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Global Exponential Synchronization with respect to Partial State Variables in a Class of Chua's System

Di Liu **and** ***Yuan Zhou***

Research Center of Analysis and Control for Complex Systems
Chongqing University of Posts and Telecommunications
Chongqing– 400065, Chongqing, China. E-mail: liud923@126.com

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Abstract. The problem of global synchronization control for a class of Chua's system is studied in this paper. Several linear controllers are proposed to realize globally exponential synchronization of two Chua's systems. Globally exponential synchronization with respect to (w.r.t) partial state variables is studied when one of the error variables is zero. Additionally, numerical simulations show the effectiveness of the proposed controllers.

Keywords: Chua's system; globally exponential synchronization; partial state variables.

1. Introduction

It is well-known that Chua's system is the first analog circuit to realize chaos in experiments. The system can show rich dynamical behaviors though it is described by the following simple ordinary differential equations

$$\begin{cases} \dot{x} = p(y - x - g(x)) \\ \dot{y} = x - y + z \\ \dot{z} = -qy \end{cases} \tag{1}$$

where x, y, z are state variables, $g(x) = G_b x + \frac{1}{2}(G_a - G_b)(|x+E| - |x-E|)$ $p > 0, q > 0$

and G_a, G_b are constants. Chua's system has the form of a simple circuit and it is easy to implement so that it results in a wide-range and in-depth research[1-2] Changing the parameters and the corresponding functions of Chua's system, the chaotic phenomenon is very rich, and it is more convenient to study the chaotic mechanism and characteristics. In resent years, Chua's circuit has a lot of deformation in the form. For example, literature [2] studied global stability issues of Chua's circuit with smooth nonlinear function $g(x) = -ax + bx^3$, literature [3] studied the chaotic oscillation characteristics of the system when $g(x) = x|x|$, In this paper, we will study the global synchronization problem of a new Chua's system which was first proposed in[1]

$$\begin{cases} \dot{x} = p(x + y - x\ln\sqrt{1+x^2}) \\ \dot{y} = x - y + z \\ \dot{z} = -qy \end{cases} \tag{2}$$

where x, y, z are state variables, $p > 0, \mathrm{q} > 0$ are constants.

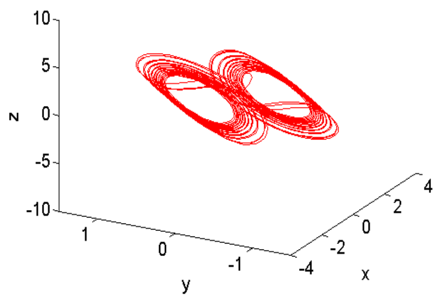

Figure 1: the phase diagram of Chua's system

The chaotic characteristics are corresponded with the parameters p, q and the initial state values of system (2). Fig.1 shows the phase diagrams of system (2) with x(0)=0, y(0)=0, z(0)=0.001, the phase diagrams of the Chua's system exhibit chaos.

The rest of this paper is organized as follows. In Section 2, several linear controllers will be designed to realize the globally exponential synchronization w.r.t partial state variables, Numerical simulations are proposed to demonstrate the correctness of our results in Section 3. And Section 4 is the conclusion.

2. Global exponential synchronization w.r.t. partial state variables

In this section, global exponential synchronization of two Chua's systems will be discussed. The drive system is given by

$$\begin{cases} \dot{x}_1 = p(x_1 + y_1 - x_1\ln\sqrt{1+x_1{}^2}) \\ \dot{y}_1 = x_1 - y_1 + z_1 \\ \dot{z}_1 = -qy_1 \end{cases} \tag{3}$$

and the response system is described as follows

$$\begin{cases} \dot{x}_2 = p(x_2 + y_2 - x_2\ln\sqrt{1+x_2{}^2}) + \mathrm{u}_1 \\ \dot{y}_2 = x_2 - y_2 + z_2 + u_2 \\ \dot{z}_2 = -qy_2 + u_3 \end{cases} \tag{4}$$

where the systems (3) and (4) denote the drive and response systems, respectively. $\mathrm{u}_i(1,2,3)$ is feedback control input which satisfies $\mathrm{u}_i(0,0,0)$.

Let $e_x = x_2 - x_1, e_y = y_2 - y_1, e_z = z_2 - z_1$, $f(e_x) = x_2 \ln\sqrt{1+x_2^2} - x_1 \ln\sqrt{1+x_1^2}$ one obtains that

$$\begin{cases} \dot{e}_x = p(e_x + e_y - f(e_x)) + u_1 \\ \dot{e}_y = e_x - e_y + e_z + u_2 \\ \dot{e}_z = -qe_y + u_3 \end{cases} \tag{5}$$

Chua's circuit is perhaps the earliest developed system from which chaos synchronization was observed [4] via a state signal taken from the transmitter system to drive the response system. Sometimes, the system have unstable states that we are not easy to find the controller to stabilize them, so partial states of them is considered. In this section, we use partial states stability theory and methodology [5] to study the globally exponential synchronization w.r.t partial state variables.

Case 1: For $x_2 = x_1$, system (5) becomes

$$\begin{aligned} \dot{e}_y &= -e_y + e_z + u_2(0, e_y, e_z) \\ \dot{e}_z &= qe_y + u_3(0, e_y, e_z) \end{aligned} \tag{6}$$

Theorem 2.1. In system(5), Let $u_2 = u_3 = 0$. Then, the zero solution of system (5) is globally exponentially stable, and thus when $x_2 = x_1$, the two systems (3) and (4) are globally exponentially synchronized between the two pairs of (y_1, y_2) and (z_1, z_2).

Proof: Construct the radially unbounded and positive definite Lyapunov function candidate as follows:

$$V_1 = e_y^2 + \frac{1}{q} e_z^2 - \varepsilon e_y e_z = \begin{bmatrix} e_y \\ e_z \end{bmatrix}^T \begin{bmatrix} 1 & -\frac{\varepsilon}{2} \\ -\frac{\varepsilon}{2} & \frac{1}{q} \end{bmatrix} \begin{bmatrix} e_y \\ e_z \end{bmatrix} \tag{7}$$

where $0 < \varepsilon \ll 1$. Derivating (7) along the solution of (6) with $u_2 = 0, u_3 = 0$, it holds that

$$\frac{dV_1}{dt} = (\varepsilon \text{q-2}) e_y^2 - \varepsilon e_z^2 + \varepsilon e_y e_z = \begin{bmatrix} e_y \\ e_z \end{bmatrix}^T \begin{bmatrix} \varepsilon \text{q-2} & \frac{\varepsilon}{2} \\ \frac{\varepsilon}{2} & -\varepsilon \end{bmatrix} \begin{bmatrix} e_y \\ e_z \end{bmatrix}$$

when $e_y^2 + e_z^2 \neq 0$ for all $0 < \varepsilon \ll 1$.

By choosing $0 < \varepsilon < \min\{\frac{2}{q}, \frac{8}{4q+1}\}$, it is easy to see that V_1 is positive while $\frac{dV_1}{dt}$ is negative definite. Thus, the zero solution of system (6) is globally exponentially stable, which implies that when $x_2 = x_1$, the two systems (3) and (4) are globally exponentially synchronized between the two pairs of (y_1, y_2) and (z_1, z_2). The proof is finished.

Case 2: For $y_2 = y_1$, the error system (5-) becomes

$$\begin{aligned}\dot{e}_x &= pe_x - pf(e_x) + u_1(e_x, 0, e_z) \\ \dot{e}_z &= u_3(e_x, 0, e_z)\end{aligned} \quad + \tag{8}$$

Where $f(\mathrm{e}_x) = x_2 \ln\sqrt{1+x_2^{\ 2}} - x_1 \ln\sqrt{1+x_1^{\ 2}}$

Theorem 2.2. In system (5), choose

$$\begin{aligned}u_1 &= -p\delta_x e_x, \quad \delta_x > 1 \\ u_3 &= -\delta_z e_z, \quad \delta_z > 0\end{aligned} \tag{9}$$

Then, the zero solution of system (5) is globally exponentially stable, and thus when $y_2 = y_1$, the two systems (3) and (4) are globally exponentially synchronized between the two pairs of (x_1, x_2) and (z_1, z_2)

Proof: Construct the radially unbounded and positive definite Lyapunov function candidate as follows:

$$V_2 = \frac{e_x^{\ 2}}{2} + \frac{e_z^{\ 2}}{2} \tag{10}$$

Derivating(10) along the solution of (8) with control input (9), it yields

$$\begin{aligned}\frac{dV_2}{dt} &= \dot{e}_x e_x + \dot{e}_z e_z \\ &= -p(\delta_x - 1)\mathrm{e}_x^{\ 2} - pe_x f(\mathrm{e}_x) - \delta_z \mathrm{e}_z^{\ 2} \\ &\le -p(\delta_x - 1)\mathrm{e}_x^{\ 2} - \delta_z \mathrm{e}_z^{\ 2}\end{aligned}$$

When $e_x^{\ 2} + e_z^{\ 2} \ne 0$. Obviously, Theorem 2.2 holds.

Case 3: For $z_2 = z_1$, the error system (5) becomes

$$\begin{aligned}\dot{e}_x &= pe_x + pe_y - pe_x f(\mathrm{e}_x) + u_1(e_x, e_y, 0) \\ \dot{e}_y &= e_x - e_y + u_2(e_x, e_y, 0)\end{aligned} \tag{11}$$

Theorem 2.3. In system (5), choose

$$\begin{aligned}u_1 &= -p\delta_x e_x, \quad \delta_x > 2 \\ u_2 &= 0\end{aligned} \tag{12}$$

Then, the zero solution of system (5) is globally exponentially stable, and thus when $z_2 = z_1$, the two systems (3) and (4) are globally exponentially synchronized between the two pairs of (x_1, x_2) and (y_1, y_2)

Proof: Construct the radially unbounded and positive definite Lyapunov function candidate as follows:

$$V_3 = \frac{{e_x}^2}{p} + {e_y}^2 \tag{13}$$

Derivating (13) along the solution of (11) with control input (12), it yields

$$\begin{aligned}\frac{dV_3}{dt} &= 2(1-\delta_x)\,{e_x}^2 + 4e_x e_y - 2{e_y}^2 - 2e_x f(e_x)\\ &\le 2(1-\delta_x)\,{e_x}^2 + 4e_x e_y - 2{e_y}^2\\ &= \begin{bmatrix} e_x \\ e_y \end{bmatrix}^T \begin{bmatrix} 2(1-\delta_x) & 2 \\ 2 & -2 \end{bmatrix} \begin{bmatrix} e_x \\ e_y \end{bmatrix}\\ &< 0\end{aligned}$$

When ${e_x}^2 + {e_y}^2 \neq 0$. Consequently, the conclusion of Theorem 6 is true.

3. Numerical simulations

In this section, several examples are proposed to illustrate the theoretical results obtained in the previous sections. A fourth order Runge-Kutta method is used to obtain the simulation results with MATLAB software. Let p=11; q=14:87, the initial state $x(0) = (0,0,0)$, When there are only partial state variables, Fig.2 shows the synchronous error of system (6) without any controller when $x_1 = x_2$. Fig.3 and Fig.4 indicate the corresponding synchronous error of systems (8) and (11) with linear controller (9) and (12), respectively. It is easy to see that the two systems are globally asymptotic synchronization.

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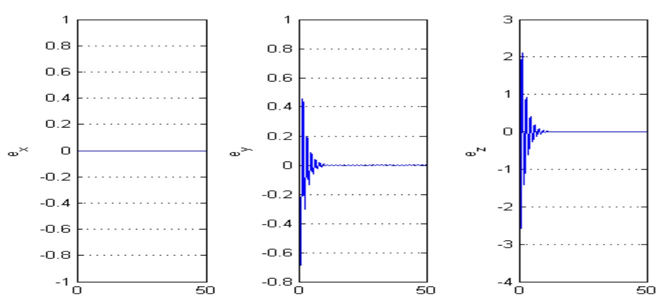

Figure 2: The synchronization errors of system (3) and (4) without any controller when $x_2 = x_1$

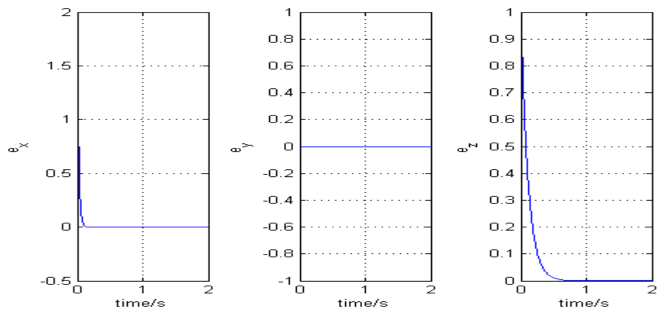

Figure 3: The synchronization errors of system (3) and (4) without any controller when $y_2 = y_1$

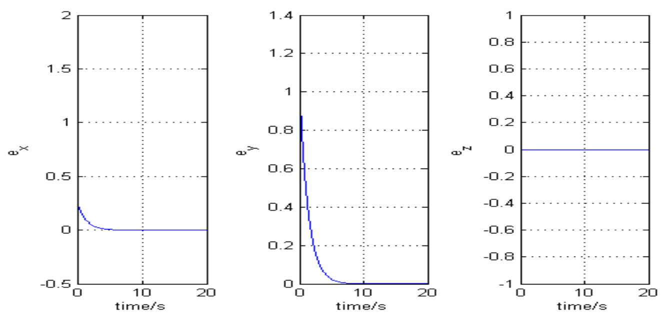

Figure 4: The synchronization errors of system (3) and (4) without any controller when $z_2 = z_1$

4. Conclusions

The problem of global synchronization control for a class of Chua's system is studied in this paper. Globally exponential synchronization with respect to (w.r.t) partial state variables is studied when one of the error variables is zero, numerical simulations show the effectiveness of the proposed theoretical results.

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