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New Stable Period Gait in Compass Model

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Abstract. This paper presents a new steady periodic gait in the compass model. It is proved that the model has period-four gait and the new gait lead to chaos through period-doubling bifurcation. This paper confirms the existence of the new gait and its bifurcation through the calculation by computer.

Keywords: new gait, compass model, bifurcation, chaos

1. Introduction

Many people have been interested in mechanical legged robots for several years. Beck et al. comes up with convincing evidence of such legged robots in [1]. The phenomenon that the swing process of humans' movement is almost passive makes people be interested in studying passive walking. Researchers have studied on passive dynamic walking robots such as in [2]–[5]. The most watched problem about passive dynamic walking robots is the energy consumption. It is expected to improve energy efficiency and give better control of dynamic walking robots through the study of passive dynamic walking. Garcia et al. proposed that the simplest model exhibits period-doubling routes to chaos [6]. On this basis, Li et al. found new bifurcations and confirmed it [7,8]. Goswami et al. studied the compass model and firstly reported the occurrence of period-doubling bifurcation [9,10]. This paper will study the phenomenon about the locomotion of a compass-gait model, and it presents a new period gait which has not been discovered. We give computer proofs about the existence of the period gait to prove our founding. Since the compass-gait model is more human-like than the simplest walking model, it is meaningful to study the dynamics found in this compass model. The new bifurcation found in this paper should be of potential implication to dynamical walking studies.

2. Period-four route to chaos in the compass model

This section is mainly about the ingredient and the theory basis of the compass model. And we also give some charts to show the new bifurcation clearly.

2.1. The compass robot

As shown in Fig.1, the compass-gait model contains two perfectly equivalent legs with mass m, and a frictionless hip with mass m_H which fitting the two legs together. In this model, the two legs are spring less and they has no knees and feet. The compass robot goes down the slope only by gravity as the power in an appropriate initial condition and a

corresponding slope φ. The support leg get away from the ground while the swing leg gets in touch with the ground, the impulsive translates. We assume the collision is perfectly nonelastic and foot and ground touch without sliding [11].
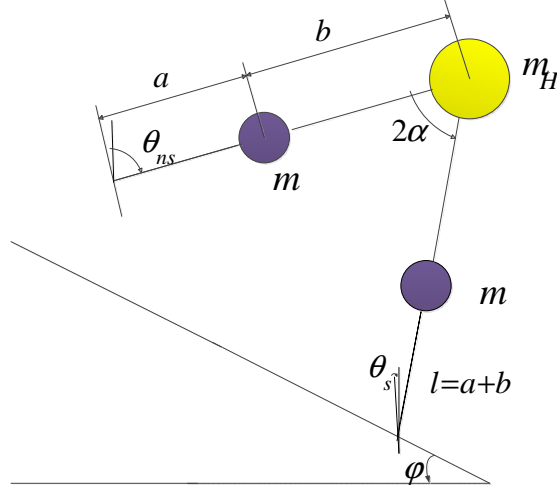

Figure 1: Diagram of the compass model

The compass model includes nonlinear differential equations of the swing phase and algebraic equations of the collision phase [11]. We make $\delta=\begin{bmatrix} \theta_{ns} & \theta_s \end{bmatrix}$ the generalized vector of the compass model [12]. This model satisfies only one constraint $h(\delta) \geq 0$. It represents that the swing leg is above the ground. The motion process of the compass model can be described by the following Lagrangian system [13]:

$$J(\delta)\ddot{\delta} + H(\delta,\ddot{\delta}) + G(\delta) = \nabla \mathrm{h}(\delta)\lambda_\delta , \tag{1}$$

$$0 \leq h(\delta) \perp \lambda_\delta \geq 0 , \tag{2}$$

$$\delta^+ = R\delta^- \text{ and } \dot{\delta}^+ = S\dot{\delta}^- . \tag{3}$$

Expression (1) represents the swing process of passive motion. In this equation J represents the inertia matrix , H consists of Coriolis and centrifugal terms , and G means gravity forces. The term $\nabla h(\delta)$ is defined by:

$$\Gamma = \{\delta \in \Re : h(\delta) = l\left(\cos\left(\theta_s + \phi\right) - \cos\left(\theta_{ns} + \phi\right)\right)\} .$$

The inequality (2) is indicated by the following extensions [13,14]:

$$\begin{cases} h(\delta) \geq 0 \\ \lambda_\delta \geq 0 \\ h(\delta)\lambda_\delta = 0 \end{cases} .$$

The constraint $h(\delta) \geq 0$ in (5) shows that there is no grazing situation. In addition, the inequality means that the collision force is unidirectional. It is called the complementarity condition when $h(\delta)\lambda_\delta = 0$ [15,16]. It means that if $h(\delta) \geq 0$ then

$\lambda_\delta = 0$, but a non-zero collision force $h(\delta) > 0$ is impossible without contact [13]. The equation (3) means it change the impulsive transition in the angular positions' vector and in the angular velocities' vector at the collision stage. In (3) the signs - and + express just before and just after the collision , respectively.

Matrices in (1) and (3) are given by:

$$J(\delta) = \begin{bmatrix} mb^2 & -mlb\cos(\theta_s - \theta_{ns}) \\ -mlb\cos(\theta_s - \theta_{ns}) & m_H l^2 + m(l^2 + a^2) \end{bmatrix},$$

$$H(\delta, \dot{\delta}) = \begin{bmatrix} mlb\dot{\theta}_s^2 \sin(\theta_s - \theta_{ns}) \\ -mlb\dot{\theta}_{ns}^2 \sin(\theta_s - \theta_{ns}) \end{bmatrix},$$

$$G(\delta) = g \begin{bmatrix} mb\sin(\theta_{ns}) \\ -(m_H l + m(a+l))\sin(\theta_s) \end{bmatrix},$$

$$R = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}, \quad S = Q_p^{-1}(\alpha)Q_m(\alpha),$$

$$Q_m(\alpha) = \begin{bmatrix} -mab & -mab + (m_H l^2 + 2mal)\cos(2\alpha) \\ 0 & -mab \end{bmatrix},$$

$$Q_p(\alpha) = \begin{bmatrix} mb(b - l\cos(2\alpha)) & ml(l - b\cos(2\alpha)) + ma^2 + m_H l^2 \\ mb^2 & -mbl\cos(2\alpha) \end{bmatrix},$$

where $l = a + b$ and $\alpha = \dfrac{1}{2}(\theta_s - \theta_{ns})$.

2.2. New bifurcation in the compass model

We can see interesting nonlinear phenomena in the following figure. Goswami et al. [17,18] and Thuilot et al. [19] first discovered the appearance of period-doubling bifurcations of the compass model. They changed the walking systems' parameters such as mass and length of the legs and even slope angle, founding that the compass-gait robot exists perioddoubling bifurcations. Then Gritli et al. found that the model exhibits a period-3 stable gait in [12]. They emphasize that the existence of stable gaits only when slopes lower than 0.0908rad. When the angle is greater than this value, the compass model falls down and is not impossible to find stable gaits. Based on the above studies, we found a new bifurcation through a large number of attempt and calculation by computer aided.

In Fig.2, the dynamical behavior of the compass model is well described. It is clear that the slope ranges becomes narrower and narrower when the period increases. To find new periodic gait and the corresponding period-doubling diagrams, we have to choose much smaller step size. We computed a step size of 4e-6 and found a new stable gait with period-four. We also found that the new gait can lead to higher periodic cycle and chaos via period-doubling .The system (1) is chaotic when $\varphi = 0.061945$ as shown in the Fig.3, which shows the phase diagram of the compass model.
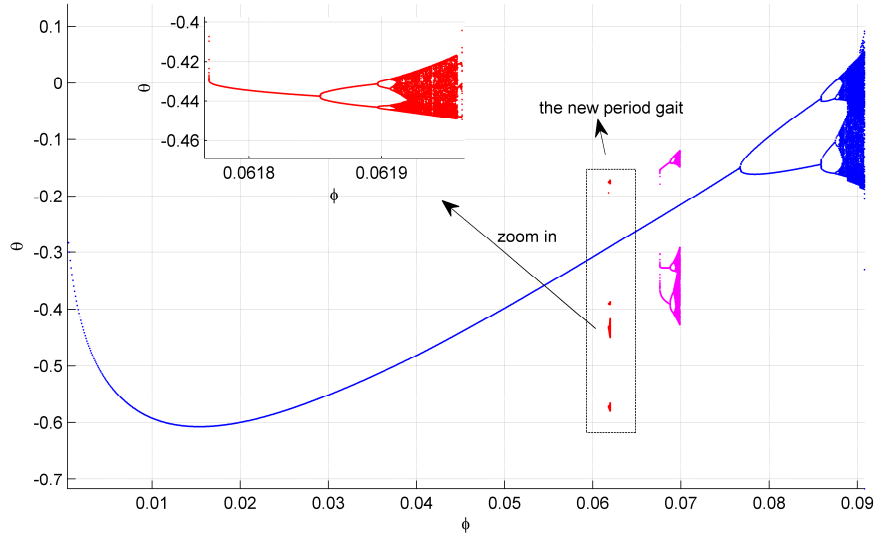
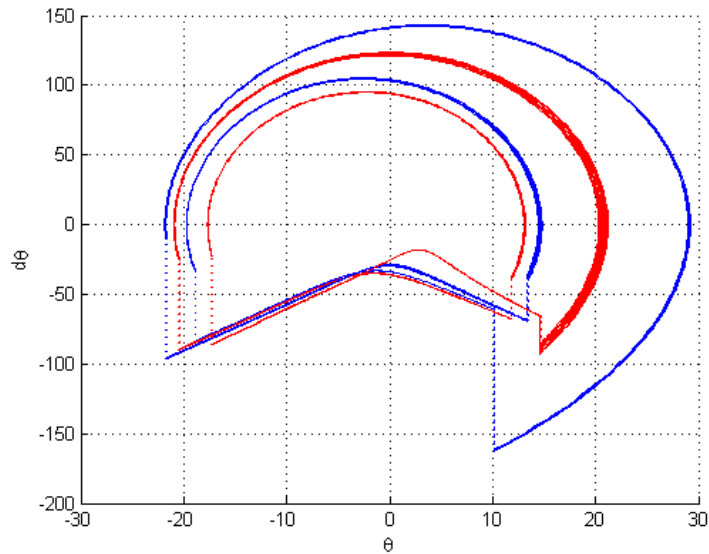**Figure 2:** The bifurcation diagram for $0 \leqslant \varphi \leqslant 0.091$

Figure 3: Phase diagram of chaotic gaits

As the ground slope φ increases, the period-four gait goes through a gentle change of period doubling bifurcations, at the same time, the period of the attractor is doubled. With the period runs to infinity, a chaotic attractor appears, and is located in the bifurcation diagram of four independent bands. The stable period-4 gait becomes unstable at $\varphi = 0.06185$ rad, where it becomes unstable and a steady period-8 gait emerges. This period-8 limit cycle becomes unstable at $\varphi = 0.0619$ rad , where a period-16 stable gait

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appears. As the slope increases, the period-doubling continues until the chaotic gait appears which is leaded by a infinite period limit cycle.

3. Conclusion

This paper is mainly about the new stable periodic gait of the four parameters exist in the compass model. We have found a period-doubling route to chaos when φ increases. And this new bifurcation phenomenon has not been reported before. The new bifurcation found in this paper is potential significance for the study of passive dynamical walking. In our investigation, the application of this discovery in biped robot is very meaningful. Firstly, the robot walking is more practicality. The using of the compass model, which compared to the simplest model, increases the degree of freedom that made the robot much more anthropomorphic which has a realistic simulation in walking robot. Meanwhile, this paper we find the new stable periodic gaits walking style which is more energy saving, so that the robot can walk passively for longer time. This article confirmed the existence of the periodic orbits is a sequence, and find more saving energy cycle track in the future.

REFERENCES

1. L.Beck, Most famous robots, http://sites.google.com/site/luisbeck007/ *humanoid*, 2009.
2. T.Kai and T.Shintani, A gait generation method for the compass-type biped robot based on discrete mechanics. *Proceedings of the 18th IFAC World Congress*, Milano, Italy, 2011;pp.8101-8107.
3. Y.Hu, G.Yan and Z.Lin, Gait generation and control for biped robots with underactution degree one, *Automatica*, 47(8) (2011) 1605-1616.
4. L.B.Freidovich, U.Mettin, A.S.Shiriaev and M.W.Spong, A passive 2-DOF walker: Hunting for gaits using virtual holonomic constraints, *IEEE Trans Robotic*, 25(5) (2009) 1202-1208.
5. J.S.Moon, M.W.Spong, Bifurcations and chaos in passive walking of a compass-gait biped with asymmetries. *Proceedings of the IEEE International Conference on Robotics and Automation*, Anchorage Alaska USA, 2010; pp. 1721-1726.
6. M.Garcia et al., The simplest walking model: Stability, complexity, and scaling, *ASME J. Biomech.* Eng., 120 (2) (1998) 281-288.
7. Q.Li, S.Tang and X.-S.Yang, New bifurcations in the simplest passive walking model, *Chaos* 23 (2013) 043-110.
8. Q.Li and X.S.Yang, New walking dynamics in the simplest passive bipedal walking model, *Appl. Math. Model.* 36(11) (2012) 5262-5271.
9. A.Goswami, B.Espiau, A.Keramane, Limit cycles and their stability in a passive bipedal gait, *Proceedings of the IEEE International Conference on Robotics and Automation*, Minneapolis, Minnesota, USA, 1996; pp. 246-251.
10. B.Thuilot, A.Goswami and B.Espiau, Bifurcation and chaos in a simple passive bipedal gait, *Proceedings of the IEEE International Conference on Robotics and Autom- ation*, Albuquerque, New Mexico, 1997; pp. 792-798.
11. A.Goswami, B.Thuilot and B.Espiau, A study of the passive gait of a compass-like biped robot: symmetry and chaos, *Int J. Robotic Res.,* 17(12) (1998) 1282-1301.

12. Hassne Gritli, Nahla Khraief and Safya Belghith, Period-three route to chaos induced by a cyclic-fold bifurcation in passive dynamic walking of a compass-gait biped robot, *Commun Nonlinear Sci Numer Simulat.,* 17 (2012) 4356-4372

13. Y.Hurmuzlua, F.Gnot and B.Broliato, Modeling stability and control of biped robots-a general framework, *Automatica,* 40 (2004) 1674-64.

14. B.Broliato, Nonsmooth Impact Dynamics: Models, Dynamics and Control. London U.K.: *Springer-Verlag* ; 1996.

15. B.Brogliato, Some perspectives on the analysis and control of complementarity systems, *IEEE Trans. Autom. Control*, 48(6) (2003) 918-935.

16. J.M.Bourgeot, Contribution la commande de systmes mcaniques nonrguliers, *Thesis*, INRIA Rhone-Alpes (2004).

17. A.Goswami, B.Espiau and A.Keramane, Limit cycles in a passive compass gait biped and passivity-mimicking control laws, *Auton. Rob.*, 4(3) (1997) 273-286.

18. A.Goswami, B.Thuilot and B.Espiau, A study of the passive gait of a compass-like biped robot: Symmetry and chaos, *Int. J. Robot. Res*., 17(12) (1998) 1282-1301.

19. B.Thuilot, A.Goswami and B.Espiau, Bifurcation and chaos in a simple passive bipedal gait, *Proceedings of the IEEE International Conference on Robotics and Automation*, Albuquerque, New Mexico, 1997; pp.792-798.