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Chaotic Dynamic Analysis of Brushless DC Motor

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Abstract. Brushless DC Motor (BLDCM) has high reliability, long service life, which, however, is a typical multivariable, strong coupled complicated nonlinear system. There peacefully coexist complex dynamical behaviors, such as limit cycles and chaotic attractors, coexists peacefully. Therefore, this paper proposes the BLDCM system, to explore these dynamics, we compute and simulate the bifurcation diagram, Lyapunov spectrum and analysis its chaotic characteristics by using the analysis method of nonlinear chaos.

Keywords: BLDCM; chaos; bifurcation; Lyapunov exponent

1. Introduction

Brushless DC Motor (BLDCM) with the function of the superior reliability, efficiency etc., is widely used in many control fields, such as the aerospace [1,2], the robot manipulator [3,4], the electric vehicle [5], and automatic manufacturing, et al. Almost two decades from then on, a number of researchers found that the BLDCM of the irregular movement, may show the aperiodic, random and uncertain parameters [6,7]. As is known to all, BLDCM is a typical multivariable, strong coupled complicated nonlinear system. A thorough study of the traditional linearization theory method cannot be carried out. Therefore, chaotic characteristics were found to exist in the system are simulation analyzed by using the method of nonlinear chaos. It can make easier for us to understand the BLDCM system.

In 1994, Hemati put forward the nonlinear mathematical model of BLDCM, identified the system parameters in the chaotic system [8]; Zhihong Yang et al. has carried the numerical simulation on the Lorenz system mathematical model of BLDCM system in the literature [9], further analysis of the nonlinear dynamic characteristics of the BLDCM; Still, Jian Wang et al. kept the further simulation experiment on the chaotic phenomenon of BLDCM, given the irregular movement of chaos explanation in the literature [10].Therefore nonlinear chaotic analysis method is widely used in this fields. In this paper, we put forward the mathematical model of BLDCM, and we find some new parameters about this system. Then, a set of the typical values of BLDCM can be taken in this paper, we compute and simulate its bifurcation diagrams, Lyapunov exponent diagrams as a parameter varies. Finally, this study is analyzed the complex dynamical behaviors by using the method of nonlinear chaos.

The rest of this paper is organized as follows. Section II builds the mathematical model of BLDCM dimensionless derivation, then analysis of the chaotic behaviors, such as limit cycles and chaotic attractors. Section III draws the conclusion.

2. Chaos analysis of brushless DC motor

In this section, we will put forward the mathematical model of BLDCM, get the bifurcation diagrams and Lyapunov exponent diagrams of BLDCM by software, and analyze that chaotic dynamics behavior of BLDCM.

The equivalent dimensionless mathematical model of Brushless DC Motor system can be described as follows[11-13],

$$\begin{cases} \dot{i}_d = u_d - a i_d + i_q \omega \\ \dot{i}_q = u_q - i_q - i_d \omega + b\omega \\ \dot{w} = c(i_q - \omega) + d i_d i_q - T_L \end{cases} \tag{1}$$

where ω is the so-called angular velocity of the motor, i_d and i_q are components of stator current vector d , q , respectively, u_d and u_q are components of stator voltage vector d , q , respectively, T_L represents the load torque, σ , δ , γ , and η are system parameters. In order to facilitate analysis, if one takes $i_d = x$, $i_q = y$, $\omega = z$, then (1) can be rewritten as

$$\begin{cases} \dot{x} = u_d - ax + yz \\ \dot{y} = u_q - y - xz + bz \\ \dot{z} = c(y - z) + dxy - T_L \end{cases} \tag{2}$$

In a wide parameter domain presents the chaotic attractor, when the parameters take different values, the result is also different in system (2), a set of parameters of BLDCM can be taken the following typical values:

$$a = 0.875,\ b = 60,\ c = 4,\ d = 0.1,\ u_d = 5,\ u_q = 0,\ T_L = 0,\ x(0) = [0.01, 0.1, 0.01]^T \tag{3}$$

We find that system (2) can demonstrate complex dynamics, such as limit cycles and chaos. In order to explore these dynamics, we compute the bifurcation diagram, Lyapunov spectrum as d varies.

For the Lyapunov spectrum, we use the QR-based method proposed in [14], where a matrix is obtained to compensate the current switching in (2). To compute bifurcation diagrams and basins of attraction, we conveniently study a Poincare map defined on the following cross-section plane

$$P = \{(\mathrm{x}, \mathrm{y}, \mathrm{z}) \mid \mathrm{x} = 60, \dot{\mathrm{x}} < 0\} \tag{4}$$

Then the corresponding Poincare map is defined as: For each $\mathrm{x} = (\mathrm{y}, \mathrm{z}) = (60, \mathrm{y}, \mathrm{z}) \in P$, $P(\mathrm{x})$ is taken to be the first return point in P under the flow with the initial condition x.

We observe the process of the system changing from the chaos to the period doubling bifurcation with d from 0 to 0.075. The bifurcation diagram and the Lyapunov spectrum are as shown in Figs. 1 and 2. Both of them show that the system can demonstrate abundant behaviors as follows, and we must emphasize that the dynamics coexist peacefully, as well as some interesting bifurcations.

At the beginning, d < 0.057, there is a chaotic attractor, a typical phase portrait is shown in Fig. 3(a) at d=0, obviously, one of the Lyapunov exponent is positive, therefore the attractor should be chaotic. Due to the symmetry of system (2), the two chaotic attractors are exactly the same as each, in other words, they are symmetry twin chaotic attractors. Both of them are large regions, which suggests that the twin attractors coexist peacefully indeed.

When d increases, there is a transition route of chaos to bifurcation. As shown in Fig. 3(b)-3(d), when d =0.044, a period-4 cycle appears, it is clear to see a gap between limit cycles and the strange attractor, which means that they coexist peacefully. When d =0.057, the period orbit becomes a period-2 orbit, and when d = 0.061, it becomes a period-1 orbit. Due to the symmetry of the system equation, there are two limit cycles are the symmetric pair of attractors. When we trace these coexisting attractors by increasing and decreasing d between 0.044 and 0.075, we have many more cases of coexisting attractors, which are similar to each. With consideration of the system, almost all kinds of attractors may coexist peacefully in this system.

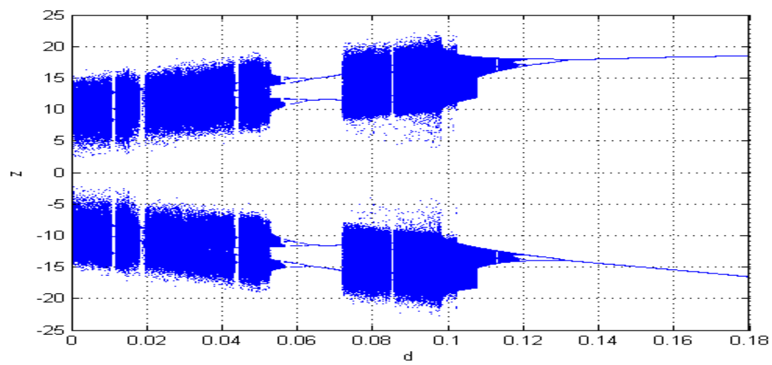

Figure 1: Bifurcation diagram of system (2) with (3) by adjusting d.

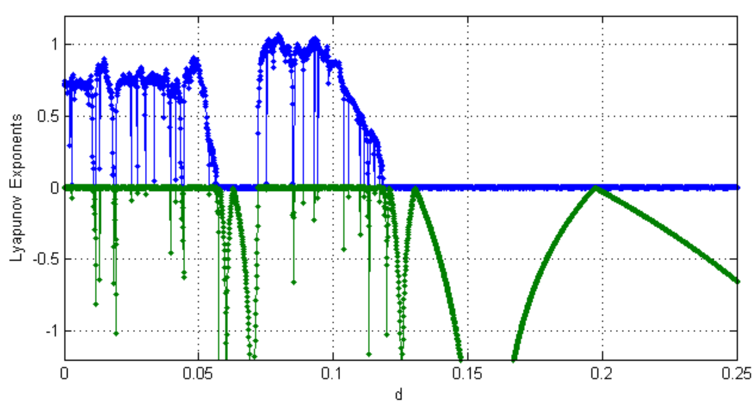

Figure 2: Lyapunov exponents of system (2) with (3) by adjusting d.

We should emphasize here that complex dynamics, such as limit cycles and chaotic attractors, coexists peacefully, which is significant difference from other chaotic systems reported before.

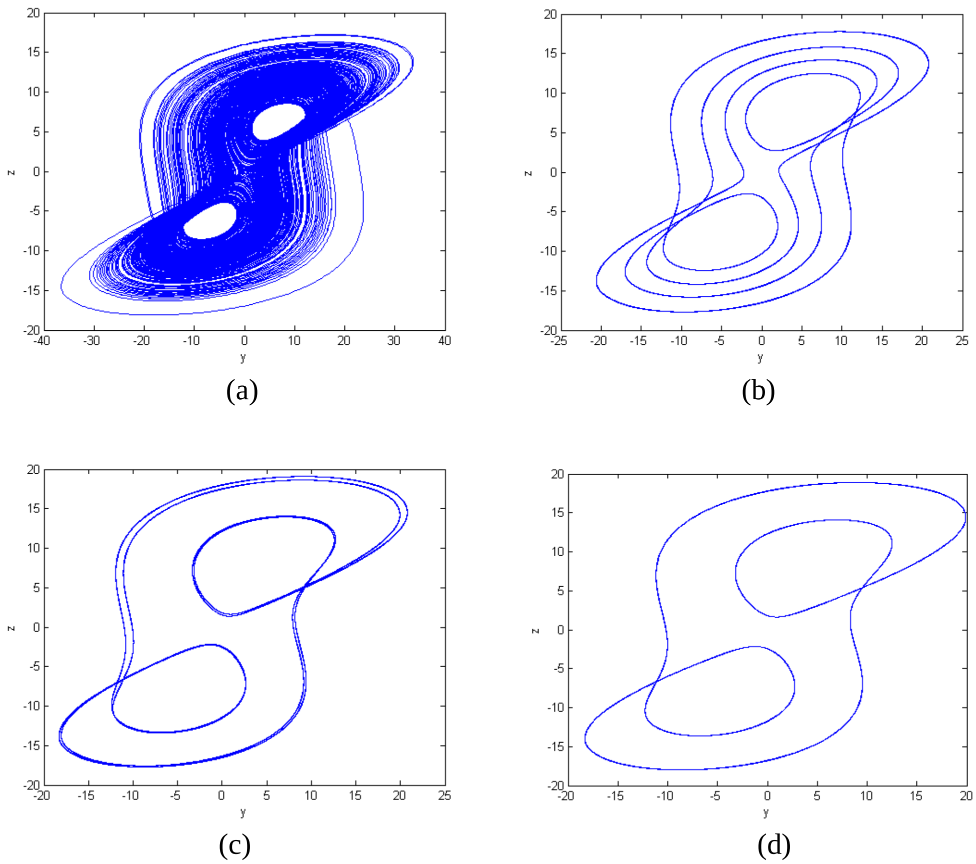

Figure 3: Typical phase portraits as d decreases . (a) Chaotic attractor at d = 0. (b) Period-4 orbit at d = 0.044. (c) Period-2 orbit at d = 0.057. (d) Period-1 orbit at d = 0.061.

3. Conclusion

The BLDCM system has peacefully coexist complex dynamical behaviors, such as period limit cycles, chaos and coexists peacefully. In this paper, the mathematical model of BLDCM dimensionless was established. Then, according to draw the phase diagrams of the bifurcation diagrams and the Lyapunov exponent figures, analyzed its chaotic dynamic behavior. This article implies that the nonlinearity of the BLDCM can introduce much more complex behaviors than we thought before. Further study of such nonlinear dynamics may provide researchers with better understanding and applications of the BLDCM in future.

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