

Pairwise Connectedness in soft biČech Closure Spaces

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Abstract. The aim of the present paper is to study the concept of pairwise connectedness in biČech closure spaces through the parameterization tool which is introduced by Molodtsov.

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1. Introduction

Čech [1] introduced the concept of closure spaces and developed some properties of connected spaces in closure spaces. According to him, a subset A of a closure space X is said to be connected in X if A is not the union of two non-empty Semi-Separated Subsets of X .

Plastria studied [2] connectedness and local connectedness of simple extensions.

Rao and Gowri [3] studied pairwise connectedness in biČech closure spaces. Gowri and Jegadeesan [7,8,9,10] introduced separation axioms in soft Čech closure spaces, soft biČech closure spaces and studied the concept of connectedness in fuzzy and soft Čech closure spaces.

In 1999, Molodtsov [4] introduced the notion of soft set to deal with problems of incomplete information. Later, he applied this theory to several directions [5] and [6].

In this paper, we introduced and exhibit some results of pairwise connectedness in soft biČech closure spaces.

2. Preliminaries

In this section, we recall the basic definitions of soft biČech closure space.

Definition 2.1. [9] Let X be an initial universe set, A be a set of parameters. Then the function $k_1: P(X_{F_A}) \rightarrow P(X_{F_A})$ and $k_2: P(X_{F_A}) \rightarrow P(X_{F_A})$ defined from a soft power set $P(X_{F_A})$ to itself over X is called Čech Closure operators if it satisfies the following axioms:

$$(C1) \quad k_1(\emptyset_A) = \emptyset_A \text{ and } k_2(\emptyset_A) = \emptyset_A .$$

$$(C2) \quad U_A \subseteq k_1(U_A) \text{ and } U_A \subseteq k_2(U_A).$$

$$(C3) \quad k_1(U_A \cup V_A) = k_1(U_A) \cup k_1(V_A) \text{ and } k_2(U_A \cup V_A) = k_2(U_A) \cup k_2(V_A).$$

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Then (X, k_1, k_2, A) or (F_A, k_1, k_2) is called a soft biČech closure space.

Definition 2.2. [9] A soft subset U_A of a soft biČech closure space (F_A, k_1, k_2) is said to be soft $k_{i=1,2}$ -closed if $k_i(U_A) = U_A, i = 1, 2$. Clearly, U_A is a soft closed subset of a soft biČech closure space (F_A, k_1, k_2) if and only if U_A is both soft closed subset of (F_A, k_1) and (F_A, k_2) .

Let U_A be a soft closed subset of a soft biČech closure space (F_A, k_1, k_2) . The following conditions are equivalent.

1. $k_2 k_1(U_A) = U_A$.
2. $k_1(U_A) = U_A$ and $k_2(U_A) = U_A$.

Definition 2.3. [9] A soft subset U_A of a soft biČech closure space $((F_A, k_1, k_2))$ is said to be soft $k_{i=1,2}$ -open if $k_i(U_A^C) = U_A^C, i = 1, 2$.

Definition 2.4. [9] A soft set $Int_{k_i}(U_A), i = 1, 2$ with respect to the closure operator k_i is defined as $Int_{k_i}(U_A) = F_A - k_i(F_A - U_A) = [k_i(U_A^C)]^C, i = 1, 2$. Here $U_A^C = F_A - U_A$.

Definition 2.5. [9] A soft subset U_A in a soft biČech closure space (F_A, k_1, k_2) is called soft $k_{i=1,2}$ neighbourhood of e_F if $e_F \in Int_{k_{i=1,2}}(U_A)$.

Definition 2.6. [9] If (F_A, k_1, k_2) be a soft biČech closure space, then the associate soft bitopology on F_A is $\tau_i = \{U_A^C : k_i(U_A) = U_A, i = 1, 2\}$.

Definition 2.7. [9] Let (F_A, k_1, k_2) be a soft biČech closure space. A soft biČech closure space (G_A, k_1^*, k_2^*) is called a soft subspace of (F_A, k_1, k_2) if $G_A \subseteq F_A$ and $k_i^*(U_A) = k_i(U_A) \cap G_A, i = 1, 2$, for each soft subset $U_A \subseteq G_A$.

3. Pairwise connectedness

In this section, we introduce pairwise soft separated sets and discuss the pairwise connectedness in soft biČech closure space.

Definition 3.1. Two non-empty soft subsets U_A and V_A of a soft biČech closure space (F_A, k_1, k_2) are said to be pairwise soft separated if and only if $U_A \cap k_1[V_A] = \emptyset_A$ and $k_2[U_A] \cap V_A = \emptyset_A$.

Remark 3.2. In other words, two non-empty U_A and V_A of a soft biČech closure space (F_A, k_1, k_2) are said to be pairwise soft separated if and only if $(U_A \cap k_1[V_A]) \cup (k_2[U_A] \cap V_A) = \emptyset_A$.

Theorem 3.3. In a soft biČech closure space (F_A, k_1, k_2) , every soft subsets of pairwise soft separated sets are also pairwise soft separated.

Proof. Let (F_A, k_1, k_2) be a soft biČech closure space. Let U_A and V_A are pairwise soft separated sets. Let $G_A \subset U_A$ and $H_A \subset V_A$. Therefore, $U_A \cap k_1[V_A] = \emptyset_A$ and $k_2[U_A] \cap V_A = \emptyset_A \dots (1)$

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$$\begin{aligned} \text{Since, } G_A \subset U_A &\Rightarrow k_2[G_A] \subset k_2[U_A] \Rightarrow k_2[G_A] \cap H_A \subset k_2[U_A] \cap H_A \\ &\Rightarrow k_2[G_A] \cap H_A \subset k_2[U_A] \cap V_A \\ &\Rightarrow k_2[G_A] \cap H_A \subset \emptyset_A \quad \text{by (1)} \\ &\Rightarrow k_2[G_A] \cap H_A = \emptyset_A. \end{aligned}$$

$$\begin{aligned} \text{Since, } H_A \subset V_A &\Rightarrow k_1[H_A] \subset k_1[V_A] \Rightarrow k_1[H_A] \cap G_A \subset k_1[V_A] \cap G_A \\ &\Rightarrow k_1[H_A] \cap G_A \subset k_1[V_A] \cap U_A \\ &\Rightarrow k_1[H_A] \cap G_A \subset \emptyset_A \quad \dots \text{ by (1)} \\ &\Rightarrow k_1[H_A] \cap G_A = \emptyset_A. \end{aligned}$$

Hence, U_A and V_A are also pairwise soft separated.

Theorem 3.4. Let (G_A, k_1^*, k_2^*) be a soft subspace of a soft biČech closure space (F_A, k_1, k_2) and let $U_A, V_A \subset G_A$, then U_A and V_A are pairwise soft separated in F_A if and only if U_A and V_A are pairwise soft separated in G_A .

Proof. Let (F_A, k_1, k_2) be a soft biČech closure space and (G_A, k_1^*, k_2^*) be a soft subspace of (F_A, k_1, k_2) . Let $U_A, V_A \subset G_A$. Assume that, U_A and V_A are pairwise soft separated in F_A implies that $U_A \cap k_1[V_A] = \emptyset_A$ and $k_2[U_A] \cap V_A = \emptyset_A$. That is, $(U_A \cap k_1[V_A]) \cup (k_2[U_A] \cap V_A) = \emptyset_A$.

$$\begin{aligned} \text{Now, } (U_A \cap k_1^*[V_A]) \cup (k_2^*[U_A] \cap V_A) &= (U_A \cap (k_1[V_A] \cap G_A)) \cup ((k_2[U_A] \cap G_A) \cap V_A) \\ &= (U_A \cap G_A \cap k_1[V_A]) \cup (k_2[U_A] \cap G_A \cap V_A) \\ &= (U_A \cap k_1[V_A]) \cup (k_2[U_A] \cap V_A) \\ &= \emptyset_A. \end{aligned}$$

Therefore, U_A and V_A are pairwise soft separated in F_A if and only if U_A and V_A are pairwise soft separated in G_A .

Definition 3.5. A soft biČech closure space (F_A, k_1, k_2) is said to be pairwise disconnected if it can be written as two disjoint non-empty soft subsets U_A and V_A such that $k_2[U_A] \cap k_1[V_A] = \emptyset_A$ and $k_2[U_A] \cup k_1[V_A] = F_A$.

Definition 3.6. A soft biČech closure space (F_A, k_1, k_2) is said to be pairwise connected if it is not pairwise disconnected.

Example 3.7. Let the initial universe set $X = \{u_1, u_2\}$ and $E = \{x_1, x_2, x_3\}$ be the parameters. Let $A = \{x_1, x_2\} \subseteq E$ and $F_A = \{(x_1, \{u_1, u_2\}), (x_2, \{u_1, u_2\})\}$. Then $P(X_{F_A})$ are,

$$\begin{aligned} F_{1A} &= \{(x_1, \{u_1\})\}, F_{2A} = \{(x_1, \{u_2\})\}, F_{3A} = \{(x_1, \{u_1, u_2\})\}, F_{4A} = \{(x_2, \{u_1\})\}, \\ F_{5A} &= \{(x_2, \{u_2\})\}, F_{6A} = \{(x_2, \{u_1, u_2\})\}, F_{7A} = \{(x_1, \{u_1\}), (x_2, \{u_1\})\}, \\ F_{8A} &= \{(x_1, \{u_1\}), (x_2, \{u_2\})\}, F_{9A} = \{(x_1, \{u_2\}), (x_2, \{u_1\})\}, \\ F_{10A} &= \{(x_1, \{u_2\}), (x_2, \{u_2\})\}, F_{11A} = \{(x_1, \{u_1\}), (x_2, \{u_1, u_2\})\}, \\ F_{12A} &= \{(x_1, \{u_2\}), (x_2, \{u_1, u_2\})\}, F_{13A} = \{(x_1, \{u_1, u_2\}), (x_2, \{u_1\})\}, \\ F_{14A} &= \{(x_1, \{u_1, u_2\}), (x_2, \{u_2\})\}, F_{15A} = F_A, F_{16A} = \emptyset_A. \end{aligned}$$

An operator $k_1: P(X_{F_A}) \rightarrow P(X_{F_A})$ is defined from soft power set $P(X_{F_A})$ to itself over X as follows.

$$\begin{aligned} k_1(F_{1A}) &= F_{1A}, k_1(F_{2A}) = F_{2A}, k_1(F_{3A}) = F_{3A}, k_1(F_{4A}) = F_{4A}, k_1(F_{5A}) = F_{5A}, \\ k_1(F_{6A}) &= F_{6A}, k_1(F_{7A}) = F_{7A}, k_1(F_{8A}) = F_{8A}, k_1(F_{9A}) = F_{9A}, k_1(F_{10A}) = F_{10A}, \\ k_1(F_{11A}) &= F_{11A}, k_1(F_{12A}) = F_{12A}, k_1(F_{13A}) = F_{13A}, k_1(F_{14A}) = F_{14A}, \end{aligned}$$

$$k_1(F_A) = F_A, k_1(\emptyset_A) = \emptyset_A.$$

An operator $k_2: P(X_{F_A}) \rightarrow P(X_{F_A})$ is defined from soft power set $P(X_{F_A})$ to itself over X as follows.

$$\begin{aligned} k_2(F_{1A}) &= k_2(F_{2A}) = k_2(F_{3A}) = F_{3A}, k_2(F_{4A}) = k_2(F_{6A}) = F_{6A}, k_2(F_{5A}) = F_{5A}, \\ k_2(F_{7A}) &= k_2(F_{9A}) = k_2(F_{11A}) = k_2(F_{12A}) = k_2(F_{13A}) = k_2(F_A) = F_A, k_2(\emptyset_A) = \emptyset_A, \\ k_2(F_{8A}) &= k_2(F_{10A}) = k_2(F_{14A}) = F_{14A}. \end{aligned}$$

Taking, $U_A = F_{4A}$ and $V_A = F_{3A}$, $k_2[U_A] \cap k_1[V_A] = \emptyset_A$ and $k_2[U_A] \cup k_1[V_A] = F_A$.

Therefore, the soft biČech closure space (F_A, k_1, k_2) is pairwise disconnected.

Example 3.8. Let us consider the soft subsets of F_A that are given in *example 3.7*. An operator $k_1: P(X_{F_A}) \rightarrow P(X_{F_A})$ is defined from soft power set $P(X_{F_A})$ to itself over X as follows.

$$\begin{aligned} k_1(F_{1A}) &= k_1(F_{3A}) = k_1(F_{4A}) = k_1(F_{7A}) = k_1(F_{9A}) = k_1(F_{13A}) = F_{13A}, \\ k_1(F_{6A}) &= k_1(F_{8A}) = k_1(F_{11A}) = k_1(F_{12A}) = k_1(F_{14A}) = k_1(F_A) = F_A, k_1(\emptyset_A) = \emptyset_A. \\ k_1(F_{2A}) &= F_{9A}, k_1(F_{10A}) = F_{12A}, k_1(F_{5A}) = F_{5A}. \end{aligned}$$

An operator $k_2: P(X_{F_A}) \rightarrow P(X_{F_A})$ is defined from soft power set $P(X_{F_A})$ to itself over X as follows.

$$\begin{aligned} k_2(F_{1A}) &= k_2(F_{7A}) = k_2(F_{8A}) = k_2(F_{11A}) = F_{11A}, \\ k_2(F_{4A}) &= k_2(F_{5A}) = k_2(F_{6A}) = F_{6A}, k_2(F_{2A}) = F_{10A}, \\ k_2(F_{9A}) &= k_2(F_{10A}) = k_2(F_{12A}) = F_{12A}, k_2(\emptyset_A) = \emptyset_A, \\ k_2(F_{3A}) &= k_2(F_{13A}) = k_2(F_{14A}) = k_2(F_A) = F_A. \end{aligned}$$

Here, the soft biČech closure space (F_A, k_1, k_2) is pairwise connected.

Remark 3.9. The following example shows that pairwise connectedness in soft biČech closure space does not preserves hereditary property.

Example 3.10. In *example 3.8.*, the soft biČech closure space (F_A, k_1, k_2) is pairwise connected. Consider (G_A, k_1^*, k_2^*) be the soft subspace of F_A such that $G_A = \{(x_1, \{u_1, u_2\})\}$. Taking, $U_A = \{(x_1, \{u_1\})\}$ and $V_A = \{(x_1, \{u_2\})\}$, $k_2^*[U_A] \cap k_1^*[V_A] = \emptyset_A$ and $k_2^*[U_A] \cup k_1^*[V_A] = G_A$. Therefore, the soft biČech closure subspace (G_A, k_1^*, k_2^*) is pairwise disconnected.

Theorem 3.11. Pairwise connectedness in soft bitopological space (F_A, τ_1, τ_2) need not imply that the soft biČech closure space (F_A, k_1, k_2) is pairwise connected.

Proof. Let us consider the soft subsets of F_A that are given in *example 3.7*. An operator $k_1(F_{1A}) = F_{1A}, k_1(F_{2A}) = k_1(F_{9A}) = F_{12A}, k_1(F_{4A}) = F_{4A}, k_1(F_{5A}) = k_1(F_{8A}) = F_{14A}, k_1(F_{7A}) = F_{7A}, k_1(F_{3A}) = k_1(F_{6A}) = k_1(F_{10A}) = k_1(F_{11A}) = k_1(F_{12A}) = k_1(F_{13A}) = k_1(F_{14A}) = k_1(F_A) = F_A, k_1(\emptyset_A) = \emptyset_A.$

An operator $k_2: P(X_{F_A}) \rightarrow P(X_{F_A})$ is defined from soft power set $P(X_{F_A})$ to itself over X as follows.

$$\begin{aligned} k_2(F_{1A}) &= k_2(F_{5A}) = F_{8A}, k_2(F_{2A}) = F_{3A}, k_2(F_{4A}) = F_{4A}, k_2(F_{7A}) = F_{7A}, \\ k_2(F_{6A}) &= k_2(F_{8A}) = k_2(F_{11A}) = F_{11A}, k_2(F_{3A}) = k_2(F_{9A}) = k_2(F_{13A}) = F_{13A}, \\ k_2(F_{10A}) &= F_{14A}, k_2(F_{12A}) = k_2(F_{14A}) = k_2(F_A) = F_A, k_2(\emptyset_A) = \emptyset_A. \end{aligned}$$

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Here, the two non empty disjoint soft subsets $U_A = \{(x_2, \{u_1\})\}$, and $V_A = \{(x_2, \{u_2\})\}$, satisfies $k_2[U_A] \cap k_1[V_A] = \emptyset_A$ and $k_2[U_A] \cup k_1[V_A] = F_A$. Therefore, the soft biČech closure space (F_A, k_1, k_2) is pairwise disconnected. But, it's associated soft bitopological space (F_A, τ_1, τ_2) is $\tau_1 = \{\emptyset_A, F_{10A}, F_{12A}, F_{14A}, F_A\}$ and $\tau_2 = \{\emptyset_A, F_{2A}, F_{5A}, F_{10A}, F_{14A}, F_A\}$. Now, $[U_A \cap \tau_1 - cl(V_A)] \cup [\tau_2 - cl(U_A) \cap V_A] = [\{(x_2, \{u_1\})\} \cap F_A] \cup [\{(x_2, \{u_1\})\} \cap \{(x_2, \{u_2\})\}] = \{(x_2, \{u_1\})\} \cup \emptyset_A \neq \emptyset_A$. Therefore, (F_A, τ_1, τ_2) is pairwise connected.

Theorem 3.12. If soft biČech closure space is pairwise disconnected such that $F_A = k_2[U_A]/k_1[V_A]$ and let G_A be a pairwise connected soft subset of F_A then G_A need not to be holds the following conditions (i) $G_A \subseteq k_2[U_A]$ (ii) $G_A \subseteq k_1[V_A]$.

Proof. Let us consider the soft subsets of F_A that are given in *example 3.7*. An operator $k_1: P(X_{F_A}) \rightarrow P(X_{F_A})$ is defined from soft power set $P(X_{F_A})$ to itself over X as follows.
 $k_1(F_{1A}) = k_1(F_{5A}) = F_{8A}, k_1(F_{2A}) = F_{3A}, k_1(F_{4A}) = F_{4A}, k_1(F_{7A}) = F_{7A},$
 $k_1(F_{6A}) = k_1(F_{8A}) = k_1(F_{11A}) = F_{11A}, k_1(F_{3A}) = k_1(F_{9A}) = k_1(F_{13A}) = F_{13A},$
 $k_1(F_{10A}) = F_{14A}, k_1(F_{12A}) = k_1(F_{14A}) = k_1(F_A) = F_A, k_1(\emptyset_A) = \emptyset_A.$

An operator $k_2: P(X_{F_A}) \rightarrow P(X_{F_A})$ is defined from soft power set $P(X_{F_A})$ to itself over X as follows.

$k_2(F_{1A}) = k_2(F_{3A}) = k_2(F_{4A}) = k_2(F_{7A}) = k_2(F_{9A}) = k_2(F_{13A}) = F_{13A},$
 $k_2(F_{6A}) = k_2(F_{8A}) = k_2(F_{11A}) = k_2(F_{12A}) = k_2(F_{14A}) = k_2(F_A) = F_A, k_2(\emptyset_A) = \emptyset_A.$
 $k_2(F_{2A}) = F_{9A}, k_2(F_{10A}) = F_{12A}, k_2(F_{5A}) = F_{5A}.$

Taking, $U_A = F_{2A}$ and $V_A = F_{5A}$ then we get, $F_A = k_2[U_A]/k_1[V_A]$.

Here, the soft biČech closure space (F_A, k_1, k_2) is pairwise disconnected. Let $G_A = F_{7A}$ be the pairwise connected soft subset of F_A . Clearly, G_A does not lie entirely within either $k_2[U_A]$ or $k_1[V_A]$.

Theorem 3.13. If the soft bitopological space (F_A, τ_1, τ_2) is pairwise disconnected then the soft biČech closure space (F_A, k_1, k_2) is also pairwise disconnected.

Proof. Let the soft bitopological space (F_A, τ_1, τ_2) is pairwise disconnected, implies that it is the union of two non empty disjoint soft subsets U_A and V_A such that $[U_A \cap \tau_1 - cl(V_A)] \cup [\tau_2 - cl(U_A) \cap V_A] = \emptyset_A$. Since, $k_{i=1,2}[U_A] \subset \tau_{i=1,2} - cl(U_A)$ for every $U_A \subset F_A$ and $\tau_2 - cl(U_A) \cap \tau_1 - cl(V_A) = \emptyset_A$ then $k_2[U_A] \cap k_1[V_A] = \emptyset_A$. Since, $U_A \cup V_A = F_A$, $U_A \subseteq k_2[U_A]$ and $V_A \subseteq k_1[V_A]$ implies that $U_A \cup V_A \subseteq k_2[U_A] \cup k_1[V_A]$, $F_A \subseteq k_2[U_A] \cup k_1[V_A]$. But, $k_2[U_A] \cup k_1[V_A] \subseteq F_A$. Therefore, $k_2[U_A] \cup k_1[V_A] = F_A$. Hence, (F_A, k_1, k_2) is also pairwise disconnected.

Definition 3.14. A soft biČech closure space (F_A, k_1, k_2) is said to be pairwise feebly disconnected if it can be written as two non-empty disjoint soft subsets U_A and V_A such that $U_A \cap k_1[V_A] = k_2[U_A] \cap V_A = \emptyset_A$ and $U_A \cup k_1[V_A] = k_2[U_A] \cup V_A = F_A$.

Result 3.15. Every pairwise disconnected soft biČech closure space (F_A, k_1, k_2) is pairwise feebly disconnected but the following example shows that the converse is not true.

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Example 3.16. In *example 3.8* Consider, $U_A = F_{8A} = \{(x_1, \{u_1\}), (x_2, \{u_2\})\}$ and $V_A = F_{2A} = \{(x_1, \{u_2\})\}$. Which satisfies the condition $U_A \cap k_1[V_A] = k_2[U_A] \cap V_A = \emptyset_A$ $U_A \cup k_1[V_A] = k_2[U_A] \cup V_A = F_A$. Therefore, the soft biČech closure space (F_A, k_1, k_2) is pairwise feebly disconnected. But, the soft biČech closure space (F_A, k_1, k_2) is pairwise connected.

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