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Generalization of Szpilrajn's Theorem on Intuitionistic Fuzzy Matrix

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Abstract. In this paper, intuitionistic fuzzy matrices are considered. Szpilrajn's theorem on orderings is generalized to intuitionistic fuzzy orderings by Zedam et al. We give another generalization in intuitionistic fuzzy matrix form and theorem is represented in terms of intuitionistic fuzzy matrix operations.

Keywords: Intuitionistic fuzzy sets, intuitionistic fuzzy matrix, generalization

AMS Mathematics Subject Classification (2010): 03E72, 15B15

1. Introduction

In real life scenario, we frequently deal with the information which is sometimes vague, sometimes inexact or imprecise and occasionally insufficient. Zadeh's classical concept of fuzzy sets [1] is strong enough to deal with such type of problems. In fuzzy set theory, the membership of an element to a fuzzy set is a single value between zero and one. However in reality, it may not be always true that the degree of non-membership of an element in a fuzzy set is equal to 1 minus the membership degree because there may be some hesitation degree. To overcome these difficulties, Atanassov [2,3] developed the theory of intuitionistic fuzzy sets (IFSs) as a generalization of fuzzy sets. Lot of research work has been done by several researchers on the field of IFSs.

Matrices play a vital role in various areas of Science and Engineering. The classical matrix theory cannot solve the problems involving various types of uncertainties. These types of problems are solved by using Fuzzy Matrix (FM) [4]. Kim and Roush [5] developed the concept of Generalized Fuzzy Matrices (GFMs). The Szpilrajn theorem is a very well-known result. Zadeh [6] studied similarity relations and fuzzy orderings. Hashimoto [7] studied Szpilrajn theorem for fuzzy matrix. Hashimoto [9] presented the concept of transitive FMs and considered the convergence of powers of transitive FMs. Hashimoto [10] studied the canonical form of a transitive FM. Xin [14,15] studied controllable FMs. Kołodziejczy [11] gave the concept of s-transitive FMs and considered the convergence of powers of s-transitive FMs. Tan [12,13] discussed the convergence of powers of transitive lattice matrices. Jiang [16] studied the transitive incline matrices. Some elementary properties and characterizations for transitive GFMs are established and transitivity of powers of a GFM was discussed [17]. Zedam, et al., [8] studied Szpilrajn theorem on intuitionistic fuzzy orderings. Pal [18] introduced intuitionistic fuzzy determinant. Pal, et al., [19] studied intuitionistic fuzzy matrices (IFMs). Khan and Pal

[20] studied on intuitionistic fuzzy tautological matrices and also studied interval-valued IFMs [21]. Bhowmik and Pal [22,23] introduced some results on IFMs and intuitionistic circulant FMs and GIFMs. Khan and Pal [24] introduced the concept of generalized inverse for IFMs. Recently, Pal [40,41] introduced new type fuzzy matrices. Hong and Nae [25] studied some properties of canonical form of transitive IFM. Some algebraic properties of GIFMs are presented over distributive lattice [26]. Some results are investigated regarding the group inverse of IFMs [27]. Several authors [28-39] worked on IFMs and obtained various interesting results which are very useful in handling uncertainty problems in our daily life. Szpilrajn theorem is an interesting problem on which many authors have worked. We generalize Szpilrajn theorem on intuitionistic fuzzy matrix.

2. Preliminaries

In this section, some preliminaries are given.

Definition 2.1. [2] An intuitionistic fuzzy set (IFS) A in X (universal set) is defined as an object of the following form $A=\{\langle x,\mu_A(x),\nu_A(x)\rangle / x\in X\}$, where the functions: $\mu_A : X\to[0,1]$ and $\nu_A : X\to[0,1]$ define the membership function and non-membership function of the element $x\in X$ respectively and for every $x\in X: 0\le\mu_A(x)+\nu_A(x)\le 1.$

Atanassov, introduced operations $\langle x,x'\rangle\vee\langle y,y'\rangle=\langle max\{x,y\},min\{x',y'\}\rangle$ and $\langle x,x'\rangle\wedge\langle y,y'\rangle=\langle min\{x,y\},max\{x',y'\}\rangle$. Moreover, the operation $\langle x,x'\rangle\leftharpoondown\langle y,y'\rangle$ defined by

$$\langle x,x'\rangle\leftharpoondown\langle y,y'\rangle=\begin{cases}\langle x,x'\rangle, & \text{if}\ \langle x,x'\rangle>\langle y,y'\rangle\\ \langle 0,1\rangle, & \text{if}\ \langle x,x'\rangle\le\langle y,y'\rangle\end{cases} \tag{1}$$

Definition 2.2. [19] Let $X=\{x_1,x_2,...x_m\}$ be a set of alternatives and $Y=\{y_1,y_2,...y_n\}$ be the attribute set of each element of X. An Intuitionistic Fuzzy Matrix (IFM) is defined by $A=(\langle(x_i,y_j),\mu_A(x_i,y_j),\nu_A(x_i,y_j)\rangle)$ for $i=1,2...m$ and $j=1,2,...n,$ where $\mu_A : X\times Y\to[0,1]$ and $\nu_A : X\times Y\to[0,1]$ satisfy the condition $0\le\mu_A(x_i,y_j)+\nu_A(x_i,y_j)\le 1.$ For simplicity we denote an intuitionistic fuzzy matrix (IFM) as a matrix of pairs $A=(\langle a_{ij},a_{ij'}\rangle)$ of a non negative real numbers satisfying $a_{ij}+a_{ij'}\le 1$ for all i, j. We denote the set of all IFM of order $m\times n$ by $\mathcal{F}_{mn}$.

For $n\times n$ IFMs $R=(\langle r_{ij},r'_{ij}\rangle)$ and $S=(\langle s_{ij},s'_{ij}\rangle)$ with their elements having values in the unit interval [0, 1], the following notations are well known:

$R\vee S=(\langle r_{ij}\vee s_{ij},r_{ij'}\wedge s_{ij'}\rangle)$

$R\wedge S=(\langle q_{ij}\wedge s_{ij},q_{ij'}\vee s_{ij'}\rangle)$

$R\overset{c}{\leftharpoondown}S=(\langle r_{ij},r'_{ij}\rangle\overset{c}{\leftharpoondown}\langle s_{ij},s_{ij'}\rangle)$ (Component wise)

$R \times S = ((\langle r_{i1} \wedge s_{1j}, r'_{i1} \vee s_{1j'} \rangle) \vee (\langle r_{i2} \wedge s_{2j}, r_{i2'} \vee s_{2j'} \rangle) \vee \ldots \vee (\langle r_{in} \wedge s_{nj}, r_{in'} \vee s_{nj'} \rangle))$,

$R^0 = I = (\langle \delta_{ij}, \delta'_{ij} \rangle)$ ($\langle \delta_{ij}, \delta'_{ij} \rangle$ is the Kronecker delta),

$R^{k+1} = R^k \times R,$

$R^+ = R \vee R^2 \vee \ldots \vee R^n$

$R^T = (\langle r_{ji}, r'_{ji} \rangle)$ (Transpose of R)

$\Delta R = R \overset{c}{\leftharpoondown} R^T$,

$\nabla R = R \wedge R^T$,

$R \leq S$ iff $r_{ij} \leq s_{ij}, r'_{ij} \geq s'_{ij}$ for all $i, j \in (1,2,3,..,n)$

$R < S \Rightarrow$ either $r_{ij} < s_{ij}$, and $r'_{ij} > s'_{ij}$ for all $i, j \in (1,2,3,..,n)$

$R^k \leq R^m$ iff $(\langle r^k_{ij}, r'^k_{ij} \rangle \leq \langle r^m_{ij}, r'^m_{ij} \rangle$ for all $i, j \in (1,2,3,..,n)$

The IFM R is called max-min transitive if $R^2 \leq R$, convergent if $R^k = R^{k+1}$ for some positive integer k, symmetric if $R = R^T$, idempotent if $R^2 = R$, nilpotent if $R^n = (\langle 0,1 \rangle)$, reflexive if $R \geq I_n$, then R where I_n the $n \times n$ identity IFM and irreflexive if all the diagonal entries are $\langle 0,1 \rangle$. The Zero matrix O is the matrix in which all the entries are $\langle 0,1 \rangle$.

3. Results

Theorem 3.1. Let $R = (\langle r_{ij}, r'_{ij} \rangle)(\langle 0,1 \rangle \leq \langle r_{ij}, r'_{ij} \rangle \leq \langle 1,0 \rangle)$ be $n \times n$ intuitionistic fuzzy transitive matrix, and let $p \neq q$ be integers such that $\langle 1,0 \rangle \geq \langle r_{pq}, r'_{pq} \rangle \geq \langle r_{qp}, r'_{qp} \rangle$. We define an $n \times n$ IFM, $T = (\langle t_{ij}, t'_{ij} \rangle)$ whose elements $\langle t_{ij}, t'_{ij} \rangle$ are given by

$$\langle t_{ij}, t'_{ij} \rangle = \begin{cases} \langle \mathrm{b}, \mathrm{b}' \rangle, > \langle r_{pq}, r'_{pq} \rangle & \text{if } (i,\ j) = (\mathrm{p}, \mathrm{q}), \\ \langle t_{ij}, t'_{ij} \rangle, & \text{if } (i,\ j) \neq (\mathrm{p}, \mathrm{q}), \end{cases} \tag{2}$$

where $\langle b, b' \rangle \in [0,1] \times [0,1]$. Then an $n \times n$ IFM S is defined by

$S = (\langle s_{ij}, s'_{ij} \rangle = T^+$ fulfills the following conditions.

(1) $S^2 \leq S$

(2) $R \leq S$

(3) $\Delta R \leq \Delta S$

(4) $\nabla R = \nabla S$,

(5) $\langle s_{pq}, s'_{pq} \rangle = \langle b, b' \rangle$ and $\langle s_{qp}, s'_{qp} \rangle = \langle r_{qp}, r'_{qp} \rangle$

Proof. (1) Since $(T^+)^2 \leq T^+$, we have $S^2 \leq S$.

(2) $R \leq T \leq T^+ = S$

(3) Suppose that $\langle r_{ij}, r'_{ij} \rangle \leftharpoondown \langle r_{ji}, r'_{ji} \rangle > \langle s_{ij}, s'_{ij} \rangle \leftharpoondown \langle s_{ji}, s'_{ji} \rangle$.

Let $\langle c, c' \rangle = \langle r_{ij}, r'_{ij} \rangle \leftharpoondown \langle r_{ji}, r'_{ji} \rangle$. Then

$\langle r_{ij}, r'_{ij}\rangle = \langle c, c'\rangle > \langle 0,1\rangle$, $\langle r_{ji}, r'_{ji}\rangle < \langle c, c'\rangle$, and $\langle s_{ji}, s'_{ji}\rangle \geq \langle c, c'\rangle$.

Therefore, there exist integers $l_1, l_2, ..., l_m \in \{1, 2, ..., n\}$ such that

$\langle t_{jl_1}, t'_{jl_1}\rangle \wedge \langle t_{l_1l_2}, t'_{l_1l_2}\rangle \wedge ... \wedge \langle t_{l_mi}, t'_{l_mi}\rangle \geq \langle c, c'\rangle$, where $m \leq n-1$.

Without loss of generality suppose that the integers $j, l_1, l_2, ..., l_n, i$ are distinct from each other. Let $l_0 = j$ and $l_{m+1} = i$. If $l_\alpha = p$ and $l_{\alpha+1} = q$ for some α, then since $\langle r_{ij}, r'_{ij}\rangle = \langle c, c'\rangle$, we have $\langle r_{qp}, r'_{qp}\rangle \geq \langle c, c'\rangle$, so that $\langle r_{pq}, r'_{pq}\rangle \geq \langle c, c'\rangle$. Then $\langle r_{ij}, r'_{ij}\rangle \geq \langle c, c'\rangle$, which contradicts the fact that $\langle r_{ji}, r'_{ji}\rangle < \langle c, c'\rangle$. Furthermore, if $l_a \neq p$ or $l_{\alpha+1} \neq q$ for every α, then $\langle r_{ji}, r'_{ji}\rangle \geq \langle c, c'\rangle$, which contradicts the fact that $\langle r_{ji}, r'_{ji}\rangle < \langle c, c'\rangle$. Hence we have,

$\langle r_{ij}, r'_{ij}\rangle \leftharpoondown \langle r_{ji}, r'_{ji}\rangle \leq \langle s_{ij}, s'_{ij}\rangle \leftharpoondown \langle s_{ji}, s'_{ji}\rangle$,

that is $\Delta R \leq \Delta S$.

(4) Since $\nabla R \leq \nabla S$ by $R \leq S$, it is sufficient to show that $\nabla S \leq \nabla R$.

Let $\langle c, c'\rangle = \langle s_{ij}, s'_{ij}\rangle \wedge \langle s_{ij}, s'_{ij}\rangle > \langle 0,1\rangle$. Then there exist integers

$i, k_1, k_2, ..., k_m, h_1, h_2, ..., h_l \in \{1, 2, ..., n\} (m, l \leq n-1)$ such that

$\langle t_{ik_1}, t'_{ik_1}\rangle \wedge \langle t_{k_1k_2}, t'_{l_1l_2}\rangle \wedge ... \wedge \langle t_{k_mj}, t'_{k_mj}\rangle \wedge \langle t_{jh_1}, t'_{jh_1}\rangle \wedge \langle t_{h_1h_2}, t'_{h_1h_2}\rangle \wedge ... \wedge \langle t_{h_li}, t'_{h_li}\rangle \geq \langle c, c'\rangle$.

Without loss of generality supposes that the integers $i, k_1, k_2, ..., k_m$ are distinct from each other and that $h_1, h_2, ... h_l, i$ are distinct from each other. Let $k_0 = h_{l+1} = i$ and $k_{m+1} = h_0 = j$.

(a) Assume that $(k_\alpha, k_{\alpha+1}) = (p, q)$ for some α and $(h_\beta, h_{\beta+1}) = (p, q)$ for every β. Then since for every α, β

$\langle r_{h_{\beta+1}h_{\beta+2}}, r'_{h_{\beta+1}h_{\beta+2}}\rangle \wedge ... \wedge \langle r_{h_1i}, r'_{h_1i}\rangle \wedge \langle r_{ik_1}, r'_{ik_1}\rangle ... \wedge \langle r_{k_{\alpha-1}k_\alpha}, r'_{k_{\alpha-1}k_\alpha}\rangle \geq \langle c, c'\rangle$,

we have $\langle r_{qp}, r'_{qp}\rangle \geq \langle c, c'\rangle$, so that $\langle r_{pq}, r'_{pq}\rangle \geq \langle c, c'\rangle$, . Then $\langle r_{ij}, r'_{ij}\rangle \wedge \langle r_{ji}, r'_{ji}\rangle \geq \langle c, c'\rangle$.

(b) Assume that $(k_\alpha, k_{\alpha+1}) = (p, q)$ for some α and $(h_\beta, h_{\beta+1}) \neq (p, q)$ for every β. Then we have $\langle r_{qp}, r'_{qp}\rangle \geq \langle c, c'\rangle$, so that

$\langle r_{pq}, r'_{pq}\rangle \geq \langle c, c'\rangle$, . Then $\langle r_{ij}, r'_{ij}\rangle \wedge \langle r_{ji}, r'_{ji}\rangle \geq \langle c, c'\rangle$.

(5) Since $T \leq S$, we have $\langle s_{pq}, s'_{pq}\rangle \geq \langle b, b'\rangle$. If $\langle s_{pq}, s'_{pq}\rangle > \langle b, b'\rangle$, then there exist integers $k_1, k_2, ..., k_m \in \{1, 2, ..., n\} (m \geq 1)$ such that

$\langle t_{pk_1}, t'_{pk_1}\rangle \wedge \langle t_{k_1,k_2}, t'_{k_1,k_2}\rangle \wedge ... \wedge \langle t_{k_mq}, t'_{k_mq}\rangle \geq \langle b, b'\rangle$.

Without loss of generality assume that integers $p, k_1, k_2, ... k_m, q$ are distinct from each other. Therefore $\langle r_{pq}, r'_{pq}\rangle > \langle b, b'\rangle$, which contradicts the fact that $\langle r_{pq}, r'_{pq}\rangle < \langle b, b'\rangle$.

Hence we have $\langle s_{pq}, s'_{pq}\rangle = \langle b, b'\rangle$. Now since $\langle r_{pq}, r'_{pq}\rangle \wedge \langle r_{qp}, r'_{qp}\rangle = \langle r_{qp}, r'_{qp}\rangle$ by (4) we have $\langle s_{pq}, s'_{pq}\rangle \wedge \langle s_{qp}, s'_{qp}\rangle = \langle r_{qp}, r'_{qp}\rangle$.
Hence $\langle s_{qp}, s'_{qp}\rangle = \langle r_{qp}, r'_{qp}\rangle$.

Corollary 3.2. If R is transitive, then there exists a matrix S such that
(1) $S^2 \le S$
(2) $R \le S$
(3) $\Delta R \le \Delta S$
(4) $\nabla R = \nabla S$,
(5) $(\langle 0,1\rangle) < I \vee S \vee S^T$.
Proof. (1)-(4) it is evident from Theorem 3.1.
(5) if $\langle r_{pq}, r'_{pq}\rangle = \langle r_{qp}, r'_{qp}\rangle = \langle 0,1\rangle (p \ne q)$, we can construct a matrix S such that $\langle s_{pq}, s'_{pq}\rangle = \langle b, b'\rangle > \langle 0,1\rangle$ and $\langle s_{qp}, s'_{qp}\rangle = \langle 0,1\rangle$ applying the Theorem 3.1. Repeating this process, we get
$(\langle 0,1\rangle) < I \vee S \vee S^T$.
Example 3.3. Let

$$R = \begin{pmatrix} \langle 0.2, 0.7\rangle & \langle 0.4, 0.5\rangle & \langle 0.3, 0.6\rangle \\ \langle 0.1, 0.8\rangle & \langle 0.1, 0.8\rangle & \langle 0.2, 0.7\rangle \\ \langle 0.0, 1.0\rangle & \langle 0.0, 1.0\rangle & \langle 0.0, 1.0\rangle \end{pmatrix}$$

Then

$$R^2 = \begin{pmatrix} \langle 0.2, 0.7\rangle & \langle 0.4, 0.5\rangle & \langle 0.3, 0.6\rangle \\ \langle 0.1, 0.8\rangle & \langle 0.1, 0.8\rangle & \langle 0.2, 0.7\rangle \\ \langle 0.0, 1.0\rangle & \langle 0.0, 1.0\rangle & \langle 0.0, 1.0\rangle \end{pmatrix} \times \begin{pmatrix} \langle 0.2, 0.7\rangle & \langle 0.4, 0.5\rangle & \langle 0.3, 0.6\rangle \\ \langle 0.1, 0.8\rangle & \langle 0.1, 0.8\rangle & \langle 0.2, 0.7\rangle \\ \langle 0.0, 1.0\rangle & \langle 0.0, 1.0\rangle & \langle 0.0, 1.0\rangle \end{pmatrix}$$

$$= \begin{pmatrix} \langle 0.2, 0.7\rangle & \langle 0.2, 0.7\rangle & \langle 0.2, 0.7\rangle \\ \langle 0.1, 0.8\rangle & \langle 0.1, 0.8\rangle & \langle 0.1, 0.8\rangle \\ \langle 0.0, 1.0\rangle & \langle 0.0, 1.0\rangle & \langle 0.0, 1.0\rangle \end{pmatrix} \le R,$$

$$\Delta R = R \overset{c}{\leftarrow} R^T = \begin{pmatrix} \langle 0.0, 1.0\rangle & \langle 0.4, 0.5\rangle & \langle 0.3, 0.6\rangle \\ \langle 0.0, 1.0\rangle & \langle 0.0, 1.0\rangle & \langle 0.2, 0.7\rangle \\ \langle 0.0, 1.0\rangle & \langle 0.0, 1.0\rangle & \langle 0.0, 1.0\rangle \end{pmatrix} \le R,$$

$$\Delta R = R \vee R^T = \begin{pmatrix} \langle 0.2, 0.7\rangle & \langle 0.1, 0.8\rangle & \langle 0.0, 1.0\rangle \\ \langle 0.1, 0.8\rangle & \langle 0.1, 0.8\rangle & \langle 0.0, 1.0\rangle \\ \langle 0.0, 1.0\rangle & \langle 0.0, 1.0\rangle & \langle 0.0, 1.0\rangle \end{pmatrix} \le R,$$

Setting $p = 2$, $q = 3$, and $\langle b, b'\rangle = \langle 0.4, 0.5\rangle$ we construct a matrix $T = \langle t_{ij}, t_{ij}\rangle$ as follows
Thus

$$T=\begin{pmatrix}\langle 0.2,0.7\rangle & \langle 0.4,0.5\rangle & \langle 0.3,0.6\rangle\\ \langle 0.1,0.8\rangle & \langle 0.1,0.8\rangle & \langle 0.4,0.5\rangle\\ \langle 0.0,1.0\rangle & \langle 0.0,1.0\rangle & \langle 0.0,1.0\rangle\end{pmatrix}$$

$$T^2=\begin{pmatrix}\langle 0.2,0.7\rangle & \langle 0.4,0.5\rangle & \langle 0.3,0.6\rangle\\ \langle 0.1,0.8\rangle & \langle 0.1,0.8\rangle & \langle 0.4,0.5\rangle\\ \langle 0.0,1.0\rangle & \langle 0.0,1.0\rangle & \langle 0.0,1.0\rangle\end{pmatrix}\times\begin{pmatrix}\langle 0.2,0.7\rangle & \langle 0.4,0.5\rangle & \langle 0.3,0.6\rangle\\ \langle 0.1,0.8\rangle & \langle 0.1,0.8\rangle & \langle 0.4,0.5\rangle\\ \langle 0.0,1.0\rangle & \langle 0.0,1.0\rangle & \langle 0.0,1.0\rangle\end{pmatrix}$$

$$=\begin{pmatrix}\langle 0.2,0.7\rangle & \langle 0.2,0.7\rangle & \langle 0.4,0.5\rangle\\ \langle 0.1,0.8\rangle & \langle 0.1,0.8\rangle & \langle 0.1,0.8\rangle\\ \langle 0.0,1.0\rangle & \langle 0.0,1.0\rangle & \langle 0.0,1.0\rangle\end{pmatrix}$$

$$T^3=\begin{pmatrix}\langle 0.2,0.7\rangle & \langle 0.4,0.5\rangle & \langle 0.3,0.6\rangle\\ \langle 0.1,0.8\rangle & \langle 0.1,0.8\rangle & \langle 0.4,0.5\rangle\\ \langle 0.0,1.0\rangle & \langle 0.0,1.0\rangle & \langle 0.0,1.0\rangle\end{pmatrix}\times\begin{pmatrix}\langle 0.2,0.7\rangle & \langle 0.2,0.7\rangle & \langle 0.4,0.5\rangle\\ \langle 0.1,0.8\rangle & \langle 0.1,0.8\rangle & \langle 0.1,0.8\rangle\\ \langle 0.0,1.0\rangle & \langle 0.0,1.0\rangle & \langle 0.0,1.0\rangle\end{pmatrix}$$

$$=\begin{pmatrix}\langle 0.2,0.7\rangle & \langle 0.2,0.7\rangle & \langle 0.2,0.7\rangle\\ \langle 0.1,0.8\rangle & \langle 0.1,0.8\rangle & \langle 0.1,0.8\rangle\\ \langle 0.0,1.0\rangle & \langle 0.0,1.0\rangle & \langle 0.0,1.0\rangle\end{pmatrix}$$

$$S=T^+=T\vee T^2\vee T^3=\begin{pmatrix}\langle 0.2,0.7\rangle & \langle 0.4,0.5\rangle & \langle 0.4,0.5\rangle\\ \langle 0.1,0.8\rangle & \langle 0.1,0.8\rangle & \langle 0.4,0.5\rangle\\ \langle 0.0,1.0\rangle & \langle 0.0,1.0\rangle & \langle 0.0,1.0\rangle\end{pmatrix}.$$

$$S^2=\begin{pmatrix}\langle 0.2,0.7\rangle & \langle 0.4,0.5\rangle & \langle 0.4,0.5\rangle\\ \langle 0.1,0.8\rangle & \langle 0.1,0.8\rangle & \langle 0.4,0.5\rangle\\ \langle 0.0,1.0\rangle & \langle 0.0,1.0\rangle & \langle 0.0,1.0\rangle\end{pmatrix}\times\begin{pmatrix}\langle 0.2,0.7\rangle & \langle 0.4,0.5\rangle & \langle 0.4,0.5\rangle\\ \langle 0.1,0.8\rangle & \langle 0.1,0.8\rangle & \langle 0.4,0.5\rangle\\ \langle 0.0,1.0\rangle & \langle 0.0,1.0\rangle & \langle 0.0,1.0\rangle\end{pmatrix}$$

$$=\begin{pmatrix}\langle 0.2,0.7\rangle & \langle 0.2,0.7\rangle & \langle 0.4,0.5\rangle\\ \langle 0.1,0.8\rangle & \langle 0.1,0.8\rangle & \langle 0.1,0.8\rangle\\ \langle 0.0,1.0\rangle & \langle 0.0,1.0\rangle & \langle 0.0,1.0\rangle\end{pmatrix}$$

$$\Delta S=\begin{pmatrix}\langle 0.2,0.7\rangle & \langle 0.4,0.5\rangle & \langle 0.4,0.5\rangle\\ \langle 0.0,1.0\rangle & \langle 0.0,1.0\rangle & \langle 0.4,0.5\rangle\\ \langle 0.0,1.0\rangle & \langle 0.0,1.0\rangle & \langle 0.0,1.0\rangle\end{pmatrix}$$

$$\nabla S=\begin{pmatrix}\langle 0.2,0.7\rangle & \langle 0.1,0.8\rangle & \langle 0.0,1.0\rangle\\ \langle 0.1,0.8\rangle & \langle 0.1,0.8\rangle & \langle 0.1,0.8\rangle\\ \langle 0.0,1.0\rangle & \langle 0.0,1.0\rangle & \langle 0.0,1.0\rangle\end{pmatrix}$$

Therefore,

$S^2\le S$, $R\le S$, $\Delta R\le \Delta S$, and $\nabla R=\nabla S$

4. Conclusion

By generalization of Hashimoto's representation the results so obtained are assumed to be valuable in discussing the intuitionistic fuzzy preferences. The results provide a structure in detail to Szpilrajn theorem on intuitionistic fuzzy matrix which is an extension of intuitionistic fuzzy orderings. The advantage of representing Szpilrajn theorem on intuitionistic fuzzy matrix form is clear and compact.

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