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Ranking of Generalized Dodecagonal Fuzzy Numbers Using Incentre of Centroids

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Abstract. This paper describes a ranking method for ordering generalized dodecagonal fuzzy numbers (DoFN) [16]. To find this ranking technique we first split the generalized DoFN into nine plane figures and then calculate the centroids of each plane figure followed by the centroid of these centroids and then find the incentre of this centroid which is a process of defuzzification proposed in this paper. This method is simple in evaluation and can rank various types of fuzzy numbers.

Keywords: Ranking function; centroid points; incentre; generalized dodecagonal fuzzy numbers.

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1. Introduction

Ranking of fuzzy number play an important role in decision making. Zadeh [22] introduced the concept of fuzzy sets to deal with imprecision, vagueness in real life situations. The method for ranking was first proposed by Jain [10]. Centroid concept in ranking fuzzy number started in 1980 by Yager [20]. Murakami et al. [13] have used both the horizontal and vertical coordinates of the centroid point as the ranking index. In Kaufmann and Gupta [11] proposed an approach for the ranking of fuzzy numbers. Campos and Gonzalez [2] proposed a subjective approach for ranking fuzzy numbers. Cheng [7] presented a method for ranking fuzzy numbers by using the distance method. Chu and Tsao [8] proposed a method for ranking fuzzy numbers with the area between the centroid point and original point. Deng and Liu [9] presented a centroid-index method for ranking fuzzy numbers. Chen and Chen [5] presented a method for ranking generalized trapezoidal fuzzy numbers. Wang and Lee [19] used the centroid concept in developing their ranking index. Chen and Tang [4] proposed a method for ranking p-norm trapezoidal fuzzy numbers. Since then several methods have been proposed by various researchers which includes ranking fuzzy numbers using maximizing and minimizing set [3] decomposition principle and signed distance [21], different heights and spreads [6], rank, mode, divergence and spread [12], area compensation distance method [15], Ordering of trapezoidal fuzzy numbers[18]. Rajarajeswari and Sudha [14] proposed a new method on the incentre of centroids and uses of Euclidean distance to ranking generalized hexagonal fuzzy numbers. Singh and Thakur [17] proposed a ranking of generalized DoFN using centroid of centroids.

2. Preliminaries

2.1. Fuzzy set

A fuzzy set $\hat{A}$ in X (set of real number) is a set of ordered pairs: $\hat{A} = \{(x, \mu_{\hat{A}}(x)) / x \in X\}$

$\mu_{\hat{A}}(x)$ is called membership function of x in $\hat{A}$ which maps X to [0,1].

2.2. Fuzzy number

A fuzzy set Ã defined on the set of real numbers R is said to be a fuzzy number if its membership function $\mu_{\hat{A}}: R \rightarrow [0,1]$ has the following characteristics

(i) Ã is normal. It means that there exists an $x \in R$ such that $\mu_{\hat{A}}(x) = 1$

(ii) Ã is convex. It means that for every $x_1, x_2 \in R$, $\mu_{\hat{A}}(\lambda x_1 + (1-\lambda)x_2) \geq \min.\{\mu_{\hat{A}}(x_1), \mu_{\hat{A}}(x_2)\}$, $\lambda \in [0,1]$

(iii) $\mu_{\hat{A}}$ is upper semi-continuous.

(iv) supp (Ã) is bounded in R.

2.3. α- cut of fuzzy set

An α-cut of fuzzy set $\tilde{A}$ is a crisp set A_α defined as $A_\alpha = \{x \in X / \mu_{\hat{A}}(x) \geq \alpha\}$.

2.4. Convex fuzzy set

A fuzzy set $\tilde{A}$ is a convex fuzzy set if and only if each of its α- cut A_α is a convex set.

2.6. Generalized dodecagonal fuzzy number

A fuzzy number $\hat{A} = (a_1,a_2,a_3,a_4,a_5,a_6,a_7,a_8,a_9,a_{10},a_{11},a_{12};u,v,w)$ is said to be generalized dodecagonal fuzzy number if its membership function is given be

$$\mu_{\hat{A}}(x) = \begin{cases} 0 & x \leq a_1 \\ u\left(\frac{x-a_1}{a_2-a_1}\right) & a_1 \leq x \leq a_2 \\ u & a_2 \leq x \leq a_3 \\ u + (v-u)\left(\frac{x-a_3}{a_4-a_3}\right) & a_3 \leq x \leq a_4 \\ v & a_4 \leq x \leq a_5 \\ v + (w-v)\left(\frac{x-a_5}{a_6-a_5}\right) & a_5 \leq x \leq a_6 \\ w & a_6 \leq x \leq a_7 \\ v + (w-v)\left(\frac{a_8-x}{a_8-a_7}\right) & a_7 \leq x \leq a_8 \\ k_2 & a_8 \leq x \leq a_9 \\ u + (v-u)\left(\frac{a_{10}-x}{a_{10}-a_9}\right) & a_9 \leq x \leq a_{10} \\ u & a_{10} \leq x \leq a_{11} \\ u\left(\frac{a_{12}-x}{a_{12}-a_{11}}\right) & a_{11} \leq x \leq a_{12} \\ 0 & a_{12} \leq x \end{cases} \quad \text{where } 0 < u < v < w \leq 1$$

3. Proposed ranking method of dodecagonal fuzzy number

The centroid of a DoFN is considered to be the balancing point of the dodecagon (Fig. 1). Divide the dodecagon into eight triangles and one hexagon ABM, BCN, CDO, DEP, HIS, IJT, JKU, KWV and EFGHRQ respectively. Let the centroids of nine trapezoids be G_1, G_2, G_3, G_4, G_6, G_7, G_8, G_9 and G_5 respectively.

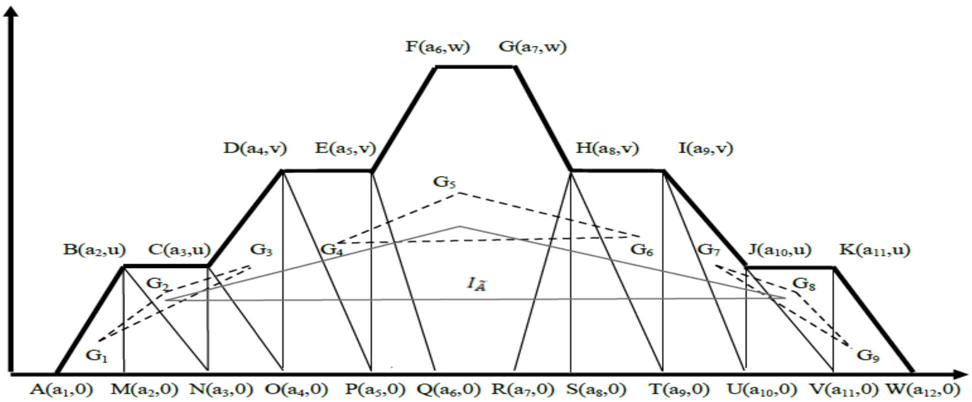

Figure 1: Generalized Dodecagonal Fuzzy Number

The centroid of the nine plane figure is

$G_1=\left(\frac{a_1+2a_2}{3},\frac{u}{3}\right)$; $G_2=\left(\frac{a_2+2a_3}{3},\frac{2u}{3}\right)$; $G_3=\left(\frac{a_3+2a_4}{3},\frac{u+v}{3}\right)$; $G_4=\left(\frac{a_4+2a_5}{3},\frac{2v}{3}\right)$;

$G_5=\left(\frac{a_5+2(a_6+a_7)+a_8}{6},\frac{v+w}{3}\right)$; $G_6=\left(\frac{2a_8+a_9}{3},\frac{2v}{3}\right)$; $G_7=\left(\frac{2a_9+a_{10}}{3},\frac{u+v}{3}\right)$; $G_8=\left(\frac{2a_{10}+a_{11}}{3},\frac{2u}{3}\right)$;

$G_9=\left(\frac{2a_{11}+a_{12}}{3},\frac{u}{3}\right)$

(a) G_1, G_2 and G_3 are non-collinear and they form triangle. We define the centroid G_1^* of the triangle with vertices G_1, G_2 and G_3 as $G_1^*=\left(\frac{a_1+3(a_2+a_3)+2a_4}{9},\frac{4u+v}{9}\right)$.

(b) G_4, G_5 and G_6 are non-collinear and they form triangle. We define the centroid G_2^* of the triangle with vertices G_4, G_5 and G_6 as
$G_2^*=\left(\frac{2(a_4+a_6+a_7+a_9)+5(a_5+a_8)}{18},\frac{5v+w}{9}\right)$

(c) G_7, G_8 and G_9 are non-collinear and they form triangle. We define the centroid G_3^* of the triangle with vertices G_7, G_8 and G_9 as $G_3^*=\left(\frac{2a_9+3(a_{10}+a_{11})+a_{12}}{9},\frac{4u+v}{9}\right)$

Also, G_1^*, G_2^* and G_3^* are non-collinear and they form triangle. We define the incentre $I_{\tilde{A}}$ of the triangle with vertices G_1^*, G_2^* and G_3^* as

$$I_{\tilde{A}}(x,y)=\left[a\left(\frac{a_1+3(a_2+a_3)+2a_4}{9}\right)+b\left(\frac{2(a_4+a_6+a_7+a_9)+5(a_5+a_8)}{18}\right)+c\left(\frac{2a_9+3(a_{10}+a_{11})+a_{12}}{9}\right),a\left(\frac{4u+v}{9}\right)+b\left(\frac{5v+w}{9}\right)+c\left(\frac{4u+v}{9}\right)\right]$$

where a = $\sqrt{\frac{[2(a_4+a_6+a_7-a_9-a_{12})+5(a_5+a_8)-6(a_{10}+a_{11})]^2+[8(u-v)+2w]^2}{18}}$;

b = $\frac{2(a_9-a_4)+3(a_{10}+a_{11}-a_2-a_3)+(a_{12}-a_1)}{9}$;

c = $\sqrt{\frac{[2(a_1+a_4-a_6-a_7-a_9)-5(a_5+a_8)+6(a_2+a_3)]^2+[8(u-v)-2w]^2}{18}}$

Hence the ranking function of the dodecagonal fuzzy number is defined as R ($\hat{A}$) = $\sqrt{x^2+y^2}$

4. Numerical example

Example 4.1. $\hat{A}$ = (0.2,0.5,0.6,0.9,1,1.2,1.4,1.6,1.8,2,2.2,2.4;$\frac{1}{3}$, $\frac{2}{3}$,1) &

$\tilde{\tilde{B}}$ = (0.2,0.4,0.6,0.8,1,1.2,1.4,1.6,1.9,2.1,2.2,2.4; $\frac{1}{3}$, $\frac{2}{3}$,1)

Solution: By using the existing method [1], we get $\hat{A} \approx \tilde{\tilde{B}}$.

By using the existing method [17], we get $\hat{A} \approx \tilde{\tilde{B}}$.

Now by using the proposed method,

$I_{\hat{A}}(x,y)$ = (10.96, 2.24)

R ($\hat{A}$) = 11.18

& $I_{\tilde{\tilde{B}}}(x,y)$ = (11.29, 2.29)

R ($\tilde{\tilde{B}}$) = 11.52

Therefore we get, $\hat{A} \prec \tilde{\tilde{B}}$.

5. Conclusion

This paper proposes a method that ranks fuzzy numbers which is simple and concrete. This method which is simple and easier for calculation not only gives satisfactory results, but also gives a correct ranking order to problems. Comparative example is used to illustrate the advantages of the proposed method.

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