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Magnetohydrodynamic Flow Past a Porous Spherical Aggregate with Stress Jump Condition

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Abstract. In this paper the effects of uniform magnetic field on a viscous, incompressible fluid past a spherical and permeable aggregate has been discussed using Brinkman's equation in the porous region and the Stokes equation in the external region. A uniform magnetic field is applied transverse to the fluid flow. At the interface of the porous region and the clear fluid stress jump boundary condition for tangential stresses, continuity of normal stress and continuity of velocity components are used. Normalized drag and normalized torque are calculated for different flows using Faxen's law. It is observed that the increase or decrease of the normalized drag and normalized torque depends on the stress jump coefficient due to the effect of magnetic field with variable permeability.

Keywords: Stokes flow, Brinkman's equation, stress jump coefficient, Normalized drag, Faxen's law.

1. Introduction

Science never gets more interesting than when it deals with objects in motion. And when it talks about the flow of a fluid, the outcome is even better. With various engineering and geophysical applications such as enhanced oil recovery, study of geothermal reservoirs, drying of porous solids, combustion in an inert porous matrix, adding to the environmental applications like study of floods, land erosion and underground spreading of chemical wastes, the flow of a viscous fluid has always remained a 'paradise' for the most brilliant heads on the planet.

Magneto-hydrodynamic (MHD) is the branch of continuum mechanics which deals with the flow of electrically conducting fluids in electric and magnetic fields. Many natural phenomena and engineering problems are worth being subjected to an MHD analysis. The phenomenon of chemical agglomeration plays a vital role in fluid dynamics due to its humungous industrial applications, when coupled with the flow of a viscous fluid through a porous medium. The term 'agglomeration' refers to the process of building larger bodies from smaller particles.

Evidently, this process of agglomeration stands as the basic need in the manufacture of many products like tablets, fertilizer pellets, fly ash and charcoal briquetted. The immersion of permeable agglomerates in their processing media results in progressive infiltration by the fluids. This phenomenon was observed and monitored earlier in the case of Silica, Calcium Carbonate, Carbon black and Titanium dioxide agglomerate [5,18]. Kinetics of dispersion of sparse agglomerates in simple shear flow was put into a deep research by Bohin et al [6]. Hydrodynamic analysis of porous spheres with infiltrated peripheral shells in linear flow fields was done by Levresse et al.[19]. A huge interest in flow past spherical boundaries emerged with the pioneering work of Hasimoto[12], who made a detailed study on axisymmetric flow past a rigid sphere. Subsequently, many scientists and tech-enthusiasts began researching and numerous papers appeared on this topic [8,31]. Several studies of the flow past and with non-porous bodies are deficient mainly to low Reynolds number. This low Reynolds number hydrodynamics was explained by Happel et al [11]. Lorentz's theorem on the Stokes' equalize was derived by Hasimoto[13]. The flow of a fluid with low Reynolds number past a porous spherical shell and the Stokes' flow past a porous particle were described in depth by Jones [15] and Higden et al [14]. Neale et al [21] and Qin et al [32] discussed the problem of creeping flow past a permeable sphere while Pop and Ingham [27] examined the problem of flow past a sphere embedded in a porous medium based on the Brinkman model. Stokes flow past a porous sphere using Brinkman's model invoked a specific interest in Padmavathi et al [24], who made a meticulous study on the same.

Beavers and Joseph [1] have proposed an empirical slip flow condition at the interface of a plane boundary for the rectilinear flow of a viscous fluid through a two dimensional parallel channel. Saffman [30] added a slip flow condition $(k \to 0)$ for the tangential velocity at the interface. Ochoa – Tapia and Whitaker [22,23] suggested, recently, a stress jump boundary condition at the fluid porous interface, where the porous region is governed by Brinkman's equation.

Kuznetsov [16,17] used this stress jump boundary condition at the interface between a porous medium and a clear fluid to discuss the flow in channels partially filled with porous medium. Recently, this stress jump condition was applied by Raja Sekhar and Sano [28] for two-dimensional viscous flow in a granular material with a void of arbitrary shape. Bhattacharyya and Raja Sekhar [2,3] have also used the stress jump condition for the viscous flow past a porous sphere with an impermeable core and a porous spherical shell. Of late, Bhattacharyya [4] discussed the effect of the moment transfer condition at the interface of creeping flow past a spherical permeable aggregate. Since then, a number of researches have explored the flow of an electrically conducting fluid through channels (ducts) because of its important application in MHD generators, pumps, accelerators, and flow meters.

In this paper, we consider a uniform magnetic field on the flow past a porous spherical aggregate using Brinkman's Model in a viscous incompressible fluid. At the interface of the porous liquid region, the stress jump boundary for the tangential stresses, the continuity of normal stresses and the continuity of velocity components are employed. Normalized drag and normalized torque are derived for various flows like uniform flow, doublet in a uniform flow and rotlet using Faxen's law. The significant effects and observations are discussed using the figures.

2. Mathematical formulation

Consider an arbitrary Stokes flow of a viscous electrically conducting incompressible fluid, past a stationary porous sphere with radius $'a'$. An uniform magnetic field is applied to the flow field with magnetic induction B_0. The governing equation of the flow inside the porous region $(r < a)$ is given by Brinkman's equation

$$\nabla p_1 = \mu \nabla^2 \boldsymbol{q}_1 - \frac{\mu}{k} q_1 - \sigma B_0^2 \boldsymbol{q}_1 \tag{1}$$

$$\nabla . \boldsymbol{q}_1 = 0 \tag{2}$$

where μ is the coefficient of viscosity, $k > 0$ is the permeability of the porous region, $\boldsymbol{q}_1$ is the volume rate of flow per unit cross section area , p_1 is the pressure, σ is the fluid conductivity and B_0 is the electromagnetic induction. The flow in the free flow region $(r > a)$ is governed by the Stokes equation

$$\nabla p_2 = \mu \nabla^2 \boldsymbol{q}_2 - \sigma B_0^2 \boldsymbol{q}_2 \tag{3}$$

$$\nabla . \boldsymbol{q}_2 = 0 \tag{4}$$

where $\boldsymbol{q}_2$ is the velocity and p_2 is the pressure. Introducing the transformation for non dimensionalize the physical quantities

$$\tilde{r} = \frac{r}{a},\ \boldsymbol{q}_x = \frac{\boldsymbol{q}_{1,2}}{\boldsymbol{q}_0},\ p_x = \frac{p_{1,2}}{\mu q_0 / a},\ M = \frac{\sigma B_0^2}{\mu} \tag{5}$$

where the subscript $x = i, e$ indicate the flow inside the porous region and outside the porous region respectively. q_0 is the velocity of the basic flow. M is the magnetic parameter.

Hence the governing equations in non-dimensional form in porous region are

$$\nabla p_i = \left(\nabla^2 - l_i^2\right)\boldsymbol{q}_i \tag{6}$$

$$\nabla . \boldsymbol{q}_i = 0 \tag{7}$$

where $l_i^2 = a^2 \left(\frac{1}{k} + M \right)$ is the characteristic measure of permeability and in the free flow region,

$$\nabla p_e = \left(\nabla^2 - l_e^2\right)\boldsymbol{q}_e \tag{8}$$

$$\nabla . \boldsymbol{q}_e = 0 \tag{9}$$

where $l_e^2 = a^2 M$ is the characteristic measure of magnetic field with characteristic radius a.

Let the velocity components in spherical coordinate system (r, θ, ϕ) be (q_r, q_θ, q_ϕ) and the corresponding stress components are given by

$$T_{rr} = -p + 2\mu \frac{\partial q_r}{\partial r} \tag{10}$$

$$T_{r\theta} = \mu \left[\frac{1}{r} \frac{\partial q_r}{\partial \theta} - \frac{q_\theta}{r} + \frac{\partial q_\theta}{\partial r} \right] \tag{11}$$

$$T_{r\phi} = \mu \left[\frac{1}{r \sin\theta} \frac{\partial q_r}{\partial \phi} - \frac{q_\phi}{r} + \frac{\partial q_\phi}{\partial r} \right] \tag{12}$$

3. Boundary condition

Beavers and Joseph [1] proposed a semi-empirical slip boundary condition that allows a non-zero velocity at the interface. They used Darcy's law to describe the flow in the porous medium which does not allow relating any boundary layer region with the porous region close to the interface. Due to this, the porous region is described by Darcy-Brinkman equation here, instead of Darcy's law. By applying a sophisticated volume averaging technique, Ochoa-Tapia and Whitaker [22] have shown that the process of matching the Brinkman-extended Darcy's law to the Stokes equation requires a discontinuity in the stress but retains the continuity of the velocity. Therefore, an appropriate stress jump boundary condition introduced by Ochoa-Tapia and Whitaker [22] for the tangential stress along with the continuity of the velocity components and that of the normal stresses are used in this problem.

i) Continuity of the velocity components on $r = a$

$$\boldsymbol{q}^e = \boldsymbol{q}^i \tag{13}$$

ii) Continuity of the normal stress on $r = a$

$$T_{rr}^e = T_{rr}^i \tag{14}$$

The stress jump boundary condition for the tangential stresses

$$\frac{\partial q_\theta^i}{\partial r} - \frac{\partial q_\theta^e}{\partial r} = \frac{\beta}{k_1} q_\theta^i \tag{15}$$

$$\frac{\partial q_\phi^i}{\partial r} - \frac{\partial q_\phi^e}{\partial r} = \frac{\beta}{k_1} q_\phi^i \tag{16}$$

where $k_1 = a / \sqrt{l_i^2 - Ma^2}$ and β is the stress jump coefficient.

iii) Condition at infinity: Let $\boldsymbol{q} = \boldsymbol{q}_0 + \boldsymbol{q}^*$ where $\boldsymbol{q}_0$ is the velocity of the basic flow and q^* is the disturbance in the presence of porous sphere, then $\boldsymbol{q}^* \to 0$ or $\boldsymbol{q} \to \boldsymbol{q}_0$ as $r \to \infty$

4. Method of solution

To solve the flow inside the porous region, Raja sekhar et al. [29] have shown the completeness of a representation of the Brinkman's equation. For the flow inside the porous region,

$$\boldsymbol{q}^i = CurlCurl(\boldsymbol{r}A^i) + Curl(\boldsymbol{r}B^i) \tag{17}$$

$$p^i = \mu \frac{\partial}{\partial r}\left(r(\nabla^2 - l_i^2) A^i \right) \tag{18}$$

where $l_i^2 = a^2\left(\frac{1}{k} + M\right)$ and A^i, B^i are scalars that satisfy $\nabla^2\left(\nabla^2 - l_i^2\right)A^i = 0$ and $\left(\nabla^2 - l_i^2\right)B^i = 0$

For the velocity and pressure outside the porous sphere, which are due to Stokes flow,

$$\boldsymbol{q}^e = CurlCurl\left(\boldsymbol{r}A^e\right) + Curl\left(\boldsymbol{r}B^e\right) \tag{19}$$

$$p^e = \mu \frac{\partial}{\partial r}\left(r(\nabla^2 - l_e^2) A^e \right) \tag{20}$$

where $l_e^2 = a^2 M$ and A^e, B^e are scalars such that $\nabla^2\left(\nabla^2 - l_e^2\right)A^e = 0$ and $\left(\nabla^2 - l_e^2\right)B^e = 0$.

The representations given in equations (17) and (19) support the following general form for the velocity components.

$$q_r = -\frac{1}{r} LA \tag{21}$$

$$q_\theta = \frac{1}{r}\frac{\partial}{\partial \theta}\frac{\partial}{\partial r}(rA) + \csc\theta \frac{\partial B}{\partial \phi} \tag{22}$$

$$q_\phi = \frac{1}{r\sin\theta}\frac{\partial}{\partial \phi}\frac{\partial}{\partial r}(rA) - \frac{\partial B}{\partial \theta} \tag{23}$$

where $L = \frac{1}{\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial}{\partial\theta}\right) + \csc^2\theta\frac{\partial^2}{\partial\phi^2}$ which is the transverse part of the Laplacian in (r,θ,ϕ) coordinate system.

Since we are considering an arbitrary Stokes flow without any singularities as basic flow, the velocity corresponding to the basic flow is

$$\boldsymbol{q}^0 = CurlCurl\left(\boldsymbol{r}A^0\right) + Curl\left(\boldsymbol{r}B^0\right) \tag{24}$$

A^0 and B^0 are given by

$$A^0(r,\theta,\phi) = \sum_{n=1}^{\infty}\left(\alpha_n r^n + \beta_n f_n(l_e r)\right)S_n(\theta,\phi) \tag{25}$$

$$B^0(r,\theta,\phi) = \sum_{n=1}^{\infty}\left(\sigma_n f_n(l_e r)\right)T_n(\theta,\phi) \tag{26}$$

where

$$S_n(\theta,\phi) = \sum_{m=0}^{n} P_n^m(\xi)\left(A_{mn}\cos m\phi + B_{mn}\sin m\phi\right) \tag{27}$$

$$T_n(\theta,\phi) = \sum_{m=0}^{n} P_n^m(\xi)\left(C_{mn}\cos m\phi + D_{mn}\sin m\phi\right) \tag{28}$$

are the spherical harmonics and $\xi = \cos\theta, P_n^m$ is the associated Legendre polynomial. $\alpha_n, \beta_n, \sigma_n,$ A_{nm} , B_{nm} , C_{nm} and D_{nm} are known constants. $f_n(l_e r)$ is the modified Bessel function of first kind which is finite at zero. A^0 and B^0 satisfy $\nabla^4 A^0 = 0$ and $\nabla^2 B^0 = 0$.

Due to the presence of the porous sphere, the modified flow in the liquid region $r > a$ is represented by

$$A^e = \sum_{n=1}^{\infty}\left(\alpha_n r^n + \beta_n f_n(l_e r) + \alpha_n' r^{-(n+1)} + \beta_n' g_n(l_e r)\right) S_n(\theta,\phi) \tag{29}$$

$$B^e = \sum_{n=1}^{\infty}\left(\sigma_n f_n(l_e r) + \sigma_n' g_n(l_e r)\right) T_n(\theta,\phi) \tag{30}$$

where α_n', β_n' and σ_n' are unknown constants to be determined from the boundary conditions. The scalars A^e and B^e for the modified flow, represent the disturbance caused to the basic flow in the region $r > a$ due to the presence of the porous sphere. The forms of A^e and B^e are assumed as in (29) and (30) by adding the perturbed terms to the basic flow. A^e and B^e satisfy the equations $\nabla^2\left(\nabla^2 - l_e^2\right)A^e = 0$ and $\left(\nabla^2 - l_e^2\right)B^e = 0$ respectively and the perturbed terms vanish as $r \to \infty$. The velocity components for the modified flow outside the sphere $(r > a)$ become

$$q_r^e = \sum_{n=1}^{\infty} n(n+1)\left(\alpha_n r^{n-1} + \frac{\beta_n}{r} f_n(l_e r) + \alpha_n' r^{-(n+2)} + \frac{\beta_n'}{r} g_n(l_e r)\right) S_n(\theta,\phi) \tag{31}$$

$$q_\theta^e = \sum_{n=1}^{\infty}\left\{\left[(n+1)\alpha_n r^{n-1} + \beta_n\left(l_e f_{n-1}(l_e r) - \frac{n}{r} f_n(l_e r)\right) - n\alpha_n' r^{-(n+2)}\right.\right.$$

$$\left.\left. - \beta_n'\left(l_e g_{n-1}(l_e r) + \frac{n}{r} g_n(l_e r)\right)\frac{\partial S_n}{\partial\theta}\right] + \csc\theta\frac{\partial T_n}{\partial\phi}\left[\sigma_n f_n(l_e r) + \sigma_n' g_n(l_e r)\right]\right\} \tag{32}$$

$$q_\phi^e = \sum_{n=1}^{\infty}\left\{\left[(n+1)\alpha_n r^{n-1} + \beta_n\left(l_e f_{n-1}(l_e r) - \frac{n}{r} f_n(l_e r)\right) - n\alpha_n' r^{-(n+2)}\right.\right.$$

$$\left.\left. - \beta_n'\left(l_e g_{n-1}(l_e r) + \frac{n}{r} g_n(l_e r)\right)\csc\theta\frac{\partial S_n}{\partial\phi}\right] - \frac{\partial T_n}{\partial\theta}\left[\sigma_n f_n(l_e r) + \sigma_n' g_n(l_e r)\right]\right\} \tag{33}$$

$g_n(l_e r)$ is the modified Bessel function of second kind. The representation of A^i and B^i in the porous region $r < a$ is

$$A^i = \sum_{n=1}^{\infty}\left(\varepsilon_n r^n + \delta_n l_n(l_i r)\right) S_n(\theta,\phi) \tag{34}$$

$$B^i = \sum_{n=1}^{\infty} (\gamma_n l_n(l_i r)) T_n(\theta,\phi) \tag{35}$$

where ε_n, δ_n and γ_n are unknown constants to be determined from the boundary conditions.

$l_n(l_i r)$ is the modified Bessel function of first kind which is finite at zero. A^i and B^i satisfy the equations $\nabla^2(\nabla^2 - l_i^2)A^i = 0$ and $(\nabla^2 - l_i^2)B^i = 0$ respectively. For the porous region $r < a$ the velocity components can be written as

$$q_r^i = \sum_{n=1}^{\infty} n(n+1)\left[\varepsilon_n r^{n-1} + \frac{\delta_n}{r} l_n(l_i r)\right] S_n \tag{36}$$

$$q_\theta^i = \sum_{n=1}^{\infty} \left\{\left[(n+1)\varepsilon_n r^{n-1} + \left(l_i l_{n-1}(l_i r) - \frac{n}{r} l_n(l_i r)\right)\delta_n\right]\frac{\partial S_n}{\partial \theta} + \csc\theta[\gamma_n l_n(l_i r)]\frac{\partial T_n}{\partial \phi}\right\} \tag{37}$$

$$q_\phi^i = \sum_{n=1}^{\infty} \left\{\left[(n+1)\varepsilon_n r^{n-1} + \left(l_i l_{n-1}(l_i r) - \frac{n}{r} l_n(l_i r)\right)\delta_n\right]\csc\theta\frac{\partial S_n}{\partial \phi} - [\gamma_n l_n(l_i r)]\frac{\partial T_n}{\partial \theta}\right\} \tag{38}$$

The unknown constants $\alpha_n', \beta_n', \sigma_n'$, ε_n δ_n, and γ_n' can be determined using the boundary conditions given in (13)-(16).

The boundary conditions given in (13)-(16) on the permeable boundary $r = a$, can be written in terms of the scalars A^e, B^e and A^i, B^i as follows

$$A^e = A^i \tag{39}$$

$$\frac{\partial A^e}{\partial r} = \frac{\partial A^i}{\partial r} \tag{40}$$

$$\frac{\partial^2 A^i}{\partial r^2} - \frac{\partial^2 A^e}{\partial r^2} = \frac{\beta}{k_1}\left(\frac{A^i}{r} + \frac{\partial A^i}{\partial r}\right) \tag{41}$$

$$r\frac{\partial^3}{\partial r^3}(A^i - A^e) = \left(l_i^2 - \frac{3\beta}{rk_1}\right)\frac{\partial}{\partial r}(rA^i) - l_e^2\frac{\partial}{\partial r}(rA^e) \tag{42}$$

$$B^e = B^i \tag{43}$$

$$\frac{\partial B^i}{\partial r} - \frac{\partial B^e}{\partial r} = \frac{\beta}{k_1} B^i \tag{44}$$

Now using these boundary conditions, the unknown co-efficients in (29)-(30) and (34)-(35) were determined. We have solved the system using Mathematica version 8.0.

5. Faxen's laws for a porous sphere

Faxen's laws provide an expression for the drag and torque acting on the rigid sphere of radius 'a' in an unbounded arbitrary stokes flow. The force D exerted on the porous sphere by the fluid in the region $r > a$ and the Torque T are given by

$$D = \int_0^{2\pi}\int_0^{\pi}\left[T_{rr}^e \hat{e}_r + T_{r\theta}^e \hat{e}_\theta + T_{r\phi}^e \hat{e}_\phi\right]_{r=a} a^2 \sin\theta d\theta d\phi \tag{45}$$

$$T = \int_0^{2\pi}\int_0^{\pi}\left[rT_{r\theta}^e \hat{e}_\phi - rT_{r\phi}^e \hat{e}_\theta\right]_{r=a} a^2 \sin\theta d\theta d\phi \tag{46}$$

$T_{rr}^e, T_{r\theta}^e$ and $T_{r\phi}^e$ are computed using the equations(5)-(7) and are used in equations (45) and (46) to get the following expressions for drag and torque respectively.

$$D = \frac{8}{3}\pi\mu\left[A_{11}\hat{i} + B_{11}\hat{j} + A_{10}\hat{k}\right]E \tag{47}$$

$$T = \frac{8}{3}\pi\mu\left[C_{11}\hat{i} + D_{11}\hat{j} + C_{10}\hat{k}\right]F \tag{48}$$

Where $E = l_e^2\left[\alpha_1 - \frac{\alpha_1'}{2} - \beta_1 f_1 + \beta_1' g_1\right]$

and $F = \sigma_1\left[-3f_1 + l_e f_0\right] + \sigma_1'\left[-3g_1 - l_e g_0\right]$

Now E and F are calculated in terms of known constants α_1, β_1 and σ_1. The value of α_1', β_1' and σ_1' are given in Appendix.

6. Examples

Here we discuss few examples.

(i) **Uniform flow**: Consider uniform flow U along x direction.

The basic flow is given by (A_0, B_0) where

$$A_0 = \frac{U}{2} r\cos\theta \tag{49}$$

$$B_0 = 0 \tag{50}$$

Comparing this (A_0, B_0) with the equations (25) and (26), we get

$$\alpha_1 = \frac{U}{2} \tag{51}$$

$$\beta_1 = 0 \tag{52}$$

Hence the normalized drag and torque can be written as

$$D_1 = \frac{D}{a\mu U} = \frac{8\pi E\hat{i}}{3\alpha_1} \tag{53}$$

$$T_1 = 0 \tag{54}$$

The variation of normalized drag with permeability for negative and positive values of stress jump coefficient β has been plotted in Fig.1 and Fig.2 respectively. These graphs have been plotted for different values of β ranging from -0.7 to 0.7 with reference to Ochoa-Tapia and Whitaker theory.

When the basic flow is uniform (in Fig.1), normalized drag decreases as the permeability increases, due to the magnetic field and negative stress jump coefficient. With the presence of a very negligible magnetic field, the relation between normalized drag and permeability depends upon the values of stress jump coefficient β. If β is positive the normalized drag generally increases with increase in permeability, but for lower values of β, a slight dip is found in normalized drag before the gradual increase, as seen in Fig.2. In Fig.3, it is seen that the normalized drag decreases with increasing permeability and increases with increasing magnetic induction.

(ii) **Doublet in a uniform flow**: A doublet of strength m is in a uniform flow U, at $(0,0,c)$. The basic flow is given by (A_0, B_0) where

$$A_0 = \frac{m}{c^2} + (\frac{m}{c^3} - \frac{U}{2}) r\cos\theta + \sum_{n=2}^{\infty} \frac{r^n}{c^{n+1}} P_n(\xi) \tag{55}$$

$$B_0 = 0 \tag{56}$$

In this case, the corresponding coefficients are given by

$$\alpha_1 = \frac{m}{c^3} - \frac{U}{2} \quad \text{and} \quad \alpha_n = \frac{1}{c^{n+1}} \quad \text{for} \quad n \geq 2 \tag{57}$$

$$\beta_1 = 0 \quad \text{for} \quad n \geq 1 \tag{58}$$

The normalized drag and torque can be written as

$$D_1 = \frac{D}{a\mu U} = \frac{4\pi E}{3} (\frac{c^3}{m - \alpha_1 c^3}) \hat{i} \tag{59}$$

$$T_1 = 0 \tag{60}$$

The behaviour of normalized drag with variation of permeability as well as stress jump coefficient in doublet in a uniform flow is almost similar to the case of uniform flow as shown in Fig.4. It is also seen from Fig.5 that the normalized drag increases gradually with increasing permeability for lower values of magnetic field induction.

(iii) **Rotlet**: Consider a rotlet of strength $\frac{F_2}{8\pi\mu}$ at $(0,0,c), (c > a)$ whose axis is along the positive direction of the y axis. The expression for A_0 and B_0 are

$$A_0 = \frac{-F_2}{8\pi\mu} \sum_{n=1}^{\infty} [\frac{r^n}{n(n+1)c^{n+1}}] P_n^1(\xi) \sin\phi \tag{61}$$

$$B_0 = \frac{-F_2}{8\pi\mu} \sum_{n=1}^{\infty} [\frac{r^n}{(n+1)c^{n+2}}] P_n^1(\xi) \sin\phi \tag{62}$$

In this case, the corresponding coefficients are given by

$$\alpha_1 = \frac{1}{2c^2}\frac{F_2}{8\pi\mu} \tag{63}$$

$$\beta_1 = 0 \tag{64}$$

$$\sigma_1 = \frac{1}{2c^3}\frac{F_2}{8\pi\mu} \tag{65}$$

Hence the normalized drag and torque can be written as

$$D_1 = \frac{D}{\frac{F_2 a}{2c^2}} = \frac{l_e^2(\alpha_1 - \alpha_1^{'}/2 + \beta_1^{'} g_1)}{3\alpha_1}\hat{i} \tag{66}$$

$$T_1 = \frac{T}{F_2 a^3/2c^3} = \frac{\sigma_1(-3f_1 + l_e f_0) + \sigma_1^{'}(-3g_1 - l_e g_0)}{3\sigma_1}\hat{j} \tag{67}$$

As seen in Fig.6, normalized drag in rotlet decreases as the permeability increases for negative stress jump coefficient values in the presence of a negligible magnetic field. The normalized drag is observed to gradually increase with increase in permeability, as shown in Fig.7, but for lower positive values of β , a slight dip is found in the normalized drag before the gradual increase, just like the one seen in uniform flow.

When a graph is plotted between normalized drag and permeability (Fig.8) in the presence of a negligible magnetic field in a rotlet, the increase in drag with respect to permeability is very negligible. However, there is a noticeable rise of drag when the magnetic field is increased gradually.

Normalized torque is observed from Fig.9 to Fig.14. Normalized torque decreases as permeability increases and the rate of decrement of torque increases with increase in stress jump coefficient owing to the magnetic field is shown in Fig.9. Fig.10 represents the increases of torque for the negative stress jump coefficient with respect to the permeability (various magnetic induction).

For the various magnetic induction, the normalized torque decreases as the permeability increases with constant positive stress jump coefficient, is represented in Fig.11. For the constant negative stress jump coefficient the normalized torque increases for various magnetic induction as the permeability increases is given in Fig.12.

Normalized torque decreases for increasing permeability as well as the coefficient of viscosity is given in Fig.13. Again the torque decreases for various electrical conductivity with respect to the permeability and the rate of decrement is depends on the decrease in electrical conductivity is shown in Fig.14.

7. Results and Conclusion

In this paper we have discussed the effect of a uniform magnetic field on the flow past a porous sphere of radius $'a'$, considering Brinkman equation in the porous region and stokes flow in the liquid region. At the porous liquid interface, the stress jump condition for tangential stresses, continuity for normal stresses and continuity of velocity components have been used. The effect of stress jump coefficient due to magnetic field on the flow quantities normalized drag and normalized torque has been observed for different flows like uniform flow, doublet in uniform flow and rotlet.

When the basic flow is uniform, the normalized drag decreases as the permeability increases in the presence of a negligible magnetic field for varying negative stress jump coefficient. But for positive stress jump coefficient, the normalized drag increases with increasing permeability with the effect of magnetic induction whereas in the case of varying magnetic induction the drag decreases as the permeability increases. For increasing magnetic induction drag increases and the graph plotted or drag against magnetic induction for various β does not give any significant difference.

The behaviour of normalized drag in doublet is almost same in uniform flow. Fig.4 shows that the normalized drag increases as permeability increases . When the basic flow is rotlet the normalized drag decreases for negative stress jump coefficient as the permeability increases. But the normalized drag increases for positive β is shown in Fig.7. The normalized drag decreases for increasing electrical conductivity with respect to the increasing permeability. Normalized torque increases for increasing stress jump coefficient is represented in Fig.9 & Fig.10. Evidently, magnetic field plays a significant role in the flow of fluids through a porous sphere, highly affecting the nature of drag.

As far as the behaviour of torque is concerned, it decreases gradually as the permeability increases in rotlet for varying values of stress jump coefficient, when exposed to a magnetic field. However, for the negative stress jump coefficient the normalized torque increases for the increasing permeability. The graph for various permeability does not affect the torque. In the case of magnetic induction torque decreases for positive stress jump coefficient and increases for negative stress jump coefficient. Normalized torque decreases for various viscous coefficient and electrical conductivity .

Hence it is observed that magnetic field plays a vital role while studying viscous flow problems for a porous sphere involving Brinkman equation in porous region and Stokes equation in the free flow region. It highly affects the physical quantities such as the drag and torque of the flow. The interest in MHD fluid flow stems because of its enormous application in distinguished devices such as MHD power generators, accelerators, centrifugal separation of matter from fluid, fluid droplet sprays, purification of crude oil, petroleum industry, polymer technology and so forth. The findings may be useful for the study of movement of oil, gas and water through the reservoirs of an oil field or a gas field, in the migration of underground water and in the filtration and water purification processes. The outcomes of this research are also of great importance in geophysics in the study of interaction of the geomagnetic field with fluid in the geothermal region.

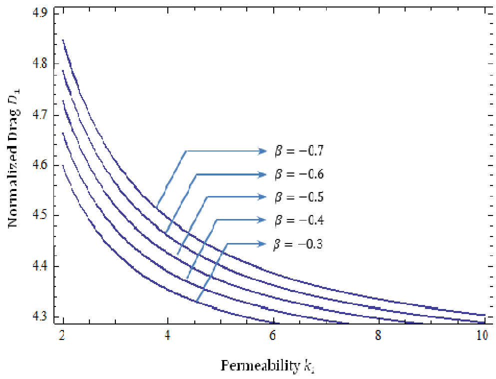


Fig. 1. Variation of normalized drag with permeability for negative stress jump coefficient with $B_0 = 1, \sigma = 1$ and $\mu = 1$ in the case of Uniform flow.

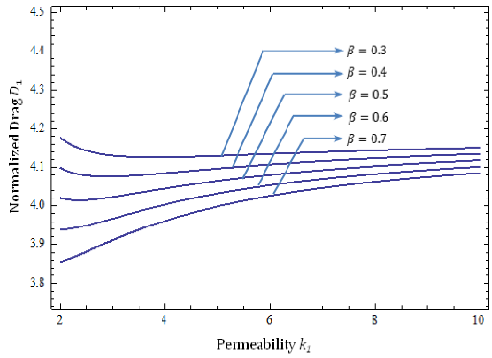


Fig. 2. Variation of normalized drag with permeability for positive stress jump coefficients with $B_0 = 1, \sigma = 1$ and $\mu = 1$ in the case of Uniform flow.

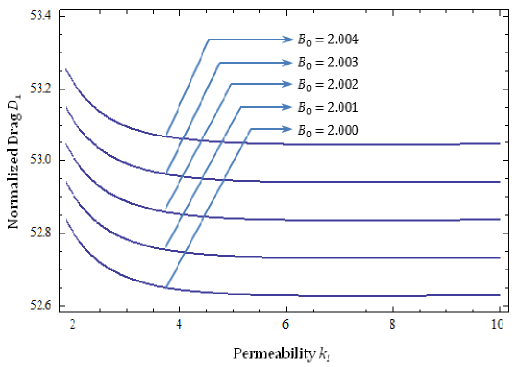


Fig. 3. Variation of normalized drag with permeability for various magnetic induction with $\beta = 0.2$, $\sigma = 1$ and $\mu = 1$ in the case of Uniform flow.

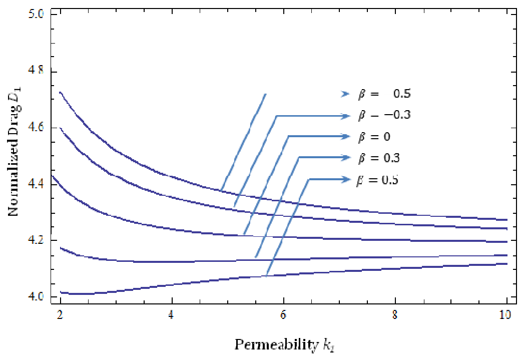


Fig. 4. Variation of normalized drag with permeability for various stress jump coefficient $B_0 = 1, \sigma = 1$ and $\mu = 1$ in the case of Doublet in a uniform flow.

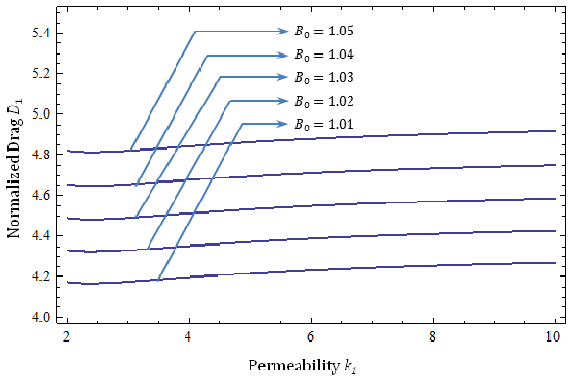


Fig. 5. Variation of normalized drag with permeability for various magnetic induction $\beta = 0.5$, $\sigma = 1$ and $\mu = 1$ in the case of Doublet in a uniform flow.

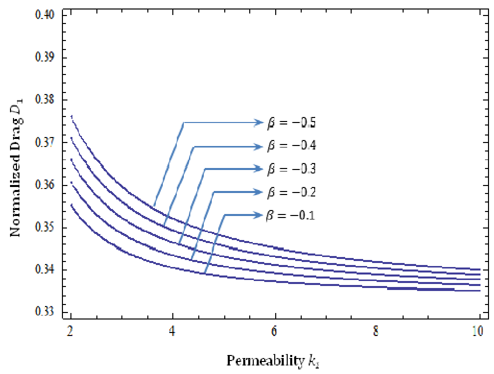


Fig. 6. Variation of normalized drag with permeability for negative stress jump coefficient with $B_0 = 1, \sigma = 1$ and $\mu = 1$ in the case of Rotlet.

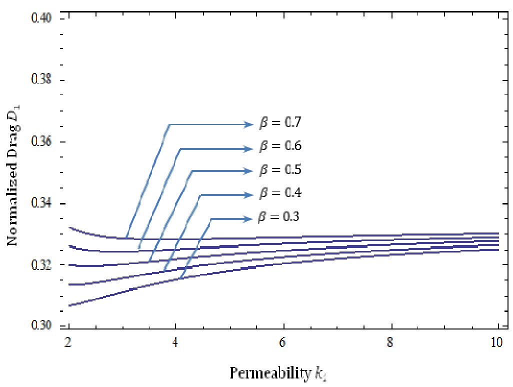


Fig. 7. Variation of normalized drag with permeability for positive stress jump coefficient with $B_0 = 1, \sigma = 1$ and $\mu = 1$ in the case of Rotlet.

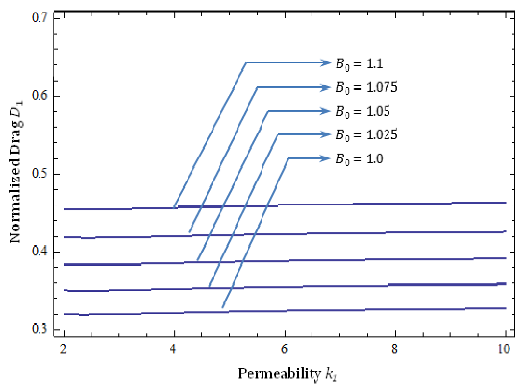
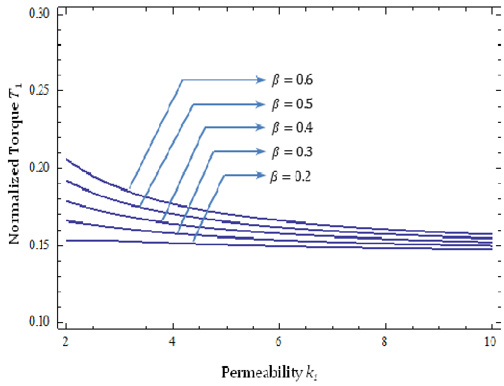


Fig. 8. Variation of normalized drag with permeability for various magnetic induction $\beta = 0.5$, $\sigma = 1$ and $\mu = 1$ in the case of Rotlet



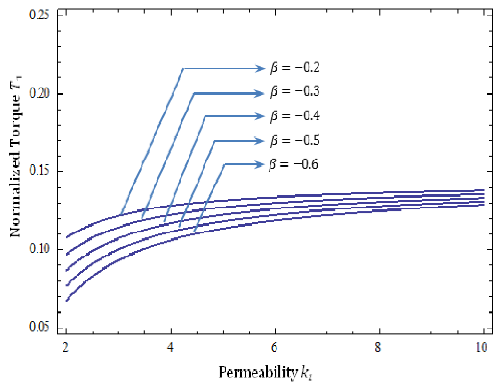
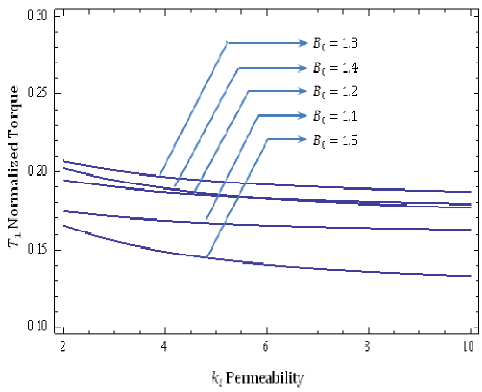


Fig. 9. Variation of normalized torque with permeability for positive stress jump coefficient with $B_0 = 1, \sigma = 1$ and $\mu = 1$ in the case of Rotlet

Fig. 10. Variation of normalized torque with permeability for negative stress jump coefficient with $B_0 = 1, \sigma = 1$ and $\mu = 1$ in the case of Rotlet



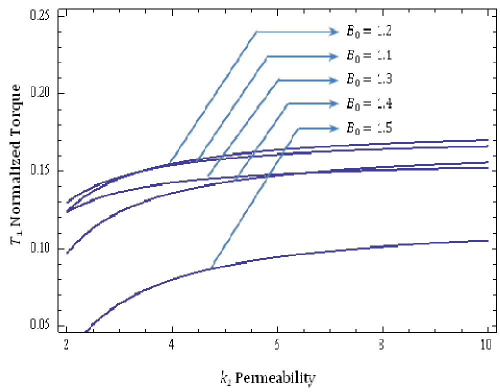


Fig. 11. Variation of normalized torque with permeability for various magnetic induction with $\beta = 0.3$, $\sigma = 1$ and $\mu = 1$ in the case of Rotlet

Fig. 12. Variation of normalized torque with permeability for various magnetic induction with $\beta = -0.3$, $\sigma = 1$ and $\mu = 1$ in the case of Rotlet

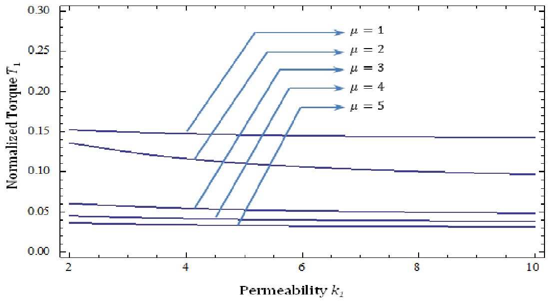


Fig.13. Variation of normalized torque with permeability for various coefficient of viscosity with $\beta = 0.3$, $\sigma = 1$ and $B_0 = 1$ in the case of Rotlet

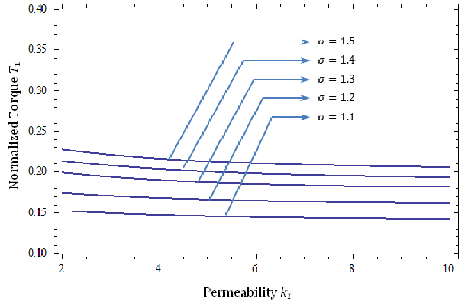


Fig. 14. Variation of normalized torque with permeability for various electrical conductivity with $\beta = 0.3$, $\sigma = 1$ and $B_0 = 1$ in the case of Rotlet

APPENDIX

$$\begin{aligned}
\epsilon_n = -1/D1(a^{-n} l_s(&-g_n(2a^2 k_1 l_i^3 l_{-1+n}(a^n(1+2n)\alpha_n + af_{-1+n} l_s \beta_n) - n(3n\beta \\
&- ak_1 l_s^2) l_n(a^n(1+2n)\alpha_n + af_{-1+n} l_s \beta_n) + ank_1 l_i^2 l_n(a^n(1+2n)\alpha_n \\
&+ af_{-1+n} l_s \beta_n) - al_i l_{-1+n}(-a^n(1+2n)(3n\beta - ak_1 l_s^2)\alpha_n + al_s(-3n\beta f_{-1+n} \\
&+ 2(3+2n) f_n k_1 l_s)\beta_n)) + ag_{-1+n} l_s(2a^2 f_n k_1 l_i^3 l_{-1+n}\beta_n - ak_1 l_i^2 l_n(3a^n(1 \\
&+ 2n)\alpha_n + (3(2+n) f_n + 2af_{-1+n} l_s)\beta_n) - al_i l_{-1+n}(3a^n(1+2n)\beta\alpha_n \\
&+ (2a\beta f_{-1+n} l_s + f_n((6+n)\beta - 2ak_1 l_s^2))\beta_n) + nl_n(3a^n(1+2n)\beta\alpha_n \\
&+ (2a\beta f_{-1+n} l_s + f_n((6+n)\beta + ak_1 l_s^2))\beta_n))))
\end{aligned}$$

$$\begin{aligned}
\delta_n = 1/D2(l_s(g_n(a^n&(1+3n+2n^2)(3n\beta + ak_1(-l_s^2 + l_i^2))\alpha_n + al_s(-2(3+8n+4n^2) f_n k_1 l_s \\
&+ f_{-1+n}(3n(1+n)\beta + ak_1(nl_s^2 + (1+n) l_i^2)))\beta_n) + ag_{-1+n} l_s(3a^n(1+3n \\
&+ 2n^2)\beta\alpha_n + (2a(1+n)\beta f_{-1+n} l_s + f_n((6+7n+n^2)\beta - ak_1((2+5n) l_s^2 \\
&+ (1+n) l_i^2)))\beta_n)))
\end{aligned}$$

$$\begin{aligned}
\alpha'_n = 1/l_s D2(a^{1+n}(&ag_{-1+n} l_s(2af_n k_1 l_s^4(al_i l_{-1+n} - (1+2n) l_n)\beta_n - 2af_{-1+n} l_s^3(a\beta l_i l_{-1+n} - (1 \\
&+ 2n)\beta l_n + ak_1 l_i^2 l_n)\beta_n + (1+n) l_i^2(-a\beta l_i l_{-1+n} + n\beta l_n + ak_1 l_i^2 l_n)(a^n\alpha_n \\
&+ f_n\beta_n) + l_s^2(2a^2 f_n k_1 l_i^3 l_{-1+n}\beta_n - ak_1 l_i^2 l_n(a^n(1+n)\alpha_n - (-5+n) f_n\beta_n) \\
&- a\beta l_i l_{-1+n}(3a^n(1+n)\alpha_n - 2(-3+n) f_n\beta_n) + (1+2n)\beta l_n(3a^n(1+n)\alpha_n \\
&- 2(-3+n) f_n\beta_n))) + g_n(a^n(1+3n+2n^2) l_i^2(-a\beta l_i l_{-1+n} + n\beta l_n \\
&+ ak_1 l_i^2 l_n)\alpha_n + a(1+n) f_{-1+n} l_s l_i^2(-a\beta l_i l_{-1+n} + n\beta l_n + ak_1 l_i^2 l_n)\beta_n \\
&+ af_{-1+n} l_s^3(-2an\beta l_i l_{-1+n} - 2a^2 k_1 l_i^3 l_{-1+n} + 2n(1+2n)\beta l_n + a(1 \\
&+ n) k_1 l_i^2 l_n)\beta_n + ak_1 l_s^4(al_i l_{-1+n} - (1+2n) l_n)(a^n(1+n)\alpha_n + 2(3 \\
&+ n) f_n\beta_n) - l_s^2(2a(3+7n+2n^2) f_n k_1 l_i^2 l_n\beta_n + an\beta l_i l_{-1+n}(3a^n(1+n)\alpha_n \\
&- 2(-3+n) f_n\beta_n) - n(1+2n)\beta l_n(3a^n(1+n)\alpha_n - 2(-3+n) f_n\beta_n) \\
&+ a^2 k_1 l_i^3 l_{-1+n}(a^n(1+n)\alpha_n - 2(1+3n) f_n\beta_n)))))
\end{aligned}$$

$$\begin{aligned}
\beta'_n = 1/l_s D2(anl_s^2(&-a\beta f_{-1+n} l_s + f_n((6+n)\beta + ak_1 l_s^2)) l_i l_{-1+n}\beta_n - n(1+2n) l_s^2(-a\beta f_{-1+n} l_s \\
&+ f_n((6+n)\beta + ak_1 l_s^2)) l_n\beta_n - a(1+n) k_1 l_i^4 l_n(a^n(1+2n)\alpha_n + af_{-1+n} l_s\beta_n) \\
&+ al_i^3 l_{-1+n}(a^n(1+3n+2n^2)\beta\alpha_n + al_s((1+n)\beta f_{-1+n} - (1 \\
&+ 3n) f_n k_1 l_s)\beta_n) - l_i^2 l_n(a^n(1+3n+2n^2)(n\beta - ak_1 l_s^2)\alpha_n + al_s(-3(2+5n \\
&+ 2n^2) f_n k_1 l_s + nf_{-1+n}(\beta + n\beta + ak_1 l_s^2))\beta_n))
\end{aligned}$$

$$D1 = (g_n l_e(an(3n\beta - ak_1 l_e^2)l_i l_{-1+n} + a^2(1+3n)k_1 l_i^3 l_{-1+n} - n(1+2n)(3n\beta - ak_1 l_e^2)l_n + an(1+2n)k_1 l_i^2 l_n) + ag_{-1+n}((1+n)l_i^2(-a\beta l_i l_{-1+n} + n\beta l_n + ak_1 l_i^2 l_n) + l_e^2(3an\beta l_i l_{-1+n} - 3n(1+2n)\beta l_n + a(2+5n)k_1 l_i^2 l_n)))$$

$$D2 = (g_n l_e(an(-3n\beta + ak_1 l_e^2)l_i l_{-1+n} - a^2(1+3n)k_1 l_i^3 l_{-1+n} + n(1+2n)(3n\beta - ak_1 l_e^2)l_n - an(1+2n)k_1 l_i^2 l_n) - ag_{-1+n}((1+n)l_i^2(-a\beta l_i l_{-1+n} + n\beta l_n + ak_1 l_i^2 l_n) + l_e^2(3an\beta l_i l_{-1+n} - 3n(1+2n)\beta l_n + a(2+5n)k_1 l_i^2 l_n)))$$

$$\sigma_n' = \frac{(f_{-1+n}k_1 l_e l_n + f_n(-k_1 l_i l_{-1+n} + \beta l_n))\sigma_n}{g_{-1+n}k_1 l_e l_n + g_n(k_1 l_i l_{-1+n} - \beta l_n)}, \gamma_n = \frac{(f_n g_{-1+n} + f_{-1+n} g_n)k_1 l_e \sigma_n}{g_{-1+n}k_1 l_e l_n + g_n(k_1 l_i l_{-1+n} - \beta l_n)}$$

where $f_n = f_n(al_e)$ and $g_n = g_n(al_i)$

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