

Common Fixed Point Theorems for Weakly Compatible Mappings using Common Property (E.A) in Intuitionistic Fuzzy Metric Space of Integral Type

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Abstract. The aim of present paper is to prove common fixed point theorems for four self mappings in intuitionistic fuzzy metric spaces using the common property (E. A.) satisfying an implicit relation of integral type.

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1. Introduction

Since the concept of fuzzy sets was introduced by Zadeh [16], there are many study results in this area. Some of them are dedicated to generalize the definition of fuzzy set. Atanassov [4] introduce the concept of intuitionistic fuzzy sets, Park [12] defined and studied a notion of intuitionistic fuzzy metric spaces, as a natural generalization of fuzzy metric spaces due to Kromosil, Michalek [9], George, Veeramani [7]. Amari and Moutawakli [1] and Liu et al. [10] respectively, define the property (E. A.) and common property (E. A.) and utilize the same to prove common fixed point theorems in metric spaces. Branciari [5] gave a fixed point result for a single mapping satisfying Banach's contraction principle for an integral type inequality. The authors [3, 5, 6, 14] proved fixed point theorems using generalized contractive conditions of integral type. In this paper the concept of implicit relation has been used for establishing various common fixed point results of integral type in intuitionistic fuzzy metric spaces. This concept plays a vital rule in the proof of the main results.

2. Basic definitions and preliminaries

Definition 2.1.[13] A binary operation $*$: $[0,1] \times [0,1] \rightarrow [0,1]$ is called a *t-norm* $*$ satisfies the following conditions:

- i. $*$ is continuous,
- ii. $*$ is commutative and associative,
- iii. $a * 1 = a$ for all $a \in [0, 1]$,

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- iv. $a * b \leq c * d$ whenever $a \leq c$ and $b \leq d$ for all $a, b, c, d \in [0,1]$.

Examples of t -norm - $a * b = ab$ and $a * b = \min\{a, b\}$.

Definition 2.2. [13] A binary operation $\diamond: [0,1] \times [0,1] \rightarrow [0,1]$ is said to be continuous t -conorm if it satisfied the following conditions:

- i. \diamond is associative and commutative,
- ii. $a \diamond 0 = a$ for all $a \in [0,1]$,
- iii. \diamond is continuous,
- iv. $a \diamond b \leq c \diamond d$ whenever $a \leq c$ and $b \leq d$ for each $a, b, c, d \in [0,1]$

Examples of t -conorm - $a \diamond b = \min(a+b, 1)$ and $a \diamond b = \max(a, b)$

Remark 2.1. [2] The concept of triangular norms (t -norm) and triangular conorms (t -conorm) are known as axiomatic skeletons that we use for characterizing fuzzy intersections and union respectively.

Definition 2.3. [2] A 5-tuple $(X, M, N, *, \diamond)$ is called intuitionistic fuzzy metric space if X is an arbitrary non empty set, $*$ is a continuous t -norm, \diamond continuous t -conorm and M, N are fuzzy sets on $X^2 \times [0, \infty]$ satisfying the following conditions:

For each $x, y, z, \in X$ and $t, s > 0$

- (IFM-1) $M(x, y, t) + N(x, y, t) \leq 1$,
- (IFM-2) $M(x, y, t) = 0$, for all x, y in X ,
- (IFM-3) $M(x, y, t) = 1$ for all x, y in X and $t > 0$ if and only if $x=y$,
- (IFM-4) $M(x, y, t) = M(y, x, t)$, for all x, y in X and $t > 0$,
- (IFM-5) $M(x, y, t) * M(y, z, s) \leq M(x, z, t + s)$,
- (IFM-6) $M(x, y, \cdot): [0, \infty] \rightarrow [0,1]$ is left continuous,
- (IFM-7) $\lim_{t \rightarrow \infty} M(x, y, t) = 1$,
- (IFM-8) $N(x, y, 0) = 1$, for all x, y in X ,
- (IFM-9) $N(x, y, t) = 0$, for all x, y in X and $t > 0$ if and only if $x = y$,
- (IFM-10) $N(x, y, t) = N(y, x, t)$, for all x, y in X and $t > 0$,
- (IFM-11) $N(x, y, t) \diamond N(y, z, s) \geq N(x, z, t + s)$,
- (IFM-12) $N(x, y, \cdot): [0, \infty] \rightarrow [0,1]$ is right continuous,
- (IFM-13) $\lim_{t \rightarrow \infty} N(x, y, t) = 0$, for all x, y in X and $t > 0$.

Then (M, N) is called an intuitionistic fuzzy metric on X . The function $M(x, y, t)$ and $N(x, y, t)$ denote the degree of nearness and degree of non nearness between x and y with respect to t , respectively.

Example 2.1. [12] Let (X, d) be a metric space. Define $a * b = ab$ and $a \diamond b = \min\{1, a+b\}$, for all $a, b \in [0, 1]$ and let M and N be fuzzy sets on $X^2 \times (0, \infty)$ defined as follows:

$$M(x, y, t) = \frac{t}{t+d(x,y)} \text{ and } N(x, y, t) = \frac{d(x,y)}{t+d(x,y)} \text{ for all } x, y \in X \text{ and all } t > 0$$

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then (M, N) is called an intuitionistic fuzzy metric space on X . We call this intuitionistic fuzzy metric induced by a metric d the standard intuitionistic fuzzy metric.

Remark 2.3. Note that the above examples holds even with the t - norm $a * b = \min\{a, b\}$ and t - conorm $a \diamond b = \max\{a, b\}$ and hence (M, N) is an intuitionistic fuzzy metric with respect to any continuous t – norm and continuous t – conorm.

Lemma 2.1.[15] Let $(X, M, N, *, \diamond)$ intuitionistic fuzzy metric space, If there exists $k \in (0, 1)$ such that for all $x, y \in X$, $M(x, y, kt) \geq M(x, y, t)$ and $N(x, y, kt) \leq N(x, y, t)$ for all $t > 0$, then $x = y$.

Definition 2.9. [11] A pair (A, S) of self-mappings of an intuitionistic fuzzy metric space $(X, M, N, *, \diamond)$ is said to satisfy the property (E.A) if there exists a sequence $\{x_n\}$ in X such that $\lim_{n \rightarrow \infty} Ax_n = \lim_{n \rightarrow \infty} Sx_n = z$ for some $z \in X$.

Definition 2.9. [11] Two pairs (A, S) and (B, T) of self-mappings of an intuitionistic fuzzy metric space $(X, M, N, *, \diamond)$ are said to satisfy the common property (E.A) if there exist two sequences $\{x_n\}$ and $\{y_n\}$ in X such that $\lim_{n \rightarrow \infty} Ax_n = \lim_{n \rightarrow \infty} Sx_n = \lim_{n \rightarrow \infty} By_n = \lim_{n \rightarrow \infty} Ty_n = z$ for some $z \in X$.

Definition 2.10. [11] A pair (f, g) of self-mappings of an intuitionistic fuzzy metric space $(X, M, N, *, \diamond)$ is said to be weakly compatible mappings if the mappings commute at all of their coincidence points, i.e., $fx = gx$ for some $x \in X$ implies $fgx = gfx$.

2. Implicit relation

Let M_5 denotes the set of all real valued continuous function ϕ and $\psi: [0,1]^5 \rightarrow \mathbb{R}$ which are non decreasing and satisfying the following conditions:

- (A) $\int_0^{\phi(u,1,u,1,u)} \phi(t)dt \geq 0$ implies $u \geq 1$
- (B) $\int_0^{\phi(u,1,1,u,u)} \phi(t)dt \geq 0$ implies $u \geq 1$
- (C) $\int_0^{\phi(u,u,1,1,u)} \phi(t)dt \geq 0$ implies $u \geq 1$
- (D) $\int_0^{\psi(v,0,v,0,v)} \psi(t)dt \leq 0$ implies $v \leq 0$
- (E) $\int_0^{\psi(v,0,0,v,v)} \psi(t)dt \leq 0$ implies $v \leq 0$
- (F) $\int_0^{\psi(v,v,0,0,v)} \psi(t)dt \leq 0$ implies $v \leq 0$

Example 2.2. Define $\phi, \psi: [0,1]^5 \rightarrow \mathbb{R}$ as

$$\phi(t_1, t_2, t_3, t_4, t_5) = 11t_1 - 12t_2 + 6t_3 - 8t_4 + 3t_5$$

and $\psi(t_1, t_2, t_3, t_4, t_5, t_6) = 10t_1 - 9t_2 + 8t_3 - 11t_4 + 2t_5$.

Clearly ϕ, ψ satisfies all condition (A), (B), (C), (D), (E) and (F). Therefore $\phi, \psi \in M_5$.

3. Main results

We now establish the following results.

Theorem 3.1. Let A, B, S and T be self-mappings of an intuitionistic fuzzy metric space $(X, M, N, *, \diamond)$ satisfying the following conditions that:

(i) the pair (A, S) (or (B, T)) satisfies the property (E.A);

(ii) for any $x, y \in X$, $\phi, \psi \in M_5$ and for all $t > 0$, there exists $k \in (0, 1)$ such that

$$\int_0^{\epsilon} \phi(M(Ax, By, \alpha t), M(Sx, Ty, t), M(Sx, Ax, t), M(Ty, By, t), M(Ax, Ty, t) * M(Sx, By, t)) \varphi(t) dt \geq 0$$

$$\int_0^{\epsilon} \psi(N(Ax, By, \alpha t), N(Sx, Ty, t), N(Sx, Ax, t), N(Ty, By, t), N(Ax, Ty, t) \diamond N(Sx, By, t)) \varphi(t) dt \leq 0$$

where $\varphi: R^+ \rightarrow R^+$ is a Lebesgue integrable mapping which is summable, non negative such that

$$\int_0^{\epsilon} \varphi(t) dt > 0 \text{ for each } \epsilon > 0$$

(ii) $A(X) \subset T(X)$ (or $B(X) \subset S(X)$).

Then the pairs (A, S) and (B, T) share the common property (E.A).

Proof: Suppose that the pair (A, S) satisfies property (E.A), then there exists a sequence $\{x_n\}$ in X such that $\lim_{n \rightarrow \infty} Ax_n = \lim_{n \rightarrow \infty} Sx_n = z$ for some $z \in X$. Since $A(X) \subset T(X)$, therefore, for each x_n , there exist y_n in X such that $Ax_n = Ty_n$. This gives,

$$\lim_{n \rightarrow \infty} Ax_n = \lim_{n \rightarrow \infty} Sx_n = \lim_{n \rightarrow \infty} Ty_n = z.$$

Now, we claim that $\lim_{n \rightarrow \infty} By_n = z$.

Applying inequality (ii), we obtain

$$\int_0^{\epsilon} \phi(M(Ax_n, By_n, \alpha t), M(Sx_n, Ty_n, t), M(Sx_n, Ax_n, t), M(Ty_n, By_n, t), M(Ax_n, Ty_n, t) * M(Sx_n, By_n, t)) \varphi(t) dt \geq 0$$

and

$$\int_0^{\epsilon} \psi(N(Ax_n, By_n, \alpha t), N(Sx_n, Ty_n, t), N(Sx_n, Ax_n, t), N(Ty_n, By_n, t), N(Ax_n, Ty_n, t) \diamond N(Sx_n, By_n, t)) \varphi(t) dt \leq 0$$

Taking limit as $n \rightarrow \infty$

$$\int_0^{\epsilon} \phi\left(M\left(z, \lim_{n \rightarrow \infty} By_n, \alpha t\right), M(z, z, t), M(z, z, t), M\left(z, \lim_{n \rightarrow \infty} By_n, t\right), M(z, z, t) * M\left(z, \lim_{n \rightarrow \infty} By_n, t\right)\right) \varphi(t) dt \geq 0$$

and

$$\int_0^{\epsilon} \psi\left(N\left(z, \lim_{n \rightarrow \infty} By_n, \alpha t\right), N(z, z, t), N(z, z, t), N\left(z, \lim_{n \rightarrow \infty} By_n, t\right), N(z, z, t) \diamond N\left(z, \lim_{n \rightarrow \infty} By_n, t\right)\right) \varphi(t) dt \leq 0$$

Since ϕ and ψ is non-decreasing in the first argument, we have

$$\int_0^{\epsilon} \phi\left(M\left(z, \lim_{n \rightarrow \infty} By_n, t\right), 1, 1, M\left(z, \lim_{n \rightarrow \infty} By_n, t\right), M\left(z, \lim_{n \rightarrow \infty} By_n, t\right)\right) \varphi(t) dt \geq 0$$

and

$$\int_0^{\epsilon} \psi\left(N\left(z, \lim_{n \rightarrow \infty} By_n, t\right), 0, 0, N\left(z, \lim_{n \rightarrow \infty} By_n, t\right), N\left(z, \lim_{n \rightarrow \infty} By_n, t\right)\right) \varphi(t) dt \leq 0$$

Using (B) and (E), we get

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$$M\left(z, \lim_{n \rightarrow \infty} By_n, t\right) \geq 1 \text{ and } N\left(z, \lim_{n \rightarrow \infty} By_n, t\right) \leq 0$$

Hence

$$M\left(z, \lim_{n \rightarrow \infty} By_n, t\right) = 1 \text{ and } N\left(z, \lim_{n \rightarrow \infty} By_n, t\right) = 0.$$

Therefore $\lim_{n \rightarrow \infty} By_n = z$.

Hence the pairs (A, S) and (B, T) share the common property.

Similarly, if the pair (B, T) satisfies property (E.A) and $B(X) \subset S(X)$, then pairs (A, S) and (B, T) share the common property (E.A).

Theorem 3.2. Let A, B, S and T be self-mappings of an intuitionistic fuzzy metric space $(X, M, N, *, \diamond)$ satisfying the following conditions that:

(i) for any $x, y \in X$, $\phi, \psi \in M_5$ and for all $t > 0$, there exists $k \in (0, 1)$ such that

$$\int_0^{\phi(M(Ax, By, at), M(Sx, Ty, t), M(Sx, Ax, t), M(Ty, By, t), M(Ax, Ty, t) * M(Sx, By, t))} \varphi(t) dt \geq 0$$

and

$$\int_0^{\psi(N(Ax, By, at), N(Sx, Ty, t), N(Sx, Ax, t), N(Ty, By, t), N(Ax, Ty, t) \diamond N(Sx, By, t))} \varphi(t) dt \leq 0$$

where $\varphi: R^+ \rightarrow R^+$ is a Lebesgue integrable mapping which is summable, non negative such that

$$\int_0^{\varepsilon} \varphi(t) dt > 0 \text{ for each } \varepsilon > 0$$

(ii) the pairs (A, S) and (B, T) share the property (E.A);

(iii) $S(X)$ and $T(X)$ are closed subsets of X .

Then each of the pairs (A, S) and (B, T) have a point of coincidence. Moreover, A, B, S and T have a unique common fixed point provided both the pairs (A, S) and (B, T) are weakly compatible.

Proof: Since the pairs (A, S) and (B, T) share the property (E.A), there exist two sequences $\{x_n\}$ and $\{y_n\}$ in X such that $\lim_{n \rightarrow \infty} Ax_n = \lim_{n \rightarrow \infty} Sx_n = \lim_{n \rightarrow \infty} Ty_n = z$ for some $z \in X$. $S(X)$ is closed subset of X , there exists a point $u \in X$ such that $z = Su$.

We, now claim that $Au = z$. By (i), we have

$$\int_0^{\phi(M(Au, By_n, at), M(Su, Ty_n, t), M(Su, Au, t), M(Ty_n, By_n, t), M(Au, Ty_n, t) * M(Su, By_n, t))} \varphi(t) dt \geq 0$$

and

$$\int_0^{\psi(N(Au, By_n, at), N(Su, Ty_n, t), N(Su, Au, t), N(Ty_n, By_n, t), N(Au, Ty_n, t) \diamond N(Su, By_n, t))} \varphi(t) dt \leq 0$$

Taking limit as $n \rightarrow \infty$,

$$\int_0^{\phi(M(Au, z, at), M(z, z, t), M(z, Au, t), M(z, z, t), M(Au, z, t) * M(z, z, t))} \varphi(t) dt \geq 0$$

and

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$$\int_0^{\infty} \psi(N(Au, z, at), N(z, z, t), N(z, Au, t), N(z, z, t), N(Au, z, t) \diamond N(z, z, t)) \varphi(t) dt \leq 0$$

As ϕ and ψ are non-decreasing in the first argument, we have

$$\int_0^{\infty} \phi(M(Au, z, t), 1, M(z, Au, t), 1, M(Au, z, t)) \varphi(t) dt \geq 0$$

and

$$\int_0^{\infty} \psi(N(Au, z, t), 0, N(z, Au, t), 0, N(Au, z, t)) \varphi(t) dt \leq 0$$

Using implicit relations (A) and (D), we have

$M(Au, z, t) \geq 1$ and $N(Au, z, t) \leq 0$.

Hence $M(Au, z, t) = 1$ and $N(Au, z, t) = 0$.

Therefore, $Au = z = Su$ which shows that u is a coincidence point of the pair (A, S).

Since $T(X)$ is also a closed subset of X , therefore, $\lim_{n \rightarrow \infty} Ty_n = z$ in $T(X)$ and hence there exists $v \in X$ such that $Tv = z = Au = Su$. Now, we show that $Bv = z$.

By using inequality (i), we have

$$\int_0^{\infty} \phi(M(Au, Bv, at), M(Su, Tv, t), M(Su, Au, t), M(Tv, Bv, t), M(Au, Tv, t) * M(Su, Bv, t)) \varphi(t) dt \geq 0$$

and

$$\int_0^{\infty} \psi(N(Au, Bv, at), N(Su, Tv, t), N(Su, Au, t), N(Tv, Bv, t), N(Au, Tv, t) \diamond N(Su, Bv, t)) \varphi(t) dt \leq 0$$

it follows

$$\int_0^{\infty} \phi(M(z, Bv, at), M(z, z, t), M(z, z, t), M(z, Bv, t), M(z, z, t) * M(z, Bv, t)) \varphi(t) dt \geq 0$$

and

$$\int_0^{\infty} \psi(N(z, Bv, at), N(z, z, t), N(z, z, t), N(z, Bv, t), N(z, z, t) \diamond N(z, Bv, t)) \varphi(t) dt \leq 0$$

As ϕ and ψ are non-decreasing in the first argument, we have

$$\int_0^{\infty} \phi(M(z, Bv, t), 1, 1, M(z, Bv, t), M(z, Bv, t)) \varphi(t) dt \geq 0$$

and

$$\int_0^{\infty} \psi(N(z, Bv, t), 0, 0, N(z, Bv, t), N(z, Bv, t)) \varphi(t) dt \leq 0$$

Using implicit relations (B) and (E), we get

$M(z, Bv, t) \geq 1$ and $N(z, Bv, t) \leq 0$.

Hence $M(z, Bv, t) = 1$ and $N(z, Bv, t) = 0$.

Therefore, $Bv = z = Tv$, which shows that v is a coincidence point of the pair (B, T).

Moreover, since the pairs (A, S) and (B, T) are weakly compatible and $Au = Su, Bv = Tv$, therefore, $Az = ASu = SAu = Sz, Bz = BTv = TBv = Tz$.

Next, we claim that $Az = z$ for showing the existence of a fixed point of A. By using inequality (i), we have

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$$\int_0^{\infty} \phi(M(Az, Bv, \alpha t), M(Sz, Tv, t), M(Sz, Az, t), M(Tv, Bv, t), M(Az, Tv, t) * M(Sz, Bv, t)) \varphi(t) dt \geq 0$$

and

$$\int_0^{\infty} \psi(N(Az, Bv, \alpha t), N(Sz, Tv, t), N(Sz, Az, t), N(Tv, Bv, t), N(Az, Tv, t) \diamond N(Sz, Bv, t)) \varphi(t) dt \leq 0$$

it follows that

$$\int_0^{\infty} \phi(M(Az, z, \alpha t), M(Az, z, t), M(Az, Az, t), M(z, z, t), M(Az, z, t) * M(Az, z, t)) \varphi(t) dt \geq 0$$

and

$$\int_0^{\infty} \psi(N(Az, z, \alpha t), N(Az, z, t), N(Az, Az, t), N(z, z, t), N(Az, z, t) \diamond N(Az, z, t)) \varphi(t) dt \leq 0$$

Since ϕ and ψ are non-decreasing in the first argument, we have

$$\int_0^{\infty} \phi(M(Az, z, t), M(Az, z, t), 1, 1, M(Az, z, t)) \varphi(t) dt \geq 0$$

and

$$\int_0^{\infty} \psi(N(Az, z, t), N(Az, z, t), 0, 0, N(Az, z, t)) \varphi(t) dt \leq 0$$

On using implicit relations (C) and (F), we get

$M(Az, z, t) \geq 1$ and $N(Az, z, t) \leq 0$.

Hence, $M(Az, z, t) = 1$ and $N(Az, z, t) = 0$. Therefore, $Az = z = Sz$.

Similarly, we can prove that $Bz = Tz = z$. Hence, $Az = Bz = Sz = Tz = z$, which implies that z is a common fixed point of A, B, S and T .

Uniqueness: Let w be another common fixed points of A, B, S and T . Then by using (i),

$$\int_0^{\infty} \phi(M(Az, Bw, \alpha t), M(Sz, Tw, t), M(Sz, Az, t), M(Tw, Bw, t), M(Az, Tw, t) * M(Sz, Bw, t)) \varphi(t) dt \geq 0$$

and

$$\int_0^{\infty} \psi(N(Az, Bw, \alpha t), N(Sz, Tw, t), N(Sz, Az, t), N(Tw, Bw, t), N(Az, Tw, t) \diamond N(Sz, Bw, t)) \varphi(t) dt \leq 0$$

it follows that

$$\int_0^{\infty} \phi(M(z, w, \alpha t), M(z, w, t), M(z, z, t), M(w, w, t), M(z, w, t) * M(z, w, t)) \varphi(t) dt \geq 0$$

and

$$\int_0^{\infty} \psi(N(z, w, \alpha t), N(z, w, t), N(z, z, t), N(w, w, t), N(z, w, t) \diamond N(z, w, t)) \varphi(t) dt \leq 0$$

Since ϕ and ψ are non-decreasing in the first argument, we have

$$\int_0^{\infty} \phi(M(z, w, t), M(z, w, t), 1, 1, M(z, w, t)) \varphi(t) dt \geq 0$$

and

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$$\int_0^{\psi(N(z,w,\alpha t),N(z,w,t),0,0,N(z,w,t))} \varphi(t) dt \leq 0$$

Using implicit relations (C) and (F), we have

$M(z, w, t) \geq 1$ and $N(z, w, t) \leq 0$.

Hence, $M(z, w, t) = 1$ and $N(z, w, t) = 0$.

Therefore, $z = w$, i.e., mappings A, B, S and T have a unique common fixed point.

Taking $B = A$ and $T = S$ in the Theorem 3.2. yields following corollary:

Corollary 3.1. Let A and S be self-mappings of an intuitionistic fuzzy metric space $(X, M, N, *, \diamond)$ satisfying the following conditions that

- (i) the pair (A, S) share the property (E.A);
- (ii) for any $x, y \in X$, $\phi, \psi \in M_5$ and for all $t > 0$, there exists $k \in (0,1)$ such that
- (iii) $\int_0^{\phi(M(Ax,Ay,\alpha t),M(Sx,Sy,t),M(Sx,Ax,t),M(Sy,Ay,t),M(Ax,Sy,t)*M(Sx,Ay,t))} \varphi(t) dt \geq 0$

and

$$\int_0^{\psi(N(Ax,Ay,\alpha t),N(Sx,Sy,t),N(Sx,Sx,t),N(Sy,Ay,t),N(Ax,Sy,t)\diamond N(Sx,Ay,t))} \varphi(t) dt \leq 0$$

where $\varphi: R^+ \rightarrow R^+$ is a Lebesgue integrable mapping which is summable, non negative such that

$$\int_0^\varepsilon \varphi(t) dt > 0 \text{ for each } \varepsilon > 0$$

- (iv) $S(X)$ is a closed subset of X .

Then A and S each have a point of coincidence. Moreover, if the pair (A, S) is weakly compatible, then A and S have a unique common fixed point.

Corollary 3.2. Let A, B, S and T be self-mappings of an intuitionistic fuzzy metric space $(X, M, N, *, \diamond)$ satisfying the following conditions that:

- (i) for any $x, y \in X$, $\phi, \psi \in M_5$ and for all $t > 0$, there exists $k \in (0,1)$ such that $\phi(M(Ax, By, \alpha t), M(Sx, Ty, t), M(Sx, Ax, t), M(Ty, By, t), M(Ax, Ty, t) * M(Sx, By, t)) \geq 0$

and

$$\psi(N(Ax, By, \alpha t), N(Sx, Ty, t), N(Sx, Ax, t), N(Ty, By, t), N(Ax, Ty, t) \diamond N(Sx, By, t)) \leq 0$$

- (ii) the pairs (A, S) and (B, T) share the property (E.A);

- (iii) $S(X)$ and $T(X)$ are closed subsets of X .

Then each of the pairs (A, S) and (B, T) have a point of coincidence. Moreover, A, B, S and T have a unique common fixed point provided both the pairs (A, S) and (B, T) are weakly compatible.

Proof: If we put $\varphi(t) = 1$ in theorem 3.2 the result follows from theorem 3.2.

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