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An EPQ Model for Deteriorating Items under Random Planning Horizon with Some Linguistic Relations Between Demand, Selling Price and Trade Credit, Ordered Quantity

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Abstract. In this paper, an environment friendly Economic Production Quantity (EPQ) model of a single item is considered in which the business in each cycle starts with shortage and ends with the end of stock. The whole problem is formulated to maximize profit of the manufacturer with random business period and the randomness is removed by chance constrained method. This model involves selling price dependent demand and purchased raw material dependent credit period which are described by two sets of linguistic relations under fuzzy logic. In addition, a new method of payment of due raw material cost (DRC) (DRC is paid as soon as it can possible) is prescribed with supported lemma and a comparative study has been done between the new method of payment and the old method of payment (DRC is paid at the end of cycle). The model is optimized by a real coded genetic algorithm (GA) developed for this purpose with tournament selection, arithmetic crossover and polynomial mutation. The model is illustrated with different sets of numerical examples for different scenarios. A practical application has also been demonstrated with real world data. Some sensitivity analysis are presented graphically.

Keywords: Fuzzy logic; genetic algorithm; construction of membership function; delay in payment; chance constrained technique

AMS Mathematics Subject Classification (2010): 90B05

1. Introduction

The concept of EPQ model with production center and sale counter together is to determine the optimum produced quantity against the customers' demand so that total cost involved in the system is minimum. In this process, normally the production firm pays the supplier for the raw materials as and when these are purchased. Now-a-days, with the advent of multi-national in the markets of developing countries like India, Nepal, China, etc, competition between the traders / suppliers is very stiff and they take up different promotional ventures / tools to push the sale. In real practice, a supplier provides forward financing to the retailers i.e. offers credit period for payment to attract more customers. In this systems, relaxed period for payment is given to the firm management if the outstanding dues are paid within a given credit period. Here credit period is treated as a promotional tool as it is one kind of price discount because paying later circuitously reduces the purchasing cost and motivates the firms to increase the ordered quantity and to go for more production.

Goyal [21] firstly explored an EPQ model under the conditions of permissible delay in payment. Chung et al. [14] derived the optimal pricing and ordering policy for an integrated inventory model when trade credit is linked to order quantity. Chen and Ouyang [10] developed an integrated fuzzy inventory model with permissible delay in payments. These models are developed under the assumption that suppliers offers a credit period to the wholesaler. This policy is named as single level credit system where in two level credit system, retailer gets a part of credit achieved by the wholesaler. Chen & Kang [7] developed an integrated inventory model considering permssible delay in payment and variant pricing strategy. Ho [6] presented integrated inventory model with price and credit linked demand under two level trade credit system. In the last two decades, the inventory models with trade credit have been widely studied by several researchers. Recently Das et al. [1] developed an integrated production inventory model under interactive fuzzy trade credit policy.

Deterioration of units is one of the most crucial factor in inventory problems for deteriorating items. Over the years, there are some investigations on inventory control / supply chain of deteriorating items with permissible delay in payment. Aggarwal and Jaggi [20] and Chu et al. [17] presented the ordering policies for deteriorating item with trade credit. Jamal et al. [18] allowed shortages in the model of Aggarwal and Jaggi. Chang and Dye [16] allowed the partial backlogged shortages with time dependent variation in deterioration rate in Jamal et al. model. Maragatham & Lakshmidevi [26] presents some fuzzy inventory model for deteriorating items with price dependent demand. Ouyang et al. [11] developed two inventory models for deteriorating items with permissible delay in payment. Some notable research papers of deteriorating items incorporating various types of assumptions are due to Bhunia et. al. [3], Sana [5], Chang et. al. [15] etc. Most of the above inventory models are developed with constant deterioration. Recently Sarkar et al. [2] developed an integrated inventory model with variable lead time, defective items and delay in payment.

In the existing literature, most of the inventory models are generally developed with the assumption of infinite planning horizon. Jaggi & Khanna [19] highlights on a Supply chain model for deteriorating items with stock-dependent consumption rate and shortages under inflation and permissible delay in payment. Gurnani [22] pointed out that an infinite planning horizon is of rare occurrence because with the passage of time, the inventory cost is likely to vary disproportionately, product specifications may be changed, etc. Also there are many real-life situations where the assumption of infinite planning horizon is not valid, i.e. the time periods of seasonal / fashionable products are normally finite and these are of single period [25] only. Moreover, the demands of customers change with time, production process improves with the improvement of technology over time, etc.

In decision making problems like inventory control systems being connected with the available data / possible values of the system parameters can not be always specified exactly i.e. deterministically. There are several reasons for that like lack of input information, multiple sources of data, fluctuating nature of parameter values, noise in data, bad statistical analysis, etc. For example, it is well known that demand of a commodity depends on its price. Now-a-days, in the volatile market, the price changes very often and thus it is almost impossible to give an exact mathematical relation between price and demand. Similar is the case with order quantity and credit period though it is a fact that offered credit period varies with order amount. But, in the society, these imprecise information / relations have been fairly communicated through human words such as high, low, large, medium, small etc. Commonly these relations are expressed as IF premise (antecedent) THEN conclusion (Consequent). These type of fuzzy relations are handled by fuzzy inference technique. The commonly used fuzzy inference techniques are- Mamdani type [23] and Takagi-Sugeno type [7, 24]. These two methods are differ in the way by which the output is calculated. There are several research papers using fuzzy logic in different areas of investigation. Ban et al. [9] discussed the stability of a simplest Takagi-Sugeno fuzzy control system. Recently Chakraborty et al. [4] used Mamdani fuzzy inference technique to solved an inventory model of deteriorating seasonal products with different price discounts.

Among the optimization techniques for the models with fuzzy logic, the evolutionary techniques are more useful. In the literature, There are several evolutionary methods such as Genetic Algorithm (GA), Particle Swarm Optimization (PSO), Ant Colony Optimization (ACO), etc. Maiti and Maiti [12] used simulated annealing (SA) and contractive mapping GA to solve a production inventory model. Gupta et al. [8] used rank based selection process in a real coded GA for solving an inventory model with interval valued inventory cost. Recently Chakraborty et al. [4] developed a real coded GA to solve an inventory model of deteriorating multi-items with price discount and variable demand defined by fuzzy inference under resource constraints.

To derive the relations between fuzzy parameters (such as demand and selling price) and to get the membership function of their different fuzzy values it requires one’s sufficient experience about the market. Normally, these values are expressed verbally by human languages and due to the complexity of human language, it is difficult to derive the image idea from the above market data. Again ideas about a fact vary from man to man. Chang [13] presented a methodology of construction membership function for group opinion aggregation based on a gradation process.

In spite of the above developments, there are some inventory control problems yet to be investigated such as till now, none has developed inventory models with trade credit defined with the help of fuzzy inference which is more realistic.

The present model defers from the others for incorporating the following new ideas.

• Normally, in the inventory models of trade credit, amount of trade credit is given deterministically through a numerical value. A relation is presented by a mathematical expression in crisp way. In practice, often this relation is expressed by "words" linguistically. Here, for the first time, linguistic relations between (price, demand) and (ordered quantity, credit period) are considered.

• A new method of payment of dues of retailer to supplier is presented and a lemma is presented which assures the validity of the new method. A comparative study has been done with the conventional method.

• The business period of the seasonal products are finite and varies every year. Thus the time period of these products are assumed as random having a probability distribution.

• The construction of membership function (MF) from the market / business data is very important for the model with fuzzy inferences. Here, a methodology is presented for the construction of MF from the marketing experts opinions.

• GA is very appropriate for the models with fuzzy logic. Here a GA has been developed for this purpose.

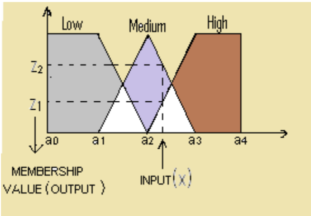
In this paper an EPQ model is considered in which the business starts with shortage. The manufacturer purchases raw material from the raw material supplier with delay in payment. The trade credit offered by the raw material supplier depends on the amount of raw material purchased through some fuzzy rules. After the end of offered credit period, the raw material supplier charges a high rate of interest on the unpaid amount. So, at the end of credit period, the cash in hand is paid to the supplier. Two methods are considered for the payment of the dues. Conventionally, dues are cleared at the end of time period. The proposed new one is clear the dues when the gross earn becomes equal to the rest unpaid amount with interest. The total raw material, required in a cycle and the produced quantities both are stored upto a certain time and their deteriorations are taken into account. It is also assumed that it requires $\delta\ (> 1)$ amount of raw material to produce a single finished unit. An environment protection cost is added with the production cost in order to reduce the carbon emission during production. Also some accessory costs due to production like laborer cost, wear & tear cost are considered. The per unit item selling price is fixed by imposing a mark-up on per unit item raw material cost and per unit time demand depends on the selling price. Also the time horizon is taken as random which follows the random distribution with known mean and standard deviation. Randomness of the cycle is removed using chance constraint technique. The whole problem is formulated to maximize the profit of the manufacturer and a real coded GA developed for this purpose. The model is illustrated numerically. For practical implication, raw data for the model parameters are collected from a manufacturing firm in India and these are represented as fuzzy numbers by constructing their membership functions. With these data, optimum inventory policy is derived for maximum profit with fuzzy selling price, demand, trade credit and ordering quantity. The difference between the conventional and new methods of clearing dues in graphically presented. Some useful relations between model parameters are also graphically depicted. The rest of this paper arranged in the following manner.

In sections -2, -3 and -4 some discussion have been made about fuzzy inference, chance constraint technique and a method of construction of fuzzy number respectively. The notations and assumptions for this model is given in section -5. The formulation of the model and the effect of trade credit on it are represented in the sections -6, -7 and -8 respectively. The fuzzy relations used in the model are given in the section -9. Section -10 contain some discussion about GA process and the optimum results with sensitivity analysis are made in section -11. Some discussion about the model and the conclusion are made in sections -12, -13 respectively.

2. Fuzzy inference methodology

The term "inference" refers to a process of obtaining new information by using existing knowledge and it is commonly referred to as IF-THEN rule-based form. It typically expresses an inference such that if we know a fact (premise, hypothesis, antecedent), then we can infer or derive another fact called a conclusion (consequent) i.e. "If x is $\tilde{A}$ Then y is $\tilde{B}$ ". Different steps of fuzzy inference process are-

Fuzzification of input value: When a value of premise is given as an input, it must correspond to some one or more linguistic fuzzy sets with some membership values.



Rule Strength Calculation: After the inputs are fuzzified, the degree to which each part of the antecedent is satisfied for each rule is known. The degree of a rule is the rule strength of the corresponding rule. If there are more than one antecedent then the rule strength is calculated by the standard min operator $\mu_{R_i} = \wedge\{\mu^{R_i}{}_{\tilde{A}}(x), \mu^{R_i}{}_{\tilde{B}}(y), \cdots\}$, where $\mu^{R_i}{}_{\tilde{A}}(x), \mu^{R_i}{}_{\tilde{B}}(y), \cdots$ are the membership values of the inputs $x, y, \cdots$ to the antecedents $\tilde{A}$, $\tilde{B}$ of the rule R_i. Thus the output is a single truth value for each rule and this is the rule strength of the corresponding rule lies between 0 and 1.

Fuzzy output: After calculation of rule strength for each rule, the fuzzy output implied by the rule is the area bounded by the line corresponding to the rule strength calculated by standard aggregation operator $\vee\{\mu_{\tilde{A}}(x), \mu_{\tilde{B}}(y), \cdots\}$.

Defuzzification: The defuzzification process consists of fuzzy output and gives a crisp value as an output. Here centroid formula, which returns the center of area under the curve, is given by, output $= \frac{\int x\, \mu_{(}x)}{\int \mu_{(}x)}$

3. Chance constraint

Chance constrained programming is one of the techniques of stochastic programming which deals with a situation where some or all parameters of the problem are described by random variables. In this discussion chance constraint is taken as

$$Prob(|N.T - \hat{H}| \leq \beta) \geq pr$$

where $\hat{H}$ is the random variable with mean $m_{\hat{H}}$ and standard deviation $\sigma_{\hat{H}}$ and NT is the deterministic from of $\hat{H}$. This can be rewrite as

$$Prob\big(N.T - \beta \leq \hat{H}\big) \geq pr \quad and \quad Prob\big(\hat{H} - N.T \leq \beta\big) \geq \text{pr}$$

From the first inequality

$$Prob\left(\frac{N.T-\beta-m_{\hat{H}}}{\sigma_{\hat{H}}} \leq \frac{\hat{H}-m_{\hat{H}}}{\sigma_{\hat{H}}}\right) \geq pr$$

Now $\frac{\hat{H}-m_{\hat{H}}}{\sigma_{\hat{H}}}$ represents the standard normal variate with mean 0 and variance 1.

$$i.e. \quad Prob(N.T - \beta \leq \hat{H}) = 1 - F\left(\frac{N.T-\beta-m_{\hat{H}}}{\sigma_{\hat{H}}}\right)$$

where $F(x)$ represents the continuous distribution function of standard normal distribution. Let ε be the standard normal value such that $F(\varepsilon) = pr = \frac{1}{\sqrt{2\Pi}} \int_{-\infty}^{\varepsilon} e^{-\frac{t^2}{2}} dt.$

Then the statement $Prob(N.T - \beta \leq \hat{H}) \geq pr$ is true if and only if

$$\frac{N.T-\beta-m_{\hat{H}}}{\sigma_{\hat{H}}} \leq -\varepsilon \Rightarrow N.T \leq m_{\hat{H}} + \beta - \varepsilon.\sigma_{\hat{H}}$$

Similarly from the 2nd inequality it can be reduced to

$$m_{\hat{H}} - \beta - \varepsilon.\sigma_{\hat{H}} \leq N.T$$

Therefore the equation $Prob(|N.T - \hat{H}| \leq \beta) \geq pr$ can be reduced to,

$$m_{\tilde{H}} - \beta - \varepsilon.\sigma_{\tilde{H}} \leq N.T \leq m_{\tilde{H}} + \beta - \varepsilon.\sigma_{\tilde{H}}$$ where,

$$pr = F(\varepsilon) = \frac{1}{\sqrt{2\Pi}} \int_{-\infty}^{\varepsilon} e^{-\frac{t^2}{2}} dt$$

is the cumulative probability $P[t \leq \varepsilon]$ available in standard statistical table for different values of ε.

4. A method of construction of a fuzzy number

In this section, triangular fuzzy number corresponding to a market parameter [such as selling price, demand etc.] is constructed from a set of data collected from some market experts following Chang [2004]. The membership function of a triangular fuzzy number $\tilde{A}(m, a, b)$ is of the from.

$$\mu_{\tilde{A}}(x) = \begin{cases} \frac{x-a}{m-a}, & for\ a \leq x \leq m \\ \frac{b-x}{b-m}, & for\ m < x \leq b \\ 0, & elsewhere \end{cases}$$

Let $g_1, g_2, \cdots, g_n$ are the assertions made by n different experts for a particular parameter [like low demand, high selling price etc]. In estimation of the fuzzy numbers, the center around which the g_i gather is to be estimated by giving more importance to the g_i's lying closer to the center. In other words, the estimation of this center is a weighted average of the g_i. To approximate the center, a distance matrix $G = [d_{ij}]_{n,n}$ of the relative distances between g_i's is calculated, where $d_{ij} = |g_i - g_j|$ and $d_{ii} = 0$, $d_{ij} = d_{ji}$. Then the average relative distances corresponding to g_i is given by $\bar{d}_i = \sum_{j=1}^{n} d_{ij}/(n-1)$. In this method the degree of importance is determined by pair-wise comparisons between g_i's which is based on the average distances. If $P = [p_{ij}]_{n,n}$ be the pair -wise comparison matrix then $p_{ij} = \bar{d}_j/\bar{d}_i$, $p_{ii} = 1$, $p_{ij} = 1/p_{ji}$. Let w_i be the true degree of importance of g_i and $0 \leq w_i \leq 1$. As P is the matrix obtained from comparison of distances, it is perfectly consistent. Therefore, $p_{ij} = w_i/w_j\ \forall i, j$. If w be the column vector of w_i, then $Pw = nw$, which implies that n is an eigenvalue of P and w is the corresponding eigenvector with $\sum_{i=1}^{n} w_i = 1$, $w_j = 1/\sum_{i=1}^{n} p_{ij}$, $j = 1,2.., n$. The importance degree w_i serves as the weight associated with g_i.

Thus the mode of the fuzzy number, $m = \sum_{i=1}^{n} w_i g_i$.

The mean deviation of the fuzzy number $\tilde{A}(m, a, b)$ is defined as $\sigma = \frac{\int_a^b |x-m|.\mu_{\tilde{A}}(x)dx}{\int_a^b \mu_{\tilde{A}}(x)dx}$.

Rewriting this equation we have $\sigma = \frac{(m-a)^2+(b-m)^2}{3(b-a)}$.

Let ξ be the ratio of left spread to right spread, that is $\xi = \frac{m-a}{b-m}$. Using the expressions for σ and ξ we have, $a = m - \frac{3(1+\xi)\xi.\sigma}{1+\xi^2}$. $b = m + \frac{3(1+\xi)\sigma}{1+\xi^2}$.

Now as the parameters σ, ξ are depends on a, b therefore σ, ξ are unknown before a, b known. So σ, ξ are approximated from the collected data as follows. σ is approximated by the average deviation where the average deviation is calculated from the collected data by the formula $\sigma = \sum_{i=1}^{n} w_i |g_i - m|$. For the approximation of ξ all g_i's are partitioned into two set such as $Let,\ \Delta = \{1,2,\ldots,n\}$, $A = \{i\ ;\ g_i < m, i \in \Delta\}$, $B = \{i\ ;\ g_i \geq m, i \in \Delta\}$.

Now calculate two values g^l, g^r defined as $g^l = \frac{\sum_{i\in A} w_i g_i}{\sum_{i\in A} w_i}$, $g^r = \frac{\sum_{i\in B} w_i g_i}{\sum_{i\in B} w_i}$

Thus the ratio of the left spread to the right spread approximated as, $\xi = \frac{m-g^l}{g^r-m}$. Then the lower end and the upper end of the fuzzy number $\tilde{A}(m,a,b)$ are calculated.

5. Notations and assumptions

In the proposed model, the following notations and assumptions are used.

- T is the length of each cycle.
- $t_1 / t_2 / t_3$ is the time from beginning of the cycle when production starts / the shortage is fully back-logged / production end.
- M is the length of credit period in each cycle.
- $\widehat{H}$ is the random time horizon which follows normal distribution with mean $m_{\widehat{H}}$ and the standard deviation $\sigma_{\widehat{H}}$.
- K is per unit time rate of production.
- D is the per unit time demand.
- Q_r $(or,\ Q_p)$ is the total purchased amount of raw material (or, quantity produced) in a cycle.
- Q_1 $(or,\ Q_2)$ is the total amount of shortage (or, total amount of stock) in each cycle.
- $q(t)$ inventory position at any time t.
- (r_0) $or\ RC$ is the (per unit item) or total raw material cost in a cycle.
- r_1 is laborer cost.
- r_2 is wear and tear cost and α is a given real numbers with $0 \leq \alpha \leq 1$.
- r_3 is environment protection cost.
- $p_3(\ or, SC)$ is per unit item (or, total) shortage cost.
- $hc_1(\ or, HC)$ is per unit item per unit time (or, total) holding cost.
- S is the per unit item selling price.
- m_s is the mark-up imposed upon the per unit item raw material cost to fix up selling price (S).
- N (≥ 1) is the number of cycles.
- $i_e(or, i_p)$ is percentage of interest earn (or, interest payed).
- θ_r $(or,\ \theta_p)$ is the rate of deterioration of raw material (or, produced quantity).
- x is the time from the starting of a cycle when the total due raw material cost(DRC) is payed to the raw material supplier. It is assumed that $x > M$.
- $RE_m/RE_x/RE_T$ $(\ or, IE_m/IE_x/IE_T)$ is the amount of revenue earned upto time $t \leq M\ /\ t \leq x\ /\ t \leq T$ (or, interest earned upto time $t \leq M\ /\ t \leq x\ /\ t \leq T$) in a cycle.
- IP_x (or, IP_T) is the interest have to pay on DRC at the time $t = x\ (or, t = T)$.
- In the proposed model it is considered that Demand depends on rate of production and the length of credit period depends on total amount of raw material purchased following some fuzzy rules.
- δ (> 1) is the rate of usefulness of raw material to produce finished goods.
- It is also considered in the model that the manufacturer does not gives any penalty for shortage and the shortage amount is fully back-logged.

6. Model illustration

In this model the business horizon $\widehat{H}$ is considered as random which follows the normal distribution with parameters $(m_{\widehat{H}}, \sigma_{\widehat{H}})$ and in deterministic form the whole planning horizon is divided into N cycles each of length T. Therefore, the chance constraint is given by $$Prob\left(\left|N.T - \widehat{H}\right| \leq \beta\right) \geq \text{pr} \qquad (1)$$

And according to the chance constraint method (section-3) it can be reduced as

$$m_{\widehat{H}} - \beta - \varepsilon.\sigma_{\widehat{H}} \leq N.T \leq m_{\widehat{H}} + \beta - \varepsilon.\sigma_{\widehat{H}} \qquad (2)$$

where, $pr = F(\varepsilon) = \frac{1}{\sqrt{2\Pi}} \int_{-\infty}^{\varepsilon} e^{-\frac{t^2}{2}} dt$

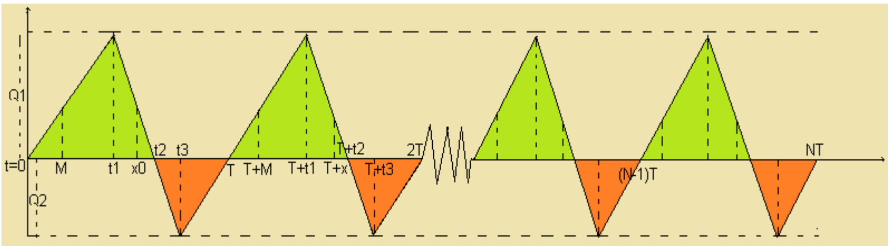


Figure 1: Business planning

In this business plain every cycle starts with shortage and ends with ending of stock. The manufacturer starts production at time $t = t_1$ from the starting of each cycle and at first shortage is fully back-logged (in time $t = t_2$) so the corresponding differential equation for each cycle is

$$\frac{dq(t)}{dt} = \begin{cases} K - D - \theta_p q(t) & for, t_2 \leq t \leq t_3 \\ -D - \theta_p q(t) & for, t_3 \leq t \leq T \end{cases} \qquad (3)$$

$$where,\ q(t) = \begin{cases} 0 & for, t = t_2 \\ Q_2 & for, t = t_3 \\ 0 & for, t = T \end{cases}$$

Also the total raw material (Q_r) decreases due to deterioration (at a rate θ_r) and for production (at a rate $\delta.K$), so the corresponding differential equation is

$$\frac{dq(t)}{dt} = -\theta_r q(t) - \delta.K, \quad t_2 \leq t \leq t_3 \qquad (4)$$

$$where,\ q(t) = \begin{cases} Q_r & for, t = t_1 \\ 0 & for, t = t_3 \end{cases}$$

Now every cycle contains the following time intervals

6.1. Shortage period [0 $\leq t \leq t_2$]

As from the starting of each cycle the shortage continuously increases up-to $t = t_1$ at demand rate and the production starts at $t = t_1$, so the total amount of shortage,

$$Q_1 = t_1.D$$

The shortage is fully back-logged with in the time $t = t_2$ in a rate $(K - D)$ so,

$$Q_1 = (t_2 - t_1)(K - D) \text{ and } t_2 = t_1 + \frac{Q1}{(K - D)}$$

6.2. Time period from end of shortage to end of production [$t_2 \leq t \leq t_3$]

In this case the corresponding differential equation is

$$\frac{dq(t)}{dt} = K - D - \theta_p.q(t).$$

Now using the conditions of equation (3), the inventory position in this time interval and the total amount of stock are reduced as follows

$$q(t) = \frac{(K-D)}{\theta_p}\left[1 - e^{\theta_p(t-t_2)}\right] \tag{5}$$

and $$Q_2 = \frac{(K-D)}{\theta_p}\left[1 - e^{\theta_p(t_3-t_2)}\right] \; where, \; t_2 \le t \le t_3$$

therefore, $$t_3 = t_2 + \frac{1}{\theta_p}\log\left(\frac{K-D}{K-D-\theta_p.Q_2}\right)$$

Solving the differential equation (4) and using the corresponding boundary conditions, the required amount of required raw material in a cycle is reduced as

$$Q_r = \frac{\delta.K}{\theta_r}\left[e^{\theta_r(t_3-t_1)} - 1\right]$$

Also the total produced amount is $Q_p = \frac{K}{\theta_r}\log\left(1 + \frac{Q_r\theta_r}{\delta.K}\right)$.

6.3. Time period from end of production to end of cycle [$t_3 \le t \le T$]

In this case the differential equation is $\frac{dq(t)}{dt} = -D - \theta_p.q(t)$, and from this differential equation the following expressions for the inventory position for this time interval and the length of the cycle are reduced using the conditions of equation (3)

$$q(t) = Q_2.e^{\theta_p(t-t_3)} - \frac{D}{\theta_p}\left[1 - e^{\theta_p(t-t_3)}\right] \tag{6}$$

$$T = t_3 + \frac{1}{\theta_p}log\left(1 + \frac{\theta_p.Q_2}{D}\right)$$

The costs arises in the model are given by the following-

6.4. Holding cost

The manufacturer holds quantities from $t = t_2$ to $t = T$ in every cycle. So using the expressions given by (5) and (6) the equation for total holding cost in cycle is reduced as follows $HC = hc_1\left[\int_{t_2}^{t_3} q(t)dt + \int_{t_3}^{T} q(t)dt\right]$

$$= \frac{hc_1}{\theta_p}\left[(K-D)\left\{(t_3-t_2) - \left(1 - e^{-\theta_p(t_3-t_2)}\right)\right\}\right.$$
$$\left. +\left(Q_2 + \frac{D}{\theta_p}\right)\left\{1 - e^{-\theta_p(T-t_3)}\right\} - D(T-t_3)\right] \tag{7}$$

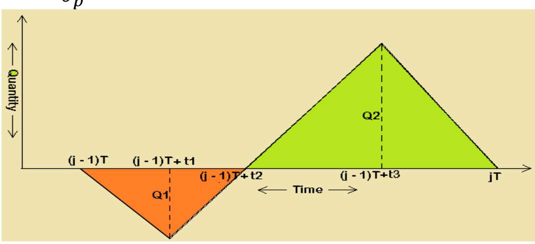


Figure 2: j-th cycle

6.5. Costs due to production

There are many accessories cost exists rather than raw material cost (RC) . Some of them namely- laborer cost, wear and tear cost and environment protection cost are considered in the proposed model under the name other cost (OC), where per unit laborer cost is inversely proportional to the rate of production, per unit wear and tear cost proportional to the rate of production and per unit item environment protection cost proportion to the rate of production up-to a certain degree α where $0 \le \alpha \le 1$. Thus the total raw material in a cycle, $RC = Q_r.r_0$ (8)

where r_0 is the per unit item raw material cost. The other cost is

$$OC = (r_1.k^{-1} + r_2.k + r_3.k^{\alpha})Q_p \ where\ 0 \le \alpha \le 1 \quad (9)$$

6.6. Shortage cost

As the total shortage amount is Q_1 and p_3 is per unit item shortage cost therefore per cycle shortage cost

$$SC = Q_1.p_3 \quad (10)$$

6.7. Set-up cost

The set-up cost is considered in two part, 1st one is constant and the 2nd one is proportion to the total quantity produced with a degree γ . Thus per cycle set-up cost is

$$SUC = su_1 + su_2.Q^{\gamma} \ where,\ 0 < \gamma < 1 \quad (11)$$

6.8. Selling price

A mark-up is imposed upon the per unit item raw material cost to fix the selling price. Therefore the selling price

$$S = m_s.r_0 \quad (12)$$

7. Credit period (Case 1: $t_1 \le M \le T$)

The raw material supplier takes interest on due payment in a rate $i_p\%$ from the producer after given some time gap of length M (credit period) from purchase and the producer earns some interest on earned revenue at a rate $i_e\%$. So, in any cycle the revenue earned by the producer in between $t = t_1$ to $t = M$ is $RE_m = S.K(t_2 - t_1) + S.D(M - t_2)$

and the earned interest on RE_m

$$IE_m = i_e.S\int_{t_1}^{t_2} K(t_2 - t)dt + i_e.S\int_{t_2}^{M} D(M - t)dt$$
$$= \frac{i_e.S}{2}[K(t_2 - t_1)^2 + D(M - t_2)^2]$$

Therefore, after end credit period the due raw material cost(DRC) is

$$DRC = RC - (RE_m + IE_m)$$
$$= p_1.K.(t_3 - t_1) - [S.K.(t_2 - t_1)\left\{1 + \frac{i_e.(t_2 - t_1)}{2}\right\}$$
$$+ S.D.(M - t_2)\{1 + \frac{i_e.(t_2 - t_1)}{2}\}] \quad (13)$$

7.1. A new strategy of due payment

As the raw material supplier offers a strategy of payment by which the buyer can pay DRC [given by (13)] at instant when earn is equal to DRC. Here x is the time from the starting of a cycle when Producer pays total DRC therefore, earned revenue(RE_x), earned interest (IE_x) and interest have to pay (IP_x) in between the time range $t = M$ to $t = x$ are

$$RE_x = S.D(x - M),\ IE_x = i_e.S.D\ \frac{(x-M)^2}{2},\ IP_x = i_p.DRC.(x - M)$$

Now according to the strategy of payment $RE_x + IE_x = DRC + IP_x$

From this relation the following quadratic equation is reduced.

$$A.(x - M)^2 + B.(x - M) + C = 0 \quad (14)$$

$$where,\ A = \frac{i_e.S.D}{2},\ B = (S.D - i_p.DRC),\ C = -DRC. \quad (15)$$

Before finding the roots of the above quadratic equation a lemma relating to the problem circumstances with the problem variables is proved below.

7.1.1. Lemma

If i_e, i_p, DRC, S and D all are positive with $i_p > i_e$ then the positive root of the quadratic equation

$$A.x^2 + B.x + C = 0 \ where, \ A = \frac{i_e.S.D}{2}, \quad B = (S.D - i_p.DRC), \quad C = -DRC$$

$$is \quad x = \frac{-B + \sqrt{B^2 - 4AC}}{2A}$$

Poof: Now $\sqrt{B^2 - 4AC} = (S.D - i_p.DRC)^2 - 4.\frac{i_e.S.D}{2}.(-DRC)$

$$= \{S.D - (i_p - i_e)(DRC)\}^2 + (DRC)^2(2i_p - i_e)i_e$$

As $i_p > i_e > 0$ therefore, $B^2 - 4AC > 0$.

Also it is given that $DRC > 0$ therefore $C < 0$. Again since $i_e, S, D > 0$ therefore $A > 0$. Hence $\sqrt{B^2 - 4AC} > |B|$ and for any value of B (positive or negative) $-B - \sqrt{B^2 - 4AC} < 0$ and $-B + \sqrt{B^2 - 4AC} > 0$. Hence the lemma.

As $x > M$, using the expressions (15) and the above lemma the solution of the equation (14) is $\quad x = M + \frac{-B + \sqrt{B^2 - 4AC}}{2A}$ (16)

7.1.2. After full payment of DRC [$x \leq t \leq T$]

After payment of dRC earned revenue($\boldsymbol{RE_T}$) and earned interest ($\boldsymbol{IE_T}$) on it are given by

$$RE_T = S.D.(T - x) \tag{17}$$

$$IE_T = i_e.S.D \int_x^T (T - t)dt$$

$$= \frac{i_e.S.D.(T - x)^2}{2} \tag{18}$$

where, x is given by equation (16).

7.1.3. Objective function

The whole time horizon is divided into N cycles of equal length, so the total profit(TP) is given by $\quad TP = N.(RE_T + IE_T - HC - OC - SC - SUC)$ (19)

where $RE_T, IE_T, HC, OC, SC \ and \ SUC$ are given by the equations (17), (18), (7), (9), (10) and (11) respectively.

7.2. Old strategy of due payment

In this case payment of due raw material cost is paid at the end of cycle. Therefore, the interest have to pay (IP_T) on DRC for the time range M to T is given by

$$IP_T = i_p.DRC(T - M), \tag{20}$$

where, DRC is given by equation (13).

Since the producer doesn't pay any amount of cash during the time gap t=M to t=T, so he gets some interest on the earned revenue. Thus the total earn in the time gap M to T (RE_T) and interest earned (IE_T) in this time gap is given by,

$$RE_T = s.D(T - M) \tag{21}$$

and $\quad IE_T = i_e.s.D\frac{(T - M)^2}{2}$ (22)

7.2.1. Objective function

Thus the profit function in this case is given by

$$TP = N.\left(RE_T + IE_T - DRC - IP_T - HC - OC - SC - SUC\right) \quad (23)$$

where $RE_T, IE_T, DRC, IP_T, HC, OC, SC$ *and* SUC are given by the equations (21), (22), (13), (20), (7), (9), (10) and (11) respectively.

8. Credit period (Case-2: $\boldsymbol{T \le M \le T + t_1}$)

In this case it is considered that the raw material supplier offers that manufacturer may place the payment at time before the placement of next order without any extra charge. As the manufacturer can earn interest on unpaid raw material cost as much as possible, so the earned revenue (RE_m) and earned interest on RE_m are given by the following.

$$RE_m = S.K.(t_2 - t_1) + S.D.(T - t_2) \quad (24)$$

$$IE_m = i_e S\{\int_{t_1}^{t_2} K(t_2 - t)dt + \int_{t_2}^{T} D(T - t)dt + K(t_2 - t_1)(M - t_2) + D(T - t_2)(M - T)\}$$

$$= i_e S[\tfrac{1}{2}\{K(t_2 - t_1)^2 + D(T - t_2)^2\} + K(t_2 - t_1)(M - t_2) + D(T - t_2)(M - T)] \quad (25)$$

8.1. Objective function

Thus in this case the total profit(TP)

$$TP = N.\left(RE_m + IE_m - RC - HC - SC - OC - SUC\right) \quad (26)$$

where $RE_m, IE_m, HC, RC, OC, SC$ *and* SUC are given by the equations (24), (25), (7), (8), (9), (10) and (11) respectively.

9. Fuzzy rules used in the model

The linguistic values considered for the problem variables selling price (S) and demand (D) are Low, $Medium$ and $High$. Also the The linguistic values considered for the problem variables raw material (Q_r) and credit period (M) are $Small$, $Medium$ and $Large$. All the linguistic values Low $(or, Small)$, $Medium$, $High$ $(or, Large)$ are handled by taking them as triangular fuzzy number of the from (l, m, u).

The model parameter demand (D) depends on the selling price (S) by the following fuzzy rules- **R-1**: If (S is Low) Then (D is $High$).

R-2: If (S is $Medium$) Then (D is $Medium$).

R-3: If (S is $High$) Then (D is Low).

Also the length of credit period (M) is depend on the purchased amount of raw material (Q_r) by the following fuzzy rules-

R-4: If (Q_r is $Small$) Then (M is $Small$).

R-5: If (Q_r is $Medium$) Then (M is $Medium$).

R-6: If (Q_r is $Large$) Then (M is $Large$).

10. GA process in the environment of the proposed model

A genetic algorithm finds the optimal solution from a given set of initial data by generating new population and evaluating them repeatedly. To generate a new population genetic algorithm uses three operators- reproduction, crossover and mutation. Initially, a population is selected and by means of above operators, the better of the population will remain, because of the survival of the fittest. By the repeated application of the operators-reproduction, crossover and mutation upto a finite number of generation (set by the user), the optimal result can be reached. The fitness function is the objective of the problem. A combination of real and natural numbers representation is used to structure a chromosome. Some discussions are made by following about the GA operators used in the present model.

Reproduction Operator: The principle object of a reproduction operator for using in GA is to make a population better than the previous one by replacing the bad solutions by duplicate copies of good solutions. There are many methods available such as tournament selection, proportionate selection and ranking selection.

Tournament Selection: In tournament selection, tournament are played between two solutions and the better solution is chosen and placed in mating pool. Similarly from other two solutions better one is chosen and placed in the matting pool. To carried out this process in similar manner we have to take each solution for tournament minimum two times. A best solution will win both times, thereby making two copies in the new population and in this way the worst solutions will be eliminated from the new population. It can also be mentioned that tournament selection revels better or same solutions than the other selection operators. Also it takes less computational time and has less complexity properties when compared to other reproduction operators. This is the cause why tournament selection operator is used here.

Arithmetic Crossover: This method works with two parent solutions and creates two offsprings. The present crossover operator obeys the interval schemata processing, in the sense that common interval schemata between the parents are preserved in the offspring. First a random number u_i between 0 and 1 is created. After that from specified probability distribution function, the ordinate β_{q_i} is calculated so that the area under the probability distribution curve from 0 to β_{q_i} is equal to the chosen random number u_i.

If $x_i^{(1,t)}$ and $x_i^{(2,t)}$ are denotes two parent solutions at the t-th generation then their two offspring $x_i^{(1,t+1)}, x_i^{(2,t+1)}$ can be calculated by the following steps.

- Choose a random number u_i in $[0,1)$.
- Calculate a number say β_{q_i} using the equation

$$\beta_{q_i} = \begin{cases} (2u_i)^{\frac{1}{DISBX+1}} & , \ u_i \le 0.5 \\ (\frac{1}{2(1-u_i)})^{\frac{1}{DISBX+1}} & , \ otherwise \end{cases}$$

where DISBX is any non-negative real number. A large value of DISBX give a higher probability for creating 'near-parent' solutions and small value allows distant solutions to be selected as offspring.

- Compute the offsprings by using the following two equations-

$$x_i^{(1,t+1)} = 0.5[(1+\beta_{q_i})x_i^{(1,t)} + (1-\beta_{q_i})x_i^{(2,t)}]$$
$$x_i^{(2,t+1)} = 0.5[(1-\beta_{q_i})x_i^{(1,t)} + (1+\beta_{q_i})x_i^{(2,t)}]$$

Polynomial Mutation: In polynomial mutation the polynomial function is used to represent the probability distribution. If $x_i^{(1,t+1)}$ be the offspring comes out after crossover and if $y_i^{(1,t+1)}$ be the muted copy then $y_i^{(1,t+1)} = x_i^{(1,t+1)} + (x_i^{(U)} - x_i^{(L)})\bar{\delta}_i$ where, $x_i^{(U)}, x_i^{(L)}$ are the variable upper bound and lower bound and the parameter $\bar{\delta_i}$ is calculated from the polynomial probability distribution $P(\delta) = 0.5(DIMUT + 1)(1 - |\delta|)^{DIMUT}$.

$$\bar{\delta_i} = \begin{cases} (2r_i)^{\frac{1}{DIMUT+1}} - 1 & , \ r_i < 0.5 \\ 1 - [2(1 - r_i)]^{\frac{1}{DIMUT+1}} & , \ r_i \geq 0.5 \end{cases}$$

where r_i is a random number in [0,1] and DIMUT is a positive real number. In this mutation operator the shape of the probability distribution is directly controlled by the external parameter DIMUT and the distribution is not dynamically changed with generations.

10.1. A routine framework for GA

At the beginning of the GA module, the different parameters of GA i.e. generation number (MAXGEN), population size (POPSIZE), probability of cross-over (PXOVER), probability of mutation (PMUT), random seed (RSEED), distribution index for SBX (DISBX) and for mutation (DIMUT) and the others. As there is no clear indication as to how large a population should be, here with $POPSIZE = no\ of\ variables \times 10$, the expected result is obtained. Here a combination of real and natural number representation is used to structure a chromosome, where a chromosome is a string of genes which are specified by the decision variables of the problem namely- length of the shortage period (t_1), mark-up (m_s) to fix up the selling price, Production rate (K), maximum amount of stock (Q_2) and the no. of cycles taken by the manufacturer (N). The variable boundaries may be fixed or flexible. The fitness function is the profit function (TP) defined by the manufacturer. An overall process of GA is given by the following algorithm.

Algorithm for the proposed GA

- Population initialization- Initializes the zero-th population.
- Run function_Model - Calculates the values of different model parameters for 0-th population.
- Set gen=0.
- check **if** $(gen < Maxgen)$, **then**

  { Run Selection operator → Run Crossover operator → Run Mutation operator.
  Create new population → Run Function_Model.
  if (all the constraints are satisfied) **then**,
  { print the result for the current generation.
  set, gen = gen+1.
  repeat step 4 }.
  $else$ repeat step 4 }. **else** Stop.

Here, the name "function_Model" reffers to all the mathematical expressions of the model with profit functions, constraints and all the concern model parameters.

11. Numerical experiment: illustration with practical data

A rice mill, Mahabir Rice Mill Company in Midnapore, West Bengal, India produces rice from raw paddy and sale to the retailers. Here, both the raw paddy and produced rice deteriorate and normally season oriented. The data from the said mill are collected and given below. For the construction of fuzzy MF, the opinions of experts / business managers in this field are taken into account.

11.1. Input data

Crisp Data: $\omega_r = 0.01,\ \omega_p = 0.005,\ \delta = 1.1,\ r_0 = 10,\ p_3 = 2,\ hc_1 = 0.15,\ r_1 = 0.5,\ r_2 = 0.005,\ r_3 = 0.5,\ su_1 = 5,\ su_2 = 2.5,\ \gamma = 0.01,\ \alpha = 0.4,\ i_e = 0.08,\ i_p = 0.12,\ \beta = 0.01,\ m_h = 48,\ \sigma = 0.16,\ \varepsilon = 1.3$.

GA Parameters: POPSIZE=50, MAXGEN=200, PXOVER=0.8, PMUT=0.2, RSEED=1.2, DISBX=2, DIMUT=100, t_1-(0.0 to 4), m_s-(1.6 to 2.3), K-(120 to 240), Q_2-(0 to 500) and N-(1 to 24).

Raw and Fuzzy Data: The raw data are collected from the market (expert's opinion) considering that the demand depends on selling price and credit period depends on total purchased amount of raw material. The data regarding the parameters (selling price, demand etc.) are given by the Table-1 and arranged maintaining the relations. The triangular fuzzy numbers Low (or, Small), Medium, High (or, Large) which are constructed from the collected raw data are given in the Table-1 and the membership functions are depicted in the Figures-3, -4, where the weight of the items are given in kilogram (kg), all the prices are given in Dollar ($)and the time intervals ar given in years.

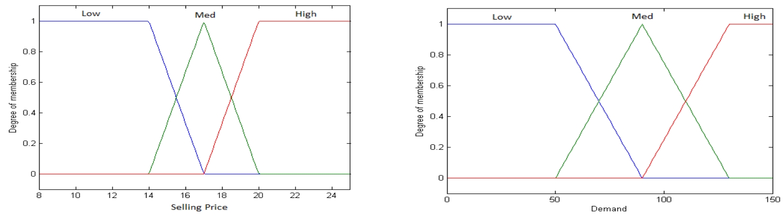


Figure 3: Membership functions of Selling price and Demand

Table 1: Collected raw data and the corresponding fuzzy numbers

name of linguistic fuzzy variable	Raw data from which Fuzzy Number is made	name of fuzzy number	Range of Fuzzy number
Selling Price	14.1 14.6 6.3 13.7 18.9	Low	(8, 14, 17)
Demand	147 154 97 122 146	High	(90, 130, 150)
Selling Price	22.8 16 17.3 17.3 16.2	Medium	(14, 17, 20)
Demand	79 104 57 104 71	Medium	(50, 90, 30)
Selling Price	17.7 18.7 19.9 19 28.2	High	(17, 20, 25)
Demand	50 81 64 33 50	Low	(0, 50, 90)
Total Quantity	627 571 1034 403 271	Small	(0, 550, 895)
Credit period	8 8 9.6 5.1 1.9	Small	(0, 7, 10)
Total Quantity	1187 168 882 871 1014	Medium	(550, 895, 1421)
Credit Period	10.4 10.6 8.7 9.6 15.8	Medium	(7, 10, 13)
Total Quantity	520 1692 1350 1386 1677	Large	(895, 1421, 2166)
Credit Period	22.8 11.9 13 12 10.9	Large	(10, 13, 20)

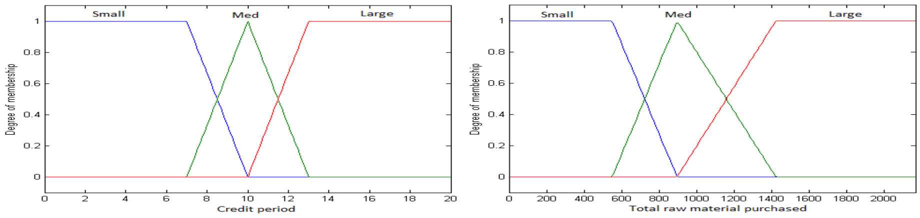


Figure 4: Membership functions of Total purchased raw material and Credit period

11.2. Optimum result

Table 2: Result Obtained using new methodology for the collected data

prob. Variable	case-1 $t_1 \leq M \leq t_2$	case-2 $t_2 \leq M \leq t_3$	case-3 $t_3 \leq M \leq T$	case-4 $T \leq M \leq T + t_1$	prob. Variable	case-1 $t_1 \leq M \leq t_2$	case-2 $t_2 \leq M \leq t_3$	case-3 $t_3 \leq M \leq T$	case-4 $T \leq M \leq T + t_1$
m_s	1.9	1.857	1.884	1.752	Q_r	559	635	738	1051
t_1	3.186	1.79	2.032	5.414	Q_d	498	563	652	929
t_2	4.835	2.781	2.719	5.414	Q_p	500	568	661	935
t_3	6.458	5.222	5.092	7.78	RC($)	5592	6353	7383	10507
T	9.56	9.557	11.95	11.95	SC($)	332	211	222	540
M	4.379	5.126	6.345	12.789	HC($)	58	131	263	160
X	5.396	5.48	6.579	–	OC($)	647	770	1064	1510
N	5	5	4	4	SUC($)	35	37	39	44
D	52	59	55	78	RDP($)	934	376	236	-
K	153	165	216	217	RE_T($)	4122	4464	5524	16285(RE_M)
Q_1	166	106	111	270	IE_T($)	687	728	1187	7863(IE_M)
Q_2	163	258	381	328	profit($)	18686	20221	20498	45548

12. Discussion

In the numerical experiment, some real life data are collected from a firm and presented in Table-1. Following the method in section-4, the membership functions for the different parameters are drawn and presented in Figs.3, 4. From these data it can be easily verified that the relations between demand is inversely proportional to selling price and purchased amount of raw material is proportional to credit period which support the rules given in section-9.

Optimum results given in Table-2 obtained using the new methodology of payment gives more profit than the results given in Table-3 obtained using the old payment policy in all cases. Also larger credit period gives more profit and from the results given in the Tables -2 and -3, it can be seen that the profit increases in the cases ($t_1 \leq$ M $\leq t_2$, $t_2 \leq$ M $\leq t_3$, $t_3 \leq$ M $\leq$ T, $T \leq$ M $\leq T + t_1$) in an ascending order of the time intervals of M. This is as per expectation. For new method of payment, it is considered that the time of payment of due cost (x) is always greater than the credit period (M). As a result increment in M reduces the gap between the time of payment for the new method (x) and the old method (T). Also the difference of profit in these results decreases as the credit period M becomes larger which is reflected in Fig.5(a).

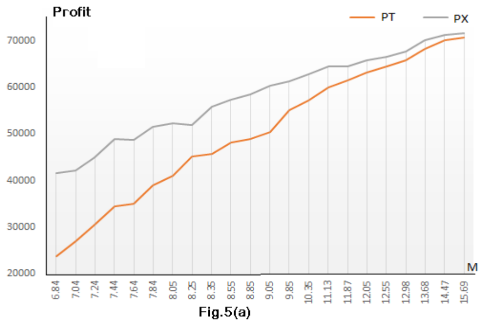
Table 3: Result obtained using old methodology for the collected data

prob. Variable	case-1 $t_1 \leq M \leq t_2$	case-2 $t_2 \leq M \leq t_3$	case-3 $t_3 \leq M \leq T$	case-4 $T \leq M \leq T + t_1$	prob. Variable	case-1 $t_1 \leq M \leq t_2$	case-2 $t_2 \leq M \leq t_3$	case-3 $t_3 \leq M \leq T$	case-4 $T \leq M \leq T + t_1$
m_s	1.9	1.857	1.884	1.752	Q_d	498	563	652	929
t_1	3.186	1.79	2.032	5.414	Q_p	500	568	661	935
t_2	4.835	2.781	2.719	5.414	RC($)	5592	6353	7383	10507
t_3	6.458	5.222	5.092	7.78	SC($)	332	211	222	540
T	9.56	9.558	11.95	11.95	HC($)	58	131	263	160
M	4.379	5.127	6.345	12.789	OC($)	647	770	1064	1510
N	5	5	4	4	SUC($)	35	37	39	44
D	52	59	55	78	RDP($)	934	376	236	-
K	153	165	216	217	IP_T($)	581	200	159	–
Q_1	166	106	111	270	RE_T($)	5129	4851	5765	16285(RE_M)
Q_2	163	258	381	328	IE_T($)	1063	860	1293	7863(IE_M)
Q_r	559	635	738	1051	profit($)	18030	19931	20302	45548

From Tables-2 and -3 it can be seen that the purchased raw material amount (Q_r) increases with cresdit period (M). This is the effect of fuzzy relations $(R_4 - R_6)$ which is also reflected in Fig.5(b). In this figure the curve of Q_r remains unchanged for each value of the M less than 7 as each value of credit period less than 7 takes a membership value 1 (c.f. Fig. 4), to the fuzzy number "Small" and therefore the rule strength of the rule R_4 (c.f. section-10) becomes 1. For this the purchased amount of raw material (Q_r) gets a constant value. Then for the next values of credit period (> 7) the amount of raw material increases as per expectation.

Figs.6(a) and 6(b) depict the variation in profit and demand with respect to the change in mark-up because the selling price is fixed by imposing a mark-up to a fixed number [given by (12)], so the change in selling price will make a same impression as the change in mark-up. Here, with the values of selling price $S\ (= m_s.r_0)$ the demand (D) changes inversely as per the relations $(R_1 - R_3)$. This is also depicted in Fig.6(a). In this figure, demand decreases as selling price (i.e. mark-up) increases and when selling price takes the value 21 (i.e. $m_s = 2.1$), the demand becomes constant as value of the fuzzy membership function for mark-up becomes 1 to the fuzzy number "High" (according to Fig.3 mark-up takes a constant membership value 1 in the range 2 - 2.5). This is also reflected in the Fig.6(b).

In the optimum results, profit decreases as mark-up increases. Normally, profit linearly related to mark-up and demand. It increases with the increase of selling price (i.e. mark-up) and / or demand. Here, with the fuzzy rules $(R_1 - R_3)$, demand decreases with selling price (i.e. mark-up). Thus selling price increases the profit and at the same time, decreases the demand which, in turn decreases the profit. On the profit, there is mixed effect due to selling price (i.e. mark-up) and demand. From Fig.6(a), it is seen that the effect of demand on profit dominates over the effect of mark-up (selling price) and for this reason, as mark-up (selling price) increases, profit decreases along with demand and for the value of m_s=2.1 (S=21), as demand becomes constant, there is only effect of mark-up (selling price) on profit and as a result, profit increases with mark-up.



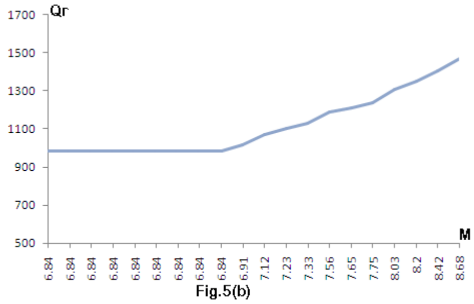
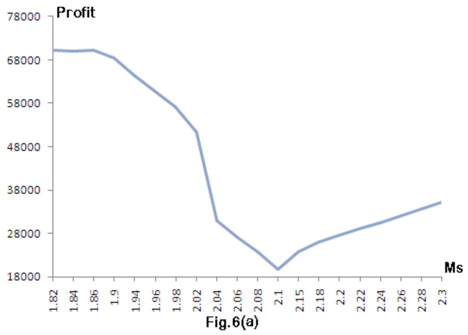


Figure 5(a): Length of credit period(M) / Difference Between Profit obtained using old method(PT) and new method(PX).
Figure 5(b): Length of credit period / Total raw material amount.



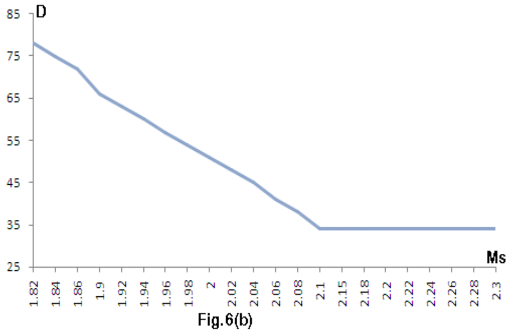


Figure 6(a): Mark-up(m_s) / Profit; **6(b):** Mark-up / Demand.

13.Conclusion

In this investigation, a practical problem for the inventory control system with trade credit is considered with some fuzzy relations between the decision variables and solved. For the first time. the membership functions for the parameters of the fuzzy relations, are formulated from some collected practical data and using fuzzy inference at two stage, optimum profits are determined and presented in Tabular and graphical form. A new method for repayment of dues is presented and compared with the conventional method. Here, fuzzy relations with single input and output have been used. Other forms of fuzzy relations can also be used. The model can be extended to include the promotional cost, profit sharing etc. among the supply chain partners.

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