest element.*

Note that $0,a,b \in G(a,b)$. The greatest element of $G(a,b)$ is denoted by $a+b$.

Proposition 2.4. [12] *If X is a c-subtraction algebra, then for all* $x,y,z \in X$, *the following hold*

(a) $x \leq x+y$ *and* $y \leq x+y$

(b) $x+0=0=0+x$

(c) $x+y=y+x$

(d) $x \leq y$ *implies* $x+z \leq y+z$

(e) $x \leq y$ *implies* $x+y=y$

(f) $x+y$ *is the least upper bound of x and y.*

Definition 2.5. [21] *Let X be a c-subtraction algebra and d is a self-map on X. Then d is called a derivation of X if it satisfies the identity* $d(x \wedge y)=(d(x) \wedge y)+(x \wedge d(y))$ *for all* $x,y \in X$.

Definition 2.6. *Let X be c-subtraction algebra and* $\Gamma: X \to X$ *be a function. A function* $d: X \to X$ *is called a two-sided Γ-derivation on X if for all* $x,y \in X$ *satisfying the following identity*

$$d(x \wedge y)=(dx \wedge \Gamma(y))+(\Gamma(x) \wedge dy).$$

Theorem 2.7. [18] *Let X be a c-subtraction algebra and d be a two-sided Γ-derivation on X. Then for all* $x,y \in X$, *the following are hold*

(a) $dx \leq \Gamma(x)$

(b) $dx \wedge dy \leq d(x \wedge y) \leq dx + dy$

(c) $d(x+y) \leq \Gamma(x)+\Gamma(y)$

(d) If X has a least element 0, then $\Gamma(0)=0$ *implies* $d0=0$.

3 .Generalized (Γ,ϒ)-derivation of c-subtraction algebras

The following definitions introduce the notion of generalized (Γ, ϒ)-derivation determined by a Γ-derivation *d* on *c*-subtraction algebra.

Definition 3.1. [27] *Let X be a c-subtraction algebra. A self-map $D:X\to X$ is called a generalized derivation on X if there exist a derivation determined by self-map $d:X\to X$ if it satisfy the following identity $D(x\wedge y)=(Dx\wedge y)+(x\wedge dy)$, where for all $x,y\epsilon X$.*

Example 3.2. [18] *Let $X=\{0,a,b,c\}$ be a c-subtraction algebra with the following Caley table:*

-	0	a	b	c
0	0	0	0	0
a	b	0	a	0
b	b	b	0	0
c	c	b	a	0

Define a mapping D: X→ X by

$$Dx=\begin{cases} 0; \text{ if } x=0 \\ a; \text{ if } x=a \\ c; \text{if } x=b;c. \end{cases}$$

Define a mapping d: X→ X by

$$dx=\begin{cases} 0; \text{if } x=0;c \\ a; \text{ if } x=a \\ b; \text{ if } x=b. \end{cases}$$

Then, it is verified that *d* is a derivation on *X*. But *D* is not a generalized derivation on *X* determined by *d*. Since $D(b\wedge a) = D(0) = 0 \neq D(b) \wedge a + b \wedge d(a) = (c\wedge a) + (b\wedge a) = a + 0= a$.
Now, we defined the generalized (Γ, ϒ)-derivation on a subtraction algebras *X* in the following definition.

Definition 3.3. *Let X be a subtraction algebras. A function Γ: X→ X is called increasing if $x \leq y$ imply $\Gamma(x) \leq \Gamma(y)$ for all $x,y\epsilon X$.*

Definition 3.4. *Let X be a c-subtraction algebra. Let Γ: X→X and Υ: X→X be two mapping and d: X→X be a two-sided Γ-derivation on X. A function D: X→ X is called a generalized (Γ,Υ)-derivation on X if for all $x,y\ \epsilon X$ satisfying the following identity*

$$D(x\wedge y)=(Dx \wedge \Gamma(y)) + (\Upsilon(x) \wedge dy).$$

If Γ=1 and ϒ=1, which imply the identity mapping on *X*, then generalized (1,1)-derivation *D* became a generalized derivation. If Γ=ϒ and *D*=*d*, then *D* will be a two-sided Γ-derivation. In this paper, we consider generalized (Γ,ϒ)- derivation whose associated derivation is a two-sided Γ-derivation.

Proposition 3.5. *Let X be a c-subtraction algebra and D be a generalized (Γ, Υ)-derivation determined by a Γ-derivation d on X such that $\Gamma(x) \leq \Upsilon(x)$. Then for all $x,y\ \epsilon X$, the following conditions are holds:*

(1) $dx \leq Dx \leq \Gamma(x)$

(2) $Dx \wedge Dy \leq D(x\wedge y) \leq Dx + Dy$

(3) If I is an ideal of X with $\Gamma(I)\subseteq X$, then $D(I)\subseteq I$

(4) If X has a least element 0, then $\Gamma(0)=0$ implies that $D0=0$

(5) If X has a greatest element 1 and Γ is an increasing function, $Dx=(D1\wedge\Gamma(x))+dx$.

Proof: (1) For all $x \in X$, then

$Dx \wedge Dx = dx \wedge D(x \wedge x) = dx \wedge ((Dx \wedge \Gamma(x) + (\Upsilon(x) \wedge dx))$. Since $\Gamma(x) \leq \Upsilon(x)$, so $dx \wedge Dx = (\Gamma(x) + dx) \wedge dx = dx$ which implies $dx = dx - (dx - Dx) \leq Dx$ by (P4). Again, $Dx + \Gamma(x) = (D(x \wedge x) + \Gamma(x) = (Dx \wedge \Gamma(x) + \Upsilon(x) \wedge dx) + \Gamma(x) = (Dx \wedge \Gamma(x) + dx) + \Gamma(x) = (Dx \wedge \Gamma(x)) + (dx + \Gamma(x)) = \Gamma(x)$. Hence, $Dx + \Gamma(x) = \Gamma(x)$ which implies $Dx \leq \Gamma(x)$. Therefore, $dx \leq Dx \leq \Gamma(x)$.

(2) Let X be a c-subtraction algebra. Since $dy \leq \Gamma(y)$, then we have $Dx - \Gamma(y) \leq Dx - dy$ and $Dx - (Dx - Dy) \leq Dx - (Dx - dy) \leq Dx - (Dx - \Gamma(y))$, hence $Dx \wedge Dy \leq Dx \wedge \Gamma(y)$. Similarly, let $Dx \leq \Upsilon(x)$, then $dy - \Upsilon(x) \leq dy - Dx$. Therefore, $dy - (dy - Dx) \leq dy - (dy - \Upsilon(x))$, i.e. $dy \wedge Dx \leq dy \wedge \Upsilon(x)$ which implies $Dy \wedge Dx \leq d\,y \wedge \Upsilon(x)$. Hence, $Dx \wedge Dy \leq Dx \wedge \Gamma(y) + dy \wedge \Upsilon(x) = D(x \wedge y)$. Also, since $Dx \wedge \Gamma(y) \leq Dx$ and $dy \wedge \Upsilon(x) \leq dy$ by (P_4), and hence $D(x \wedge y) = Dx \wedge \Gamma(y) + dy \wedge \Upsilon(x) \leq Dx + dy \wedge \Upsilon(x) \leq Dx + dy \leq Dx + Dy$.

(3) Let X be a c-subtraction algebra and Dx be a generalized (Γ,Υ)-derivation on X. Let $\Gamma(x) \in I$ for $x \in I$. Since $Dx \leq \Gamma(x)$, then $Dx - \Gamma(x) = 0 \in I$. Then by definition of ideal of X, $Dx \in I$ for all $x \in I$. Thus, $D(I) \subseteq I$.

(4) If 0 is the least element of X and $\Gamma(0) = 0$. Since d is a two-sided Γ-derivation on X, then $d0 = 0$ and so, $D0 = D(0 \wedge 0) = (D0 \wedge \Gamma(0)) + (\Upsilon(0) \wedge d0) = D0 \wedge 0 + \Upsilon(0) \wedge 0 = 0 + 0 = 0$. Thus, $D0 = 0$.

(5)For every $x \in X$, we have $dx \leq \Gamma(x) \leq \Gamma(1) \leq \Upsilon(1)$, then $Dx = D(1 \wedge x) = (D1 \wedge \Gamma(x)) + (\Upsilon(1) \wedge dx) = (D1 \wedge \Gamma(x)) + dx$. □

Proposition 3.6. *Let X be a c-subtraction algebra and D be a generalized (Γ, Υ)-derivation determined by a Γ-derivation d on X. Then the following conditions hold:*

(1) If $x \leq y$ then $D(x+y) = Dy$ for all $x,y \in X$

(2) $D(G(a,b)) \subseteq G(a,b)$

(3) $G(Da,Db) \subseteq G(a,b)$.

Proof: (1) We have $x+y=y$, so we get $D(x+y)=Dy$.

(2) For all $x \in G(a,b)$, we have $x-a \leq b$. From Proposition 3.5(1), we have $Dx \leq \Gamma(x)$, and so $Dx - \Gamma(a) \leq x - a \leq b$ by (p9). Therefore, $Dx \in G(a,b)$.

(3) For all $x \in G(Da, Db))$, we have $x - Da \leq Db \leq \Gamma(b)$. Then, $x - b \leq Da \leq \Gamma(a)$, so $x - a \leq b$. Hence $x \in G(a, b)$. □

Corollary 3.7. *Let X be a c-subtraction algebra and D be a generalized (Γ, Υ)-derivation determined by a Γ-derivation d on X. Then the following properties hold:*

(1) $D(a+b) \leq a+b$

(b) $Da+Db \leq \Gamma(a)+\Gamma(b) \leq a+b$.

Proof: (1) It is trivial from Proposition 3.6(2).

(2) Since the greatest element of $G(Da, Db)$ is $Da + Db$ and the greatest of $G(a, b)$ is $a + b$, then we get $Da + Db \leq \Gamma(a) + \Gamma(b) \leq a+b$, by using the Proposition 3.6(2).

Proposition 3.8. *Let X be a c-subtraction algebra and D be a generalized (Γ, Υ)-derivation on X with greatest element 1. Let D be a generalized (Γ, Υ)-derivation and d be a Γ-derivation on X such that Γ is an increasing function satisfying the condition $\Gamma(x) \leq \Upsilon(x)$. Then following conditions are hold:*

(1) If $\Gamma(x) \leq D1$, then $Dx = \Gamma(x)$.

(2) If $\Gamma(x) \leq D1$, then $D1 \leq Dx$.

(3) If $y \leq x$ and $Dx= \Gamma(x)$, then $Dy= \Gamma(y)$.

Proof: Let X be a c-subtraction algebra and D be a generalized (Γ, Υ)-derivation on X with greatest element 1. We have $Dx = D(1 \wedge x) = D1\wedge \Gamma(x) +\Upsilon (1) \wedge dx = D1\wedge \Gamma(x) + dx$.

(1) If $\Gamma(x) \leq D1$, then $\Gamma(x) - Dx = 0$, so we have $\Gamma(x) - (\Gamma(x) - D1)= \Gamma(x) \wedge D1$. Hence, by Proposition 3.5(1) and (5), $Dx = \Gamma(x)$.

(2) If If $\Gamma(x) \geq D1$, then $D1- \Gamma(x) = 0$ and hence, $D1- (D1- \Gamma(x))=D1 \wedge \Gamma(x) \leq Dx$.

(3) If $y \leq x$, then $y=x \wedge y$. We get $Dy = D(x \wedge y) = (Dx \wedge \Gamma(y)) + (\Upsilon(x) \wedge dy) = \Gamma(y) + dy = \Gamma(y)$.

Proposition 3.9. *Let X be a c-subtraction algebra and D be a (Γ, Υ)-derivation determined by a Γ-derivation d on X. Let Γ be a increasing function such that $\Gamma(x)\leq \Upsilon(x)$ for all $x \in X$ and $y \leq x$. Then satisfying the following condition:*

(1) $Dx=(D(x+y)\wedge \Gamma(x))+dx$.

Proof: Let X be a c-subtraction algebra and D be a (Γ, Υ)-derivation determined by a Γ-derivation d on X. Then for all $x,y \in X$ and by using the Proposition 2.4(e), we have

$Dx = D((x+y) \wedge x) = (D(x+y) \wedge \Gamma(x)) + (\Upsilon(x+y) \wedge dx) = D(x+y) \wedge \Gamma(x) + dx$.

Definition 3.10. *Let X be a c-subtraction algebra and D be a generalized (Γ, Υ)-derivation determined by a Γ-derivation d on X. If $x \leq y$ implies $Dx \leq Dy$, then D is called an isotone generalized (Γ, Υ)-derivation.*

Proposition 3.11. *Let X be a c-subtraction algebra and D be a generalized (Γ, Υ)-derivation determined by a Γ-derivation d on X. Then for all $x,y \in X$, the followings hold*

(1) If $D(x\wedge y)=Dx \wedge Dy$, then D be an isotone generalized (Γ,Υ)-derivation

(2) If $D(x+y) = Dx + Dy$, then D is also an isotone generalized (Γ, Υ)-derivation.

Proof: Let X be a c-subtraction algebra and D be a generalized (Γ , Υ)-derivation on X.

(1) Let $x \leq y$. Then by (p4), $Dx = D(x \wedge y) = Dx \wedge Dy \leq Dy$.

(2) Let $x \leq y$. Then since $x+y = y$ from Proposition 2.4 (e), $Dy = D(x+y) = Dx + Dy$. Hence, we get $Dx \leq Dy$.

Definition 3.12. [4] *Let X be a subtraction algebra. Then X is called bounded subtraction algebra if there is an element 1 of X satisfying the condition $x \leq 1$ for all $x \in X$.*

Proposition 3.13. *Let X be a bounded c-subtraction algebra with greatest element 1 and D be a generalized (Γ, Υ)-derivation determined by a Γ-derivation d on X. Then $Dx=\Gamma(x)$ if and only if $D1=\Gamma(1)$.*

Proof: Let $Dx= \Gamma(x)$, then it is obvious that $D1=\Gamma(1)$. Conversely, let $D1= \Gamma(1)$. Then from Proposition 3.8(3) and $x \leq 1$ gives, $Dx = D(1\wedge x) = (D1 \wedge \Gamma(x)) + (\Upsilon(1) \wedge dx) = (D1 \wedge \Gamma(x)) + dx = (\Gamma(1) \wedge \Gamma(x)) + dx = \Gamma(x) + d x= \Gamma(x)$. □

Theorem 3.14. *Let X be a c-subtraction algebra with greatest element 1. Let D be a generalized (Γ, Υ)-derivation determined by a Γ-derivation d on X. If $\Gamma: X \rightarrow X$ be a endomorphism such that $\Gamma(x) \leq \Upsilon(x)$ for all $x \epsilon X$. Then followings are equivalent:*

(1) D is isotone generalized (Γ,Υ)-derivation

(2) $Dx = \Gamma(x) \wedge D1$

(3) $D(x \wedge y) = Dx \wedge Dy$

(4) $Dx + Dy \leq D(x+y)$.

Proof: Let D be a generalized (Γ,Υ)-derivation on c-subtraction algebra X.

(1) implies (2). Since D is generalized isotone derivation. Therefore, $Dx \leq D1$, then $Dx \leq \Gamma(x)$. Also Proposition 3.5 (5) gives $Dx = (D1 \wedge \Gamma(x)) + dx$ which indicate $\Gamma(x) \wedge D1 \leq Dx$. Thus, $Dx = \Gamma(x) \wedge D1$.

(2) implies(3). Assume that (2) holds, then $Dx \wedge Dy = (\Gamma(x) \wedge D1) \wedge (\Gamma(y) \wedge D1) = (\Gamma(x) \wedge \Gamma(y)) \wedge D1 = D(x \wedge y)$.

(3) implies (1). Assume that (3) holds. Let $x \leq y$, then $x \wedge y = x$, and hence $Dx = D(x \wedge y) = Dx \wedge Dy$ which implies $Dx \leq Dy$.

(1) implies (4). Assume that (1) holds. We have $Dx \leq D(x+y)$ and $Dy \leq D(x+y)$, so we get $Dx+Dy \leq D(x+y)$.

(4) implies (1). Assume that (4) holds. Let $x \leq y$, then $Dx + Dy \leq D(x+y) = Dy$ which imply that $Dx \leq Dy$. □

Remark 3.15. *Let X be c-subtraction algebra with least element 0 and D be a generalized (Γ, Υ)-derivation determined by a Γ-derivation d on X. Let $\Gamma: X \rightarrow X$ be isomorphism on X and $\Upsilon: X \rightarrow X$ be a function. Then Γ is a one to one and onto generalized (Γ, Υ)-derivation determined by a Γ-derivation $0: X \rightarrow X$ such that $0(x)=0$ for all $x \epsilon X$.*

Theorem 3.16. *Let X be a c-subtraction algebra and D be a generalized (Γ, Υ)-derivation determined by Γ-derivation d on X. Then $Dx = \Gamma(x)$ hold if and only if $D(x+y) = (\Gamma(x)+Dy) \wedge (Dx + \Gamma(y))$.*

4. Conclusion

In this paper, we have considered the notion of generalized (Γ,Υ)-derivation on complicated (c-subtraction) subtraction algebra determined by a Γ-derivation d on c-subtraction algebra and investigated some useful properties of this notion on c-subtraction algebra. In our opinion, these result can be similarly extended to the other algebraic structure such as B-algebras, BG-algebras, BF-algebras, MV-algebras, d-algebras, Q-algebras, BL-algebras, Lie algebras and so forth. The study of generalized (Γ, Υ)-derivations on different algebraic structures may have a lot of applications in different branches of theoretical physics, engineering, information theory, cryptanalysis and computer science etc. It is our hope that this work would serve as a foundation for further study in the theory of derivations of subtraction algebras.

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