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Common Coupled Fixed Point Theorems in Intuitionistic Fuzzy Metric Spaces

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Abstract. In this paper, we prove some coupled fixed point theorems, which generalized the result of Hu [6], Hu et al.[7] from fuzzy metric spaces to intuitionistic fuzzy metric spaces for semi-compatible mappings, which is weaker form of compatible mappings.

Keywords: Coupled fixed point, intuitionistic fuzzy metric space, coupled common fixed point, semi-compatible maps.

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1. Introduction.

Atanassov [2] introduced and studied the concept of intuitionistic fuzzy sets as a generalization of fuzzy sets. Motivated by the idea of intuitionistic fuzzy sets Alaca et al. [1] define the concept of intuitionistic fuzzy metric spaces using continuous t-norms and continuous t-conorms. Turkoglu et al. [14] formulated the definition of weakly commuting and R-weakly commuting mappings in intuitionistic fuzzy metric spaces. Turkoglu et al. [15] introduced the concept of compatible maps and compatible maps of types (α) and (β) in intuitionistic fuzzy metric spaces and gave some relations between the concepts of compatible maps and compatible maps of types (α) and (β). On the other hand, Bhaskar and Lakshmikantham [3], Lakshmikantham and Ciric [8], gave some coupled fixed point theorems in partially ordered metric spaces. In 2010, Sedghi et al. [12] proved common coupled fixed point theorems for contraction in fuzzy metric spaces for commuting mappings. Motivated by the results of Fang [4], Hu [6] proved a coupled fixed point theorem for compatible mappings satisfying $\emptyset$-contractive conditions in fuzzy metric spaces with continuous t-norm of H-type and generalized the result of Sedghi et al. [12]. Inspired by the work of Hu [6], Hu et al. [7], we prove common coupled fixed point theorems for pair of mappings satisfying a general contractive condition in intuitionistic fuzzy metric space, by using the notion of semi-compatibility.

2. Preliminaries

First, we start with some basic definitions.

Definition 2.1. [11] A binary operation $*: [0,1]\times[0,1] \rightarrow [0,1]$ is continuous t-norm if $*$ is satisfying the following conditions:

(i) $*$ is associative and commutative;
(ii) $*$ is continuous;
(iii) $a * 1 = a$ for all $a\in[0,1]$;
(iv) $a * b \leq c * d$ whenever $a \leq c$ and $b \leq d$, for each a, b, c, d $\in$ [0, 1].

Definition 2.2. [11] A binary operation $\Diamond: [0,1]\times[0,1] \rightarrow [0,1]$ is continuous t-conorm if $\Diamond$ is satisfying the following conditions:

(i) $\Diamond$ is associative and commutative;
(ii) $\Diamond$ is continuous;
(iii) $a \Diamond 0 = a$ for all $a\in[0,1]$;
(iv) $a \Diamond b \leq c \Diamond d$ whenever $a \leq c$ and $b \leq d$, for each a, b, c, d $\in$ [0, 1].

Definition 2.3. [5] Let $\sup_{0<t<1}\Delta(t,t) = 1$. A t-norm Δ is said to be of H-type if the family of functions $\{\Delta^m(t)\}_{m=1}^{\infty}$ is equicontinuous at t = 1, where

$$\Delta^1(t) = t\,\Delta t\,, \Delta^{m+1}(t) = t\,\Delta\big(\Delta^m(t)\big), \quad m = 1,2,\ldots, t \in [0,1]. \tag{2.1}$$

The t-norm Δ_M = min is an example of t-norm of H-type, but there are some other t-norms Δ of H-type.

Obviously, Δ is a t-norm of H-type if and only if for any $\lambda \in (0, 1)$, there exists $\delta(\lambda) \in (0, 1)$ such that $\Delta^m(t) > 1-\lambda$ for all $m\in N$, when $t > 1-\delta$.

Definition 2.4. [13] Let $\inf_{0<t<1} \Diamond\,(t,t) = 1$. A t-conorm $\Diamond$ is said to be of H-type if the family of functions $\{\Diamond^m\,(t)\}_{m=1}^{\infty}$ is equicontinuous at t = 0, where

$$\Diamond^1\,(t) = t \Diamond t\,, \Diamond^{m+1}\,(t) = t \Diamond \big(\Diamond^m\,(t)\big), \quad m = 1,2,\ldots, t \in [0,1]. \tag{2.2}$$

The t-conorm $\Diamond_M$ = max is an example of t-conorm of H-type, but there are some other t-conorms $\Diamond$ of H-type.

Obviously, $\Diamond$ is a t-conorm of H-type if and only if for any $\lambda \in (0, 1)$, there exists $\delta(\lambda) \in (0, 1)$ such that $\Diamond^m(t) < \lambda$ for all $m \in N$, when $t < \delta$.

Definition 2.5. [1] A 5-tuple $(X,M,N,*,\Diamond)$ is said to be an intuitionistic fuzzy metric space if X is an arbitrary nonempty set, $*$ is a continuous t-norm, $\Diamond$ is a continuous t-conorm and M,N are fuzzy sets on $X^2 \times [0,+\infty)$ satisfying the following conditions:

(i) M(x, y, t) + N(x, y, t) $\leq$ 1 for all x, y, z $\in$ X and t > 0;
(ii) M(x, y, t) = 0 for all x, y, z $\in$ X;
(iii) M(x, y, t) = 1 for all x, y, z $\in$ X and t > 0 if and only if x = y;
(iv) M(x,y,t) = M(y,x,t) for all x, y, z $\in$ X and t > 0;
(v) M(x, y, t)$*$M(y, z, s) $\leq$ M(x,z,t+s) for all x, y, z $\in$ X and s, t > 0;
(vi) for all x, y, z $\in$ X, M(x, y, $\cdot$) : $[0,\infty) \rightarrow [0,1]$ is left continuous;
(vii) $\lim_{n\rightarrow\infty}$M(x, y, t) = 1 for all x, y, z $\in$ X and t > 0;
(viii) N(x, y, 0) = 1 for all x, y, z $\in$ X;
(ix) N(x, y, t) = 0 for all x, y, z $\in$ X and t > 0 if and only if x = y;

(x) N(x, y, t) = N(y, x, t) for all x, y, z ∈ X and t > 0;
(xi) N(x, y, t) ◊N(y, z, s) ≥ N(x, z, t + s) for all x, y, z ∈ X and s, t > 0;
(xii) for all x, y, z ∈ X, N(x, y,·) : [0,∞) → [0,1] is right continuous;
(xiii) $\lim_{n\to\infty}$N(x, y, t) = 0 for all x, y, z ∈ X.

Then (M, N) is called an intuitionistic fuzzy metric on X. The functions M(x, y, t) and N(x, y, t) denote the degree of nearness and the degree of non-nearness between x and y with respect to t, respectively.

Remark 2.1. [9] Every fuzzy metric space(X, M, ∗) is an intuitionistic fuzzy metric space of the form (X,M,1−M,∗,◊) such that t-norm ∗ and t-conorm ◊ are associated, i.e., x◊y = 1−((1 − x) ∗ (1 − y)) for all x, y ∈ X.

Remark 2.2. [9] In intuitionistic fuzzy metric space X, M(x, y,·) is non-decreasing and N(x, y,·) is an non-increasing for all x, y ∈ X.

Example 2.1. [12] Let (X, d) be a metric space. Define t-norm a∗ b = min{a, b} and t-conorm a◊b = max{a, b}and for all x, y ∈ X and t > 0,

$$M(x, y, t) = \frac{t}{t+d(x,y)}, \quad N(x, y, t) = \frac{d(x,y)}{t+d(x,y)}.$$

Then (X, M, N, ∗,◊) is an intuitionistic fuzzy metric space induced by the metric d. It is obvious that N(x, y, t) = 1−M(x, y, t).

Definition 2.6. [1] Let (X, M,N,∗,◊) be an intuitionistic fuzzy metric space.

(i) A sequence $\{x_n\}$ in X is said to be convergent to a point x ∈ X if, for all t > 0 $\lim_{n\to\infty}M(x_n,x,t) = 1$, $\lim_{n\to\infty}N(x_n,x,t) = 0$.
(ii) A sequence $\{x_n\}$ in X is said to be Cauchy sequence if, for any t > 0 and p > 0, $\lim_{n\to\infty} M(x_{n+p}, x_n, t) = 1$, $\lim_{n\to\infty} N(x_{n+p}, x_n, t) = 0$.

Since ∗ and ◊ are continuous, the limit is uniquely determined from (vii) and (xiii) respectively.

Definition 2.7. [1] An intuitionistic fuzzy metric space (X, M,N,∗,◊) is said to be complete if and only if every Cauchy sequence in X is convergent.

Definition 2.8. [1] An intuitionistic fuzzy metric space (X, M,N,∗,◊) is said to be compact if every sequence in X contains a convergent subsequence.

Lemma 2.1. [1] Let (X, M,N,∗,◊)be an intuitionistic fuzzy metric space and $\{y_n\}$ be a sequence in X. If there exists a number k ∈ (0,1) such that

$M(y_{n+2}, y_{n+1}, kt) \geq M(y_{n+1}, y_n, t)$,
$N(y_{n+2}, y_{n+1}, kt) \leq N(y_{n+1}, y_n, t)$ for all t > 0 and n = 1,2,…,

then $\{y_n\}$ is a Cauchy sequence in X.

Lemma 2.2. [1] Let (X, M,N,∗,◊) be an intuitionistic fuzzy metric space and for all x, y ∈ X, t > 0 and if for a number k ∈ (0,1),

M(x, y, kt) ≥ M(x, y, t) and N(x, y, kt) ≤ N(x,y,t) then x = y.

We define $\Phi = \{\emptyset: R^{+} \to R^{+}\}$, where $R^{+} = [0, +\infty)$ and each $\emptyset \in \Phi$ satisfies the following conditions:

(A_1) $\emptyset$ is non-decreasing,

(A_2) $\emptyset$ is upper semi continuous from the right,

(A_3) $\sum_{n=0}^{\infty} \emptyset^{n}(t) < +\infty$ for all $t > 0$, where $\emptyset^{n+1}(t) = \emptyset(\emptyset^{n}(t)), n \in N$.

It is easy to prove that if $\emptyset \in \Phi$, then $\emptyset(t) < t$ for all $t > 0$.

We define n-property in intuitionistic fuzzy metric spaces.

Definition 2.9. Let $(X, M, N, *, \lozenge)$ be an intuitionistic fuzzy metric space. M and N are said to satisfy the n-property on $X^2 \times [0, \infty)$ if

$$\lim_{n\to\infty}[M(x, y, k^{n}t)]^{n^{p}} = 1, \lim_{n\to\infty}[N(x, y, k^{n}t)]^{n^{p}} = 0$$

whenever $x, y \in X$, $k > 1$ and $p > 0$.

Lemma 2.3. [5] Let $(X,M,*)$ be a fuzzy metric space and M satisfy the n-property; then $\lim_{n\to\infty} M(x, y, t) = 1$, for all $x, y \in X$. (2.3)

We give the following lemma in intuitionistic fuzzy metric spaces.

Lemma 2.4. Let $(X, M, N, *, \lozenge)$ be an intuitionistic fuzzy metric space and M and N satisfies the n-property; then

$$\lim_{n\to\infty} M(x, y, t) = 1,$$
$$\lim_{n\to\infty} N(x, y, t) = 0, \text{for all } x, y \in X.$$

Proof: If not, since $M(x, y,\cdot)$ is non-decreasing and $0 \le M(x, y,\cdot) \le 1$, and $N(x, y,\cdot)$ is non-increasing, $1 \ge N(x, y,\cdot) \ge 0$, there exists $x_0, y_0 \in X$ such that $\lim_{t\to+\infty} M(x_0, y_0, t) = \lambda < 1$, and $\lim_{t\to+\infty} N(x_0, y_0, t) = \lambda > 1$ then for $k > 1, k^{n}t \to +\infty$ when $n \to \infty$ as $t > 0$ and we get $\lim_{n\to\infty}[M(x, y, k^{n}t)]^{n^{p}} = 0$, and $\lim_{n\to\infty}[N(x, y, k^{n}t)]^{n^{p}} = 1$,which is a contradiction.

Lemma 2.5. [5] Let $(X, M, *)$ be a fuzzy metric space, where $*$ is a continuous t-norm of H-type. If there exists $\emptyset \in \Phi$ such that if

$$M(x, y, \emptyset(t)) \ge M(x, y, t), \quad \text{for all } t > 0, \text{ then } x = y.$$

We give following lemma in intuitionistic fuzzy metric spaces.

Lemma 2.6. [5] Let $(X, M, N, *, \lozenge)$ be an intuitionistic fuzzy metric space, where $*$ and $\lozenge$ is a continuous t-norm and continuous t-conorm of H-type. If there exists $\emptyset \in \Phi$ such that if

$$M(x, y, \emptyset(t)) \ge M(x, y, t),$$
$$N(x, y, \emptyset(t)) \le N(x, y, t), \text{ for all } t > 0, \text{ then } x = y.$$

Definition 2.10. [8] An element $(x, y) \in X \times X$ is called a coupled fixed point of the mappings $F: X \times X \to X$ if $F(x, y) = x$, $F(y, x) = y$.

Definition 2.11. [8] An element $(x, y) \in X \times X$ is called a coupled coincidence point of the mappings $F: X \times X \to X$ and $g: X \to X$ if $F(x, y) = g(x)$, $F(y, x) = g(y)$.

Definition 2.12. [8] An element $(x, y) \in X \times X$ is called a common coupled fixed point of the mappings $F: X\times X \to X$ and $g: X \to X$ if $x = F(x, y) = g(x)$, $y = F(y, x) = g(y)$.

Definition 2.13. [8] An element $x \in X$ is called a common fixed point of the mappings $F: X \times X \to X$ and $g: X \to X$ if $x = g(x) = F(x, x)$.

Definition 2.14. [13] The mappings $F: X\times X \to X$ and $g: X \to X$ are said to be compatible if

$$\lim_{n\to\infty} M\big(gF(x_n, y_n), F\big(g(x_n), g(y_n)\big), t\big) = 1,$$
$$\lim_{n\to\infty} M\big(gF(y_n, x_n), F\big(g(y_n), g(x_n)\big), t\big) = 1 \text{ and}$$
$$\lim_{n\to\infty} N\big(gF(x_n, y_n), F\big(g(x_n), g(y_n)\big), t\big) = 0,$$
$$\lim_{n\to\infty} N\big(gF(y_n, x_n), F\big(g(y_n), g(x_n)\big), t\big) = 0$$

for all $t > 0$ whenever $\{x_n\}$ and $\{y_n\}$ are sequences in X, such that

$$\lim_{n\to\infty} F(x_n, y_n) = \lim_{n\to\infty} g(x_n) = x, \quad \lim_{n\to\infty} F(y_n, x_n) = \lim_{n\to\infty} g(y_n) = y$$

for all $x, y \in X$ are satisfied.

Definition 2.15. [8] The mappings $F: X\times X \to X$ and $g: X\to X$ are called commutative if

$$g(F(x, y)) = F(gx, gy), \text{ for all } x, y \in X.$$

Definition 2.16. [7]The mappings $F: X\times X \to X$ and $g: X\to X$ are called weakly compatible mappings if $F(x, y) = g(x)$, $F(y, x) = g(y)$ implies that $gF(x, y) = F(gx, gy)$, $gF(y, x) = F(gy, gx)$ for all $x, y \in X$.

We introduced the concept of semi-compatible mappings in intuitionistic fuzzy metric spaces.

Definition 2.17. The mappings $F: X\times X\to X$ and $g: X\to X$ are called semi compatible if

$$\lim_{n\to\infty} M(gF(x_n, y_n), F(x, y), t) = 1, \quad \lim_{n\to\infty} M(gF(y_n, x_n), F(y, x), t) = 1$$
and
$$\lim_{n\to\infty} N(gF(x_n, y_n), F(x, y), t) = 0, \quad \lim_{n\to\infty} N(gF(y_n, x_n), F(y, x), t) = 0.$$

for all $t > 0$ whenever $\{x_n\}$ and $\{y_n\}$ are sequences in X, such that

$$\lim_{n\to\infty} F(x_n, y_n) = \lim_{n\to\infty} g(x_n) = x, \lim_{n\to\infty} F(y_n, x_n) = \lim_{n\to\infty} g(y_n) = y.$$

for all $x, y \in X$.

Theorem 2.1. (1 of [6]). Let $(X, M, *)$ be a complete FM-space, where $*$ is a continuous t-norm of H-type satisfying (2.1). Let $F: X \times X \to X$ and $g: X \to X$ be two mappings, and there exists $\emptyset \in \Phi$ such that

$$M(F(x, y), F(u, v), \emptyset(t)) \geq M(g(x), g(u), t) * M(g(y), g(v), t)$$

for all $x, y, u, v \in X$ and $t > 0$.

Suppose that $F(X\times X) \subseteq g(X)$, g is continuous, F and g are compatible. Then there exist $x, y \in X$ such that $x = g(x) = F(x, x)$; that is, F and g have a unique common fixed point in X.

Now we give our result in intuitionistic fuzzy metric spaces.

Theorem 2.2. (3.2 of [7]). Let $(X, M, *)$ be FM-space, where $*$is a continuous t-norm of H-type satisfying (2.1). Let $F : X \times X \to X$ and $g : X \to X$ be two weakly compatible mappings, and there exists $\emptyset \in \Phi$ such that

$M(F(x, y), F(u, v), \emptyset(t)) \geq M(g(x), g(u), t) * M(g(y), g(v), t)$

for all x, y, u, v ∈ X and t > 0.

Suppose that F(X×X) ⊆ g(X), and F(X×X) or g(X) is complete. Then F and g have a unique common fixed point in X.

3. Main results

For simplicity, denote

$$[M(x,y,t)]^n = \underbrace{M(x,y,t) * M(x,y,t) * \dots * M(x,y,t)}_{n}$$

$$[N(x,y,t)]^n = \underbrace{N(x,y,t) \lozenge N(x,y,t) \lozenge \dots \lozenge N(x,y,t)}_{n} \quad \text{for all } n \in N.$$

Now we give our main results in intuitionistic fuzzy metric spaces.

Theorem 3.1. Let (X, M, N, $*$,◊)be an intuitionistic fuzzy metric space, where $*$ is a continuous t-norm and ◊ is a continuous t-co-norm of H-type defined by $t * t \geq t$ and $(1-t) \lozenge (1-t) \leq (1-t)$ satisfying(2.4). Let F:X×X→X and g:X→X be two mappings and there exists $\emptyset \in \Phi$ such that

$$M(F(x, y),F(u, v),\emptyset(t)) \geq M(g(x),g(u),t) * M(g(y),g(v),t);$$
$$N(F(x,y),F(u,v),\emptyset(t)) \leq N(g(x),g(u),t) \lozenge N(g(y),g(v),t); \tag{3.1}$$

for all x, y, u, v ∈ X and t > 0.

Suppose that F(X×X) ⊆ g(X), and g is continuous, F and g are semi-compatible, then there exist x, y ∈ X such that x = g(x) = F(x, x), that is, F and g have a unique common fixed point in X.

Proof: Let $x_0, y_0 \in X$ be two arbitrary points in X. Since F(X×X) ⊆ g(X), we can choose $x_1, y_1 \in X$ such that $g(x_1) = F(x_0, y_0)$ and $g(y_1) = F(y_0, x_0)$. Continuing in this way we can construct two sequences $\{x_n\}$ and $\{y_n\}$ in X such that

$$g(x_{n+1}) = F(x_n,y_n),\ g(y_{n+1}) = F(y_n,x_n), \text{for all } n \geq 0.$$

The proof is divided into four steps.

Step I. First we Prove $\{gx_n\}$ and $\{gy_n\}$ are Cauchy sequences.

Since $*$ and ◊ is a t-norm and t-conorm of H-type, for any λ > 0, there exists a $\mu > 0$ such that

$$\underbrace{(1-\mu) * (1-\mu) * \dots * (1-\mu)}_{k} \geq 1-\lambda,$$

And
$$\underbrace{(1-\mu) \lozenge (1-\mu) \lozenge \dots \lozenge (1-\mu)}_{k} \leq 1-\lambda \tag{3.2}$$

for all k∈ N. Since M(x, y,·) and N(x, y, ·) is continuous and $\lim_{t\to+\infty} M(x,y,t) = 1$ and $\lim_{t\to+\infty} N(x,y,t) = 0$, for all x, y ∈ X, there exists $t_0 > 0$ such that

$$M(gx_0, gx_1, t_0) \geq 1-\mu, \text{ and } N(gx_0, gx_1, t_0) \leq 1-\mu$$
$$M(gy_0, gy_1, t_0) \geq 1-\mu, \text{and } N(gy_0, gy_1, t_0) \leq 1-\mu \tag{3.3}$$

On the other hand, since $\emptyset \in \Phi$, by condition (A_3), we have

$$\sum_{n=1}^{\infty} \emptyset^n(t_0) < \infty. \text{Then for any } t > 0,$$

There exists $n_0 \in N$ such that

$$t > \sum_{k=n_0}^{\infty} \emptyset^k(t_0) \tag{3.4}$$

From condition (3.1), we have

$$M(gx_1, gx_2, \emptyset(t_0)) = M(F(x_0, y_0), F(x_1, y_1), \emptyset(t_0)) \geq M(gx_0, gx_1, t_0) * M(gy_0, gy_1, t_0),$$

and $$N(gx_1, gx_2, \emptyset(t_0)) = N(F(x_0, y_0), F(x_1, y_1), \emptyset(t_0)) \leq N(gx_0, gx_1, t_0) \lozenge N(gy_0, gy_1, t_0),$$

$$M(gy_1, gy_2, \emptyset(t_0)) = M(F(y_0, x_0), F(y_1, x_1), \emptyset(t_0)) \geq M(gy_0, gy_1, t_0) * M(gx_0, gx_1, t_0).$$

and $$N(gy_1, gy_2, \emptyset(t_0)) = N(F(y_0, x_0), F(y_1, x_1), \emptyset(t_0)) \leq N(gy_0, gy_1, t_0) \lozenge N(gx_0, gx_1, t_0).$$

Similarly, we can also get

$$\begin{aligned} M(gx_2, gx_3, \emptyset^2(t_0)) &= M(F(x_1, y_1), F(x_2, y_2), \emptyset^2(t_0)) \\ &\geq M(gx_1, gx_2, \emptyset(t_0)) * M(gy_1, gy_2, \emptyset(t_0)), \\ &\geq [M(gx_0, gx_1, t_0)]^2 * [M(gy_0, gy_1, t_0)]^2, \end{aligned}$$

and $$\begin{aligned} N(gx_2, gx_3, \emptyset^2(t_0)) &= N(F(x_1, y_1), F(x_2, y_2), \emptyset^2(t_0)) \\ &\leq N(gx_1, gx_2, \emptyset(t_0)) \lozenge N(gy_1, gy_2, \emptyset(t_0)), \\ &\leq [N(gx_0, gx_1, t_0)]^2 \lozenge [N(gy_0, gy_1, t_0)]^2 \end{aligned}$$

$$M(gy_2, gy_3, \emptyset^2(t_0)) = M(F(y_1, x_1), F(y_2, x_2), \emptyset^2(t_0)) \geq [M(gy_0, gy_1, t_0)]^2 * [M(gx_0, gx_1, t_0)]^2 .$$

and $$N(gy_2, gy_3, \emptyset^2(t_0)) = N(F(y_1, x_1), F(y_2, x_2), \emptyset^2(t_0)) \leq [N(gy_0, gy_1, t_0)]^2 \lozenge [N(gx_0, gx_1, t_0)]^2$$

Continuing in this process, we can get

$$M(gx_n, gx_{n+1}, \emptyset^n(t_0)) \geq [M(gx_0, gx_1, t_0)]^{2^{n-1}} * [M(gy_0, gy_1, t_0)]^{2^{n-1}},$$

and $$N(gx_n, gx_{n+1}, \emptyset^n(t_0)) \leq [N(gx_0, gx_1, t_0)]^{2^{n-1}} \lozenge [N(gy_0, gy_1, t_0)]^{2^{n-1}}$$

$$M(gy_n, gy_{n+1}, \emptyset^n(t_0)) \geq [M(gy_0, gy_1, t_0)]^{2^{n-1}} * [M(gx_0, gx_1, t_0)]^{2^{n-1}},$$

and $$N(gy_n, gy_{n+1}, \emptyset^n(t_0)) \leq [N(gy_0, gy_1, t_0)]^{2^{n-1}} \lozenge [N(gx_0, gx_1, t_0)]^{2^{n-1}}$$

So, from (3.3) and (3.4), for $m > n \geq n_0$, we have

$$\begin{aligned} M(gx_n, gx_m, t) &\geq M(gx_n, gx_m, \textstyle\sum_{k=n_0}^{\infty} \emptyset^k(t_0)) \\ &\geq M(gx_n, gx_m, \textstyle\sum_{k=n}^{m-1} \emptyset^k(t_0)) \\ &\geq M(gx_n, gx_{n+1}, \emptyset^n(t_0)) * M(gx_{n+1}, gx_{n+2}, \emptyset^{n+1}(t_0)) * \\ &\quad \ldots * M(gx_{m-1}, gx_m, \emptyset^{m-1}(t_0)) \\ &\geq [M(gy_0, gy_1, t_0]^{2^{n-1}} * [M(gx_0, gx_1, t_0)]^{2^{n-1}} * [M(gy_0, gy_1, t_0)]^{2^n} \\ &\quad [M(gx_0, gx_1, t_0)]^{2^n} * \ldots * [M(gy_0, gy_1, t_0)]^{2^{m-2}} * [M(gx_0, gx_1, t_0)]^{2^{m-2}} \\ &\geq [(1-\mu)]^{2^{n-1}} * [(1-\mu)]^{2^{n-1}} * [(1-\mu)]^{2^n} * [(1-\mu)]^{2^n} \\ &\quad * \ldots * [(1-\mu)]^{2^{m-2}} * [(1-\mu)]^{2^{m-2}} \\ &\geq \underbrace{(1-\mu) * (1-\mu) * \ldots * (1-\mu)}_{2^{2(m+2n-3)}} \geq 1-\lambda, \end{aligned}$$

Similarly

$$N(gx_n, gx_m, t) \leq \underbrace{(1-\mu) \lozenge (1-\mu) \lozenge \ldots \lozenge (1-\mu)}_{2^{2(m+2n-3)}} \leq 1-\lambda,$$

which implies that

$M(gx_n, gx_m, t) > 1 - \lambda$,

and $N(gx_n, gx_m, t) < 1 - \lambda$ (3.5)

For all m, n∈ N with $m > n \geq n_0$ and $t > 0$, so $\{g(x_n)\}$ is a Cauchy sequence.

Similarly, we can get that $\{g(y_n)\}$ is also a Cauchy sequence.

Step II: To prove F and g have coupled coincidence point;

i.e. F(x, y) = g(x), F(y, x) = g(y).

By the completeness of X, there exists x, y ∈ X such that

$\lim_{n\to\infty} F(x_n, y_n) = \lim_{n\to\infty} g(x_n) = x,\ \lim_{n\to\infty} F(y_n, x_n) = \lim_{n\to\infty} g(y_n) = y$

Since F and g are semi compatible, we get

$\lim_{n\to\infty} M(gF(x_n, y_n), F(x,y), t) = 1$ and $\lim_{n\to\infty} M(gF(y_n, x_n), F(y, x), t) = 1$

$\lim_{n\to\infty} N(gF(x_n, y_n), F(x,y), t) = 0$ and $\lim_{n\to\infty} N(gF(y_n, x_n), F(y,x), t) = 0$ (3.6)

By the continuity of the mapping g, we have

$\lim_{n\to\infty} gF(x_n, y_n) = \lim_{n\to\infty} gg(x_n) = g(x)$,

$\lim_{n\to\infty} gF(y_n, x_n) = \lim_{n\to\infty} gg(y_n) = g(y)$ (3.7)

Thus equation (3.6) and (3.7) yields that

$\lim_{n\to\infty} M(g(x), F(x,y), t) = 1$ and $\lim_{n\to\infty} M(g(y), F(x,y), t) = 0$,

and

$\lim_{n\to\infty} M(g(x), F(x,y), t) = 1$ and $\lim_{n\to\infty} N(g(y), F(x,y), t) = 0$

which implies that gx = F(x, y) and gy = F(y, x).

Step III. Now we prove that gx = y and gy = x.

Since $*$ and ◊ is a t-norm and t-conorm of H-type, by using (3.2)

Since M(x, y,·) and N(x,y,·) is continuous and

$\lim_{t\to+\infty} M(x,y,t) = 1$ and $\lim_{t\to+\infty} N(x,y,t) = 0$

for all x, y ∈ X,there exists $t_0 > 0$ from equation (3.3)

On the other hand, since $\emptyset \in \Phi$, by condition (A_3)

we have $\sum_{n=1}^{\infty} \emptyset^n(t_0) < \infty$.

Then for any t > 0, there exists $n_0 \in N$ such that

$t > \sum_{k=n_0}^{\infty} \emptyset^k(t_0)$.

Since $M(gx, gy_{n+1}, \emptyset(t_0)) = M(F(x,y), F(y_n, x_n), \emptyset(t_0))$

$\geq M(gx, gy_n, t_0) * M(gy, gx_n, t_0)$,

and

$N(gx, gy_{n+1}, \emptyset(t_0)) = N(F(x,y), F(y_n, x_n), \emptyset(t_0))$

$\leq N(gx, gy_n, t_0) \lozenge N(gy, gx_n, t_0)$

letting n→ ∞, we get

$M(gx, y, \emptyset(t_0)) \geq M(gx, y, t_0) * M(gy, x, t_0)$,

and $N(gx, y, \emptyset(t_0)) \leq N(gx, y, t_0) \lozenge N(gy, x, t_0)$ (3.8)

Similarly, we can get

$M(gy, x, \emptyset(t_0)) \geq M(gy, x, t_0) * M(gx, y, t_0)$,

and $N(gy, x, \emptyset(t_0)) \leq N(gy, x, t_0) \lozenge N(gx, y, t_0)$. (3.9)

From (3.8) and (3.9), we have

$M(gx, y, \emptyset(t_0)) * M(gy, x, \emptyset(t_0)) \geq [M(gx, y, t_0)]^2 * [M(gy, x, t_0)]^2$

and $N(gx, y, \emptyset(t_0)) \lozenge N(gy, x, \emptyset(t_0)) \leq [N(gx, y, t_0)]^2 \lozenge [N(gy, x, t_0)]^2$

By this way, we get for all n ∈ N,

$$M(gx, y, \emptyset^n(t_0)) * M(gy, x, \emptyset^n(t_0)) \geq \left[M\left(gx, y, \emptyset^{n-1}(t_0)\right)\right]^2 * \left[M\left(gy, x, \emptyset^{n-1}(t_0)\right)\right]^2$$
$$\geq [M(gx, y, t_0)]^{2^n} * [M(gy, x, t_0)]^{2^n}$$

and

$$N(gx, y, \emptyset^n(t_0)) \lozenge N(gy, x, \emptyset^n(t_0)) \leq \left[N\left(gx, y, \emptyset^{n-1}(t_0)\right)\right]^2 \lozenge \left[N\left(gy, x, \emptyset^{n-1}(t_0)\right)\right]^2$$
$$\leq [N(gx, y, t_0)]^{2^n} \lozenge [N(gy, x, t_0)]^{2^n}$$

Since $t > \sum_{k=n_0}^{\infty} \emptyset^k(t_0)$, then, we have

$$\begin{aligned} M(gx, y, t) * M(gy, x, t) &\geq M\left(gx, y, \textstyle\sum_{k=n_0}^{\infty} \emptyset^k(t_0)\right) * M\left(gy, x, \textstyle\sum_{k=n_0}^{\infty} \emptyset^k(t_0)\right) \\ &\geq M(gx, y, \emptyset^{n_0}(t_0)) * M(gy, x, \emptyset^{n_0}(t_0)) \\ &\geq [M(gx, y, t_0)]^{2^{n_0}} * [M(gy, x, t_0)]^{2^{n_0}} \\ &\geq \underbrace{(1-\mu) * (1-\mu) * \ldots * (1-\mu)}_{2^{2n_0}} \geq 1-\lambda. \end{aligned}$$

Similarly

$$N(gx, y, t) \lozenge N(gy, x, t) \leq \underbrace{(1-\mu) \lozenge (1-\mu) \lozenge \ldots \lozenge (1-\mu)}_{2^{2n_0}} \leq 1-\lambda.$$

So for any $\lambda > 0$ we have

$$M(gx, y, t) * M(gy, x, t) \geq 1-\lambda, \quad \text{and} \quad N(gx, y, t) \lozenge N(gy, x, t) \leq 1-\lambda,$$

for all $t > 0$. Hence we get $gx = y$ and $gy = x$.

Step IV. To Prove $x = y$.

Using condition (3.2), (3.3) and (3.4) of step 1,
we have consider

$$\begin{aligned} M(gx_{n+1}, gy_{n+1}, \emptyset(t_0)) &= M(F(x_n, y_n), F(y_n, x_n), \emptyset(t_0)) \\ &\geq M(gx_n, gy_n, t_0) * M(gy_n, gx_n, t_0), \end{aligned}$$

and

$$\begin{aligned} N(gx_{n+1}, gy_{n+1}, \emptyset(t_0)) &= N(F(x_n, y_n), F(y_n, x_n), \emptyset(t_0)) \\ &\leq N(gx_n, gy_n, t_0) \lozenge N(gy_n, gx_n, t_0) \end{aligned}$$

Letting $n \to \infty$ yields

$$M(x, y, \emptyset(t_0)) \geq M(x, y, t_0) * M(y, x, t_0).$$

and $N(x, y, \emptyset(t_0)) \leq N(x, y, t_0) \lozenge N(y, x, t_0)$

Thus, we have

$$\begin{aligned} M(x, y, t) &\geq M\left(x, y, \textstyle\sum_{k=n_0}^{\infty} \emptyset^k(t_0)\right) \\ &\geq M(x, y, \emptyset^{n_0}(t_0)) \\ &\geq [M(x, y, t_0)]^{2^{n_0}} * [M(y, x, t_0)]^{2^{n_0}} \\ &\geq \underbrace{(1-\mu) * (1-\mu) * \ldots * (1-\mu)}_{2^{2n_0}} \geq 1-\lambda. \end{aligned}$$

and

$$\begin{aligned} N(x, y, t) &\leq N\left(x, y, \textstyle\sum_{k=n_0}^{\infty} \emptyset^k(t_0)\right) \\ &\leq N(x, y, \emptyset^{n_0}(t_0)) \\ &\leq [N(x, y, t_0)]^{2^{n_0}} \lozenge [N(y, x, t_0)]^{2^{n_0}} \\ &\leq \underbrace{(1-\mu) \lozenge (1-\mu) \lozenge \ldots \lozenge (1-\mu)}_{2^{2n_0}} \leq 1-\lambda. \end{aligned}$$

which implies that $x = y$.

Thus we have proved that f and g have a unique common fixed point in X.

This completes the proof.

Theorem 3.2. Let (X, M, N, $*$,◊) be an intuitionistic fuzzy metric space, where $*$ is a continuous t-norm and ◊ is a continuous t-co-norm of H-type defined by $t * t \geq t$ and $(1-t) \lozenge (1-t) \leq (1-t)$ satisfying (2.1) and (2.2).Let F:X×X→X and g:X→X be two mappings and there exists $\emptyset \in \Phi$ such that

$M(F(x, y),F(u, v),\emptyset(t)) \geq M(g(x),g(u),t) * M(g(y),g(v),t)$,

$N(F(x, y),F(u, v),\emptyset(t)) \leq N(g(x),g(u),t) \lozenge N(g(y),g(v),t)$

for all x, y, u, v ∈ X, t > 0.

Suppose that F(X×X) ⊆ g(X), and g is continuous, F and g are compatible Then there exist x, y ∈ X such that x = g(x) = F(x, x), that is, F and g have a unique common fixed point in X.

Theorem 3.3. Let (X, M, N, $*$,◊) be an intuitionistic fuzzy metric space, where $*$ is a continuous t-norm and ◊ is a continuous t-co-norm of H-type defined by $t * t \geq t$ and $(1-t) \lozenge (1-t) \leq (1-t)$ satisfying (2.1) and (2.2). Let F: X×X→X and g:X→X be two weakly compatible mappings and there exists $\emptyset \in \Phi$ such that

$M(F(x,y),F(u,v),\emptyset(t)) \geq M(g(x),g(u),t) * M(g(y),g(v),t)$,

$N(F(x, y),F(u, v),\emptyset(t)) \leq N(g(x),g(u),t) \lozenge N(g(y),g(v),t)$,

for all x, y, u, v ∈ X and t > 0.

Suppose that F(X×X) ⊆ g(X), F(X×X) or g(X) is complete. Then F and g have a unique common fixed point in X.

Taking g = I (the identity mapping) in Theorem 3.3, we get the following consequence.

Corollary 3.1. Let (X, M, N, $*$,◊) be an intuitionistic fuzzy metric space, where $*$ and ◊ is a continuous t-norm and continuous t-conorm of H-type satisfying (2.1) and (2.2). Let F:X×X→X and there exists $\emptyset \in \Phi$ such that

$M(F(x, y),F(u, v),\emptyset(t)) \geq M(x, u, t) * M(y,v,t)$,

and $N(F(x, y),F(u, v),\emptyset(t)) \leq N(x,u,t) \lozenge N(y,v,t)$, for all x, y, u, v ∈ X and t > 0.

Then there exist x ∈ X such that x = F(x, x), that is, F admits a unique fixed point in X.

Let $\emptyset(t) = kt$, where 0 < k < 1,the following by Lemma 1,we get the following

Proof:-If set g = I Identity map in Theorem 3.3 then the proof is complete.

Corollary 3.2. Let $a * b \geq ab$ for all a, b∈ [0,1]and (X, M,N,$*$,◊) be an intuitionistic fuzzy metric space such that M and N has n-property. Let F: X×X→X and g: X→X be two functions such that

$M(F(x, y),F(u, v),kt) \geq M(gx,gu,t) * M(gy,gv,t)$,

and $N(F(x, y),F(u, v),kt) \leq N(gx,gu,t) \lozenge N(gy,gv,t)$

for all x, y, u, v ∈ X, where 0 < k < 1, F(X× X) ⊂ g(X) and g is continuous and commutes with F. Then there exist a unique x∈X such that x = g(x) = F(x, x).

4. Conclusion

Theorem 3.1 is a generalization of result of Hu [6] in fuzzy metric spaces to intutitionistic fuzzy metric spaces. Theorem 3.2 is a generalization of result of Hu et. al. [7] in fuzzy metric spaces to intutitionistic fuzzy metric spaces and corollary 3.1 is a generalization of corollary 3.3 of Hu et. al. [7] and corollary 3.2 is a generalization of corollary 2 of Hu [6] and corollary 2.6 of Sedghi [12].

REFERENCES

1. C.Alaca, D.Turkoglu and C.Yildiz, Fixed points in intuitionistic fuzzy metric spaces, *Chaos, Solitons & Fractals,* 29 (2006) 1073-1078.
2. K. Atanassov, Intuitionistic fuzzy sets, *Fuzzy Sets and Systems,* 20 (1986) 87-96.
3. T.G.Bhaskar and V.Lakshmikantham, Fixed point theorems in partially ordered metric spaces and applications, *Nonlinear Anal.*, 65 (2006) 1379-1393.
4. J.X.Fang, Common fixed point theorems of compatible and weakly compatible maps in menger spaces, *Nonlinear Analysis Theory, Methods &Applications,*71 (5-6) (2009) 1833-1843.
5. O.Hadzic and E.Pap, *Fixed point theory in probabilistic metric space*, Vol. 536 of mathematics and its applications, Kluwer Academic, Dordrecht, The Netherlands, 2001.
6. X.Q.Hu, Common coupled fixed point theorems for contractive mappings in fuzzy metric spaces, *Fixed Point Theory and Application*, (2011) Article ID 363716, 14 pages.
7. X.Q.Hu, M.X.Zheng, B.Damjanovic and X.F.Shao, Common coupled fixed point theorems for weakly compatible mappings in fuzzy metric spaces, *Fixed Point Theory and Applications*, 220 (2013).
8. V.Lakshmikantham and L.Ḉiriḉ, Coupled fixed point theorems for nonlinear contraction in partially ordered metric spaces, *Nonlinear Analysis Theory, Methods & Applications,*70 (12) (2009) 4341-4349.
9. R.Lowen, *Fuzzy Set Theory*, Kluwer Academic Pub., Dordrecht 1996.
10. J.H.Park, Intuitionistic fuzzy metric spaces, *Chaos, Solitons & Fractals*, 22 (2004) 1039-1046.
11. B.Schweizer and A.Sklar, Statistical metric spaces, *Pacific J. Math.*, 10 (1960) 314-334.
12. S.Sedghi, I.Altun and N.Shobe, Coupled fixed point theorems for contractions in fuzzy metric spaces, *Nonlinear Anal.*, 72 (2010) 1298-1304.
13. Sumitra, Coupled fixed point theorem for weakly compatible mappings in intuitionistic fuzzy metric space, *International Journal of Computer Application*, 50(23) (2012) 1-5.
14. D.Turkoglu, C.Alaca, Y.J.Cho and C.Yildiz, Common fixed point theorems in intuitionistic fuzzy metric spaces, *J. Appl. Math. and Computing*, 22 (2006) 411-424.
15. D.Turkoglu, C.Alaca and C.Yildiz, Compatible maps and compatible maps of types (α) sand (β) in intuitionistic fuzzy metric spaces, *Demonsstratio Math.*, 39 (2006) 671-684.