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MHD Casson Fluid Flow Past a Non-Isothermal Porous Linearly Stretching Sheet

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Abstract. Magnetohydrodynamic (MHD) Casson fluid flow past a non-isothermal porous linearly stretching sheet is investigated. At first the system of governing equations are transformed into similarity ordinary differential equations. These equations are solved numerically using the Nactsheim-Swigert Shooting iteration technique together with Runge-Kutta six order iteration. Numerical results are obtained for the velocity in x and y directions and temperature at the linear stretching sheet. Self-similar solutions are obtained and related results are presented graphically and discussed quantitatively with respect to variation in Casson flow parameter as well as other fluid flow parameters.

Keywords: MHD, Casson fluid, stretching sheet, Non-isothermal.

1. Introduction

The study of non-Newtonian fluids has attracted much attention because of their extensive variety of applications in engineering and industry especially in extraction of crude oil from petroleum products. In the category of non-Newtonian fluids, Casson fluid has distinct features. This model was presented by Casson for the flow of viscoelastic fluids in 1995. This model is cast off by fuel engineers in the description of adhesive slurry and is improved for forecasting high shear-rate viscosities when only low and transitional shear-rate data are accessible. The fluid flow over a stretching surface is significant in solicitations such as extrusion, cord depiction, copper spiraling, warm progressing, and melts of high molecular weight polymers. Initial work for the boundary layer flow on continuous surfaces was discussed by Sakiadis [1] and Tsou et al. [2]. Magnetohydrodynamic three-dimensional flow and heat transfer over a stretching surface in a viscoelastic fluid are discussed by Ahmad and Nazar [3]. Saidul Islam et. al.[4] investigate the MHD Free Convection and Mass Transfer Flow with Heat Generation through an Inclined Plate. Abdur Rahman et. al. [5] studied the thermophoresis Effect on MHD Forced Convection on a Fluid over a Continuous Linear Stretching Sheet in Presence of Heat Generation and Power-Law Wall Temperature. Hasanuzzaman et al.[6] studied the of similarity solution of unsteady combined free and force convective laminar boundary layer flow about a vertical porous surface with suction and blowing. Very

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freshly in another article, Nadeem et al. [7] examined the magnetohydrodynamic (MHD) boundary layer flow of a Casson fluid over an exponentially penetrable shrinking sheet. Nadeem et.al. examine the MHD three- dimensional Casson fluid flow past a porous linearly stretching sheet[8].

Focal impartial of the current study is to deliberate the three-dimensional examination for the Casson fluid model arrogant the exponential stretching sheet. Through smearing the resemblance alteration, we diminish the system of nonlinear partial differential equations into the system of nonlinear ordinary differential equations. Non-dimensionalized corporeal constraints namely Casson fluid parameter β , Hartmann number M, and porosity parameter λ seem after applying the similarity transformations. Nonlinear coupled equations are then attempted numerically to get the solutions, and then, fleshly behaviors of each of the parameter are exposed graphically.

2. Mathematical model

Consider three-dimensional (3D) incompressible flow past non isothermal a stretching sheet. It is consider that sheet is stretched along the xy - plane, while fluid is placed along the *z*-axis. Moreover, it is consider that constant magnetic field is applying normal to the fluid flow, and the induced magnetic field



assumed to be negligible. Here, we assumed that sheet has stretched with the linear velocities u = ax and v = by along the xy-plane, respectively.

The rheological equation of state for an isotropic flow of a Casson fluid can be expressed as

$$\tau_{ij} = \begin{cases} 2\left(\mu_B + \frac{p_z}{\sqrt{2\pi}}\right)e_{ij}, \pi > \pi_c \\ 2\left(\mu_B + \frac{p_z}{\sqrt{2\pi}}\right)e_{ij}, \pi < \pi_c \end{cases}$$
(1)

In the above equation $\pi = e_{ij}e_{ij}$ and e_{ij} denotes the $(i, j)^{th}$ component of the deformation rate, π be the product of the component of deformation rate itself, π_c be a critical value of this product based on the non-Newtonian model, μ_B be the plastic dynamic viscosity of the non-Newtonian fluid and p_z be the yield stress of the fluid.

From the equation (1) we obtained,

$$\therefore \mu_B = \frac{1}{2} \frac{\tau_{ij}}{e_{ij}} - \frac{p_z}{\sqrt{2\pi}}, \text{ we have, } v = \frac{\mu_B}{\rho} \text{ and } \beta = \mu_B \frac{\sqrt{2\pi_c}}{P_z}$$

The boundary layer equations of three-dimensional incompressible Casson fluid are stated as follows:

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$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial x} + \frac{\partial w}{\partial x} = 0$$
(2)

$$u\frac{\partial u}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial w}{\partial z} = v\left(1 + \frac{1}{\beta}\right)\left(\frac{\partial^2 u}{\partial z^2}\right) - \frac{\sigma B_0^2}{\rho}u - \frac{\upsilon}{k}u + g\beta\left(T - T_\infty\right)$$
(3)

$$u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z} = v\left(1 + \frac{1}{\beta}\right)\left(\frac{\partial^2 v}{\partial z^2}\right) - \frac{\sigma B_0^2}{\rho}v - \frac{\upsilon}{k}v$$
(4)

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} + w\frac{\partial T}{\partial z} = \frac{k_1}{\rho c_p} \left(\frac{\partial^2 T}{\partial z^2}\right) + \frac{\sigma B_0^2}{\rho} \left(u^2 + v^2\right)$$
(5)

where u, v, and w denote the respective velocities in the x, y and z-directions, respectively, β is the Casson fluid parameter, v is kinematic viscosity, B_0 is the magnetic induction, and k is the porous medium permeability. The associated boundary conditions of equations (3) and (4) are as follows:

$$u = u_w(x) = ax, \quad v = v_w(x) = by, \quad T = T_w \quad \text{at} \ z = 0$$

$$u \to 0, \quad v \to 0, \quad T \to T_\infty \quad \text{as} \ z \to \infty$$
(6)

In the above expressions, *a* and *b* are positive constant, and u_w and v_w are stretching velocities in *x* and *y* directions, respectively. Introducing the following similarity transformations;

$$u = axf'(\eta), \ v = byg'(\eta), \ w = -(av)^{\frac{1}{2}}(f(\eta) + cg(\eta)), \ \eta = \left(\frac{a}{v}\right)^{\frac{1}{2}}z$$
(7)

where, $c = \frac{b}{a}$ is the ratio of the velocities in y and x-directions, and prime denote differentiation with respect to η . Making use of equation (7) equation of continuity is identically satisfied, and equations (3) and (4) along with (6) take the following derivation.

From equation (3), (4) and (5) we get,

$$f''' = \frac{\beta}{\left(1+\beta\right)} \left\{ \left(f'\right)^2 - \left(f+cg\right)f'' + \left(M^2 + \lambda\right)f' - g\theta G_r \right\}$$

$$\tag{8}$$

$$g''' = \left(\frac{\beta}{1+\beta}\right) \left\{ \left(g'\right)^2 - \left(f+cg\right)g'' + \left(M+\lambda\right)g' \right\}$$
(9)

$$\theta'' = -\theta' P_r \left(f + cg \right) - M^2 E_c P_r \left\{ f'^2 + g'^2 \right\}$$
(10)

Expression for skin friction coefficient C_f on the surface along the x-and y-directions, which are denoted by C_{fx} and C_{fy} respectively, are defined as follows:

$$\operatorname{Re}_{x}^{\frac{1}{2}}C_{fx} = (1 + \frac{1}{\beta})f''(0) , \operatorname{Re}_{x}^{\frac{1}{2}}C_{fy} = \left(1 + \frac{1}{\beta}\right)\left(c\frac{y}{x}\right)g''(0)$$
(11)

where, $\operatorname{Re}_{x} = u_{x}(x) \frac{x}{v}$ is the local Reynolds number for the stretching velocity $u_{w}(x)$.

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3. Numerical solution

The system of coupled nonlinear coupled differential equations (8) and (9) along with the boundary conditions (11) and (12) is solved numerically using shooting method. The step size $\Delta \eta = 0.001$ is used to obtain the numerical solution with η_{max} , and accuracy to the fifth decimal place is chosen as the criterion of convergence.

4. Result and discussions

In this section, we have discussed the velocity profiles $f'(\eta)$, $g'(\eta)$ and temperature profiles $\theta(\eta)$ for various physical parameters such as Casson fluid parameter β , Hartmann number M, porosity parameter λ , and stretching parameter c, Grashof number G_r , Eckert number E_c and Prandtl number P_r . In **Fig. 2-4**, magnitude of velocity profiles $f'(\eta)$, $g'(\eta)$, temperature profiles $\theta(\eta)$ and boundary layer thickness decreases with the increase of non-Newtonian parameter β , present phenomena obviously reduced to Newtonian fluid.









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From Fig. 5-7 it is observed that for higher values of M, both boundary layer thickness and the magnitude of the velocity profiles $f'(\eta)$ and $g'(\eta)$ decreases and temperature profile $\theta(\eta)$ increases. Physically present phenomena occur when magnetic field can induce current in the conductive fluid, and then it creates a resistive-type force on the fluid. So finally, it is conclude that magnetic field is used to control boundary layer separation. From Fig. 4-10 it is observed that for increasing values of porosity parameter

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 λ , with in the boundary layer, it decreases the velocity profiles $f'(\eta)$, $g'(\eta)$ and increases temperature profile $\theta(\eta)$. Moreover, these graphs show that boundary layer thickness also decreases for higher values of λ . From Fig. 11-13 it is observed that for increasing values of stretching ratio c, it decreases the velocity $f'(\eta)$, while $g'(\eta)$ increases and the temperature profile $\theta(\eta)$ decreases. It notice that for c=0, present phenomena reduce the case of two dimensional linear stretching, while for c=1, sheet will stretched along the both directions with same ratio, and third and last case relate to stretching ratio parameter c other than 0 and 1; then the flow behavior along both the direction will be different. From Fig. 14-16 it is observed that for increasing values of Grashof number G_r , with in the boundary layer, the velocity profile $f'(\eta)$ increase, velocity profile $g'(\eta)$ decreases and temperature profile $\theta(\eta)$ decreases. Moreover, these graphs show that boundary layer thickness also decreases for higher values of G_r . From **Fig. 17-19** a minor change is observed of the velocity profiles $f'(\eta)$, $g'(\eta)$ and temperature profile $\theta(\eta)$ for increasing values of Eckert number E_c , with in the boundary layer. From Fig. 20-22 it is observed that for increasing values of Prandtl number P_r , with in the boundary layer, it decreases the velocities profiles $f'(\eta)$, $g'(\eta)$ and temperature profile $\theta(\eta)$. Moreover, these graphs show that boundary layer thickness also decreases for higher values of P_r .

5. Conclusions

The main results of present analysis can be listed below:

- Casson fluid parameter β reduces the velocity profiles in both x and y direction and the temperature profiles.
- Hartmann number *M* decreases the velocity profiles in both *x* and *y* direction and increases the temperature profiles.
- Porosity parameter λ decreases the velocity profiles in both x and y directions and increases temperature profile.
- Stretching parameter *c* decreases the velocity in *x* direction, while increases the velocity in *y* directions and decreases the temperature profile.
- Grashof number G_r increases the velocity profile in x direction, decreases velocity profile in y directions and decreases temperature profile.
- Eckert number E_c gives a minor change of the velocity profiles in both x and y directions and temperature profile.
- Prandtl number P_r decreases the velocities profiles in both x and y directions and temperature profile.

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