

A New Fuzzy Method for Assessing Human Skills

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Received 12 November 2014; accepted 20 December 2014

Abstract. In this paper we develop a method for assessing the overall performance of groups of individuals participating in any kind of human activities. For this, we represent each of the group under assessment as a fuzzy subset of a set U of linguistic labels characterizing its members' performance and we apply a recently developed Trapezoidal Fuzzy Assessment Model (TRFAM) for converting the fuzzy data collected from the corresponding activity to a crisp number. The TRFAM is a variation of the popular in fuzzy mathematics centre of gravity (COG) defuzzification technique, which has been properly adapted and used as an assessment method in earlier papers. According to the TRFAM the higher is an individual's performance the more its "contribution" to the corresponding group's overall performance (weighted performance). Two real life applications are also presented, related to the bridge players' performance and to the students' assessment respectively, illustrating our assessment method in practice.

Keywords: Fuzzy sets, centre of gravity (COG) defuzzification technique, trapezoidal fuzzy assessment model, contract bridge, student assessment

1. Introduction

There used to be a tradition in science and engineering of turning to probability theory when one is faced with a problem in which uncertainty plays a significant role. This transition was justified when there were no alternative tools for dealing with the uncertainty. Today this is no longer the case. *Fuzzy logic*, which is based on fuzzy sets theory introduced by Zadeh [18] in 1965, provides a rich and meaningful addition to standard logic and an alternative tool for dealing with uncertainty.

A real test of the effectiveness of an approach to uncertainty is its capability to solve problems which involve different facets of uncertainty. Fuzzy logic has a much higher problem solving capability than standard probability theory. Most importantly, it opens the door to construction of mathematical solutions of computational problems which are stated in a natural language. The applications which may be generated from or adapted to fuzzy logic are wide-ranging and provide the opportunity for modelling under conditions which are inherently imprecisely defined, despite the concerns of classical logicians (e.g. see Chapter 6 of [7,12,13] and its relevant references, [14], etc).

The methods of assessing the individuals' performance usually applied in practice are based on principles of the bivalent logic (yes-no). However these methods are not the most suitable ones when dealing with ambiguous cases. In Education, for example, a

Michael Gr. Voskoglou

teacher is frequently not absolutely sure about a particular numerical grade characterizing a student's performance. Fuzzy logic, due to its nature of characterizing such ambiguous cases with multiple values, offers a wider and richer field of resources for this purpose.

In this paper we shall use principles of fuzzy logic for developing a general method for assessing the individual skills in any human activity. The rest of the paper is organized as follows: In the next section we develop our fuzzy assessment method. In section three we present two real life applications illustrating our method in practice. Finally the last section is devoted to conclusions and discussion on the future perspectives of research in this area.

For general facts on fuzzy sets we refer to the book [7]

2. The fuzzy assessment method

Let us consider a group, say H , of n individuals, where n is a positive integer, participating in a human activity (e.g. problem-solving, decision making, football match, a chess tournament, etc). A classical way for assessing the overall group's performance with respect to the corresponding activity is to express the individuals performance in numerical values and then to calculate the mean of their performance in terms of these values (*mean group's performance*).

Here, we shall use principles of fuzzy logic for developing an alternative method of assessment, according to which the higher is an individual's performance, the more its "contribution" to the group's total performance (*weighted group's performance*). For this, let $U = \{A, B, C, D, F\}$ be a set of linguistic labels characterizing the individuals' performance with respect to the above activity, where A stands an excellent performance, B for a very good, C for a good, D for a fair and F stands for an unsatisfactory performance. Obviously, the above characterizations are fuzzy depending on the user's personal criteria, which however must be compatible to the common logic, in order to be able to model the real situation in a worthy of credit way. We represent H as a fuzzy subset of U in the form: $H = \{(x, m(x)) : x \in U\}$, where $m : U \rightarrow [0, 1]$ $m : U \rightarrow [0, 1]$ is the corresponding *membership function*.

A very popular in fuzzy logic method for converting the fuzzy data collected from the corresponding activity to a crisp number is the *centre of gravity (COG) defuzzification technique* [11]. According to this technique the fuzzy data is represented by the pair of numbers (x_c, y_c) as the coordinates of the COG, say F_c , of the level's section contained between the graph of the corresponding membership function and the OX axis. In earlier papers Subbotin and Voskoglou [8, 15, 16, etc] have adapted the COG technique to be used as an assessment method. For this, an individual's performance is characterized as unsatisfactory (F), if $x \in [0, 1)$, as fair (D), if $x \in [1, 2)$, as good (C), if $x \in [2, 3)$, as very good (B), if $x \in [3, 4)$ and as excellent (A), if $x \in [4, 5]$ respectively. In other words, if $x \in [0, 1)$, then $y_1 = m(x) = m(F)$, if $x \in [1, 2)$, then $y_2 = m(x) = m(D)$, etc. In this case the graph of the membership function attached to H takes the form of the bar graph of Figure 1 consisting of five rectangles, say S_i , $i=1,2,3,4,5$, whose sides lying on the X axis have length 1.

A New Fuzzy Method for Assessing Human Skills

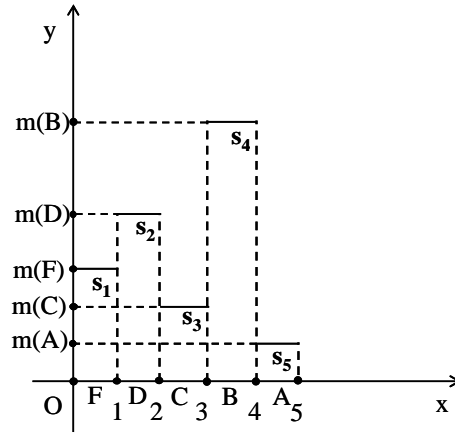


Figure 1: Bar graphical data representation

Then the coordinates of the COG of the resulting bar graph can be easily calculated using known from Mechanics formulas and a criterion can be obtained for the assessment of the group's performance ; e.g. see [16], section 4.

In this paper we shall use the Trapezoidal Fuzzy Assessment Model (TRFAM) instead of the above method, which is a recently developed variation of the COG technique [9-10]. The important novelty of this approach is in the replacement of the rectangles appearing in the graph of the membership function of the COG technique by isosceles trapezoids sharing common parts. In the TRFAM's scheme (Figure 2) we have five trapezoids, corresponding to the above defined grades F, D, C, B and A respectively of the individuals' performance. Without loss of generality and for making our calculations easier we consider isosceles trapezoids with bases of length 10 units lying on the OX axis. The height of each trapezoid is equal to the percentage of individuals who achieved the corresponding grade for their performance, while the parallel to its base side is equal to 4 units. We allow for any two adjacent trapezoids to have 30% of their bases (3 units) belonging to both of them. In this way we cover the ambiguous cases of individuals' scores being at the boundaries between two successive grades. For students' assessment, for example, it is a very common approach to divide the interval of the specific grades in three parts and to assign the corresponding grade using + and - . For example, $75 - 77\% = B-$, $78 - 81\% = B$, $82 - 84\% = B+$. However, this consideration does not reflect the common situation, where the teacher is not sure about the grading of the students whose performance could be assessed as marginal between and close to two adjacent grades; for example, something like $84 - 85$ being between $B+$ and A. The TRFAM fits this situation.

An individuals' group can be represented, as in the COG method, as a fuzzy set in U , whose membership function $y=m(x)$ has as graph the line $OB_1C_1H_1B_2C_2H_2B_3C_3H_3B_4C_4H_4B_5C_5D_5$ of Figure 2, which is the union of the line segments OB_1 , B_1C_1 , C_1H_1 , ..., B_5C_5 , C_5D_5 . However, in case of the TRFAM and in contrast to the COG technique the analytic form of $y = m(x)$ is not needed for calculating the COG of the resulting area. In fact, since the boundary cases of the individuals' scores are considered as common parts for any pair of the adjacent trapezoids, it is logical to count these parts twice; e.g. placing the ambiguous cases $B+$ and $A-$ in both regions B

Michael Gr. Voskoglou

and A. In other words, the COG method, which uses the analytic form of $y = m(x)$ for calculating the coordinates of the COG of the area between the graph of the membership function and the OX axis, thus considering the areas of the “common” triangles $A_2H_1D_1$, $A_3H_2D_2$, $A_4H_3D_3$ and $A_5H_4D_4$ only once, is not the proper one to be applied in the above situation.

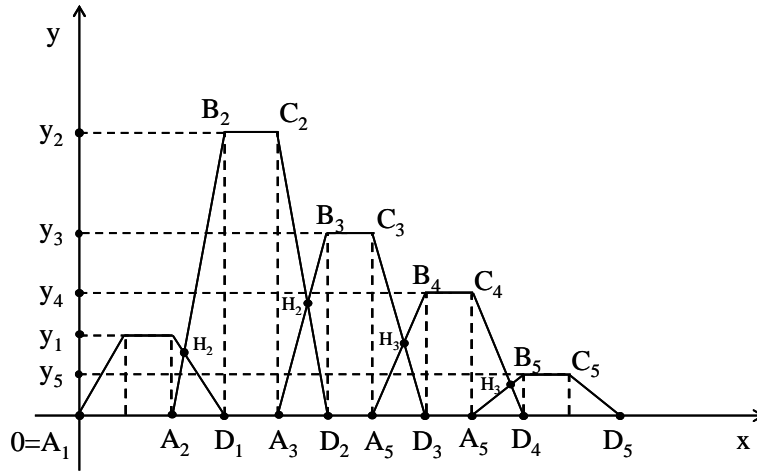


Figure 2: The TRAFM's scheme

Therefore, in this case we represent each one of the five trapezoids of Figure 3 by its COG F_i , $i=1, 2, 3, 4, 5$ and we consider the entire area, i.e. the sum of the areas of the five trapezoids, as the system of these points-centers. More explicitly, the steps of the whole construction of the TRFAM are the following:

1. Let y_i , $i=1, 2, 3, 4, 5$ be the percentages of the individuals' whose performance was characterized by F, D, C, B, and A respectively; then $\sum_{i=1}^5 y_i = 1$ (100%).

2. We consider the isosceles trapezoids with heights equal to y_i , $i=1, 2, 3, 4, 5$, in the way that has been illustrated in Figure 2.

3. We calculate the coordinates (x_{c_i}, y_{c_i}) of the COG F_i , $i=1, 2, 3, 4, 5$, of each trapezoid as follows: It is well known that the COG of a trapezoid lies along the line segment joining the midpoints of its parallel sides a and b at a distance d from the longer side b given by $d = \frac{h(2a+b)}{3(a+b)}$, where h is its height (e.g. see [23]). Therefore in our case

we have

$$y_{c_i} = \frac{y_i(2*4+10)}{3*(4+10)} = \frac{3y_i}{7}.$$

Also, since the abscissa of the COG of each trapezoid is equal to the abscissa of the midpoint of its base, it is easy to observe that $x_{c_i} = 7i - 2$.

4. We consider the system of the COG's F_i , $i=1, 2, 3, 4, 5$ and we calculate the coordinates (X_c, Y_c) of the COG F_c of the whole area S considered in Figure 2

A New Fuzzy Method for Assessing Human Skills

by the following formulas, derived from the commonly used in such cases definition (e.g. see [17]):

$$X_c = \frac{1}{S} \sum_{i=1}^5 S_i x_{c_i}, Y_c = \frac{1}{S} \sum_{i=1}^5 S_i y_{c_i} \quad (1)$$

In formulas (1) S_i , $i = 1, 2, 3, 4, 5$ denote the areas of the corresponding trapezoids.

Thus, $S_i = \frac{(4+10)y_i}{2} = 7y_i$ and $S = \sum_{i=1}^5 S_i = 7 \sum_{i=1}^5 y_i = 7$. Therefore, from formulas (1)

we finally get that

$$X_c = \frac{1}{7} \sum_{i=1}^5 7y_i(7i-2) = (7 \sum_{i=1}^5 i y_i) - 2 \quad \text{and} \quad Y_c = \frac{1}{7} \sum_{i=1}^5 7y_i \left(\frac{3}{7} y_i \right) = \frac{3}{7} \sum_{i=1}^5 y_i^2 \quad (2)$$

5. We determine the area where the COG F_c lies as follows: For $i, j = 1, 2, 3, 4, 5$, we have that $0 \leq (y_i - y_j)^2 = y_i^2 + y_j^2 - 2y_i y_j$, therefore $y_i^2 + y_j^2 \geq 2y_i y_j$, with the equality holding if, and only if, $y_i = y_j$. Thus,

$$1 = \left(\sum_{i=1}^5 y_i \right)^2 = \sum_{i=1}^5 y_i^2 + 2 \sum_{\substack{i,j=1, \\ i \neq j}}^5 y_i y_j \leq \sum_{i=1}^5 y_i^2 + 2 \sum_{\substack{i,j=1, \\ i \neq j}}^5 (y_i^2 + y_j^2) = 5 \sum_{i=1}^5 y_i^2 \quad \text{or} \quad \sum_{i=1}^5 y_i^2 \geq \frac{1}{5} \quad (3)$$

with the equality holding if and only if $y_1 = y_2 = y_3 = y_4 = y_5 = \frac{1}{5}$. In the case of equality

the first of formulas (2) gives that $X_c = 7\left(\frac{1}{5} + \frac{2}{5} + \frac{3}{5} + \frac{4}{5} + \frac{5}{5}\right) - 2 = 19$. Further,

combining the inequality (3) with the second of formulas (2) one finds that $Y_c \geq \frac{3}{35}$

Therefore the unique minimum for Y_c corresponds to the COG $F_m(19, \frac{3}{35})$. The ideal case

is when $y_1 = y_2 = y_3 = y_4 = 0$ and $y_5 = 1$. Then from formulas (3) we get that $X_c = 33$ and $Y_c = \frac{3}{7}$. Therefore the COG in this case is the point $F_i(33, \frac{3}{7})$. On the other hand, the worst

case is when $y_1 = 1$ and $y_2 = y_3 = y_4 = y_5 = 0$. Then from formulas (3), we find that the COG is the point $F_w(5, \frac{3}{7})$. Therefore the area where the COG F_c lies is the area of the triangle

$F_w F_m F_i$ (see Figure 4).

6. We formulate our criterion for comparing the performances of two (or more) different groups' as follows: From elementary geometric observations (see Figure 3) it follows that for two groups the group having the greater X_c performs better. Further, if the two groups have the same $X_c \geq 19$, then the group having the COG which is situated closer to F_i is the group with the greater Y_c . Also, if the two groups have the same $X_c < 19$, then the group having the COG which is situated farther to F_w is the group with the smaller Y_c . Based on the above considerations we obtain the following criterion:

- *Between two groups the group with the greater value of X_c demonstrates the better performance.*
- *If two groups have the same $X_c \geq 19$, then the group with the greater value of Y_c demonstrates the better performance.*
- *If two groups have the same $X_c < 19$, then the group with the smaller value of Y_c demonstrates the better performance.*

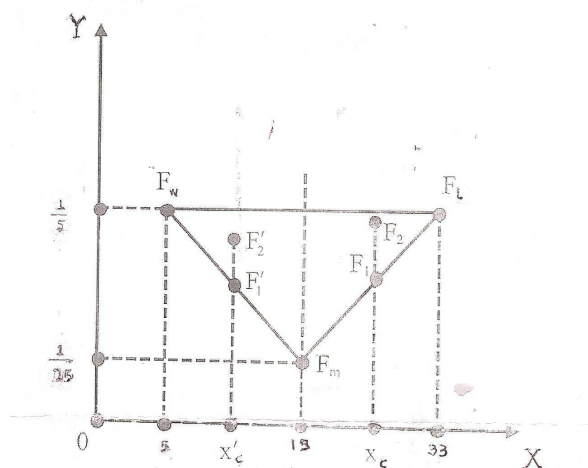


Figure 3: The area where the COG lies

Once developed, the TRFAM is very easy to be applied in practice, because it needs no complicated calculations in its final step.

Observing the first of formulas (1) we can see that the TRFAM assigns to the abscissa of the COG greater coefficients to the higher scores. Therefore, since the value of the COG's abscissa measures in the group's performance (see the above criterion), we conclude that the TRFAM focuses on the *quality performance* rather and not on the *mean performance* of the corresponding group.

3. Applications

In this section we shall present two real life applications illustrating in practice the importance of our results obtained in the previous section. The first of these applications concerns a new assessment method of bridge players' performance, while the second one is related to the assessment of students' performance.

3.1. Anew assessment method of the bridge players' performance

Contract bridge is a card game belonging to the family of trick-taking games. It occupies nowadays a position of great prestige being, together with chess, the only *mind sports* (i.e. games or skills where the mental component is more significant than the physical one) officially recognized by the International Olympic Committee. Millions of people play bridge worldwide in clubs, tournaments and championships, but also on line (e.g. [1]) and with friends at home, making it one of the world's most popular card games.

A match of bridge can be played either among *teams* (two or more) of four players (two partnerships), or among *pairs*. For a pairs event a minimum of three tables (6 pairs, 12 players) is needed, but it works better with more players. At the end of the match in the former case the result is the difference in *International Match Points (IMPs)* between the competing teams and then there is a further conversion, in which some fixed number of *Victory Points (VPs)* is appointed between the teams. It is worthy to notice that the table converting IMPs to VPs has been obtained through a rigorous mathematical manipulation [4].

A New Fuzzy Method for Assessing Human Skills

On the contrary, the usual method of scoring in a pairs' competition is in *match points*. Each pair is awarded two match points for each pair who scored worse than them on each game's session (*hand*), and one match point for each pair who scored equally. The total number of match points scored by each pair over all the hands played is calculated and it is converted to a percentage. However, IMPs can also be used as a method of scoring in pair events. In this case the difference of each pair's IMPs is usually calculated with respect to the mean number of IMPs of all pairs.

For the fundamentals and the rules of bridge, as well as for the conventions usually played between the partners we refer to the famous book [6] of *Edgar Kaplan* (1925-1997), who was an American bridge player and one of the principal contributors to the game. Kaplan's book was translated in many languages and was reprinted many times since its first edition in 1964. There is also a fair amount of bridge-related information on the Internet, e.g. see web sites [2, 3], etc.

The *Hellenic Bridge Federation (HBF)* organizes, on a regular basis, *simultaneous* bridge tournaments (pair events) with pre-dealt boards, played by the local clubs in several cities of Greece. Each of these tournaments consists of six in total events, played in a particular day of the week (e.g. Wednesday), for six successive weeks. In each of these events there is a local scoring table (match points) for each participating club, as well as a central scoring table, based on the local results of all participating clubs, which are compared to each other. At the end of the tournament it is also formed a total scoring table in each club, for each player individually. In this table each player's score equals to the mean of the scores obtained by him/her in the five of the six in total events of the tournament. If a player has participated in all the events, then his/her worst score is dropped out. On the contrary, if he/she has participated in less than five events, his/her name is not included in this table and no possible extra bonuses are awarded to him/her.

In case of a pairs' competition with match points as the scoring method and according to the usual standards of contract bridge, one can characterize the players' performance, according to the percentage of success, say p , achieved by them, as follows:

- Excellent (A), if $p > 65\%$.
- Very good (B), if $55\% < p \leq 65\%$.
- Good (C), if $48\% < p \leq 55\%$.
- Fair (D), if $40\% \leq p \leq 48\%$.
- Unsatisfactory (F), if $p < 40\%$.

Our application presented here is related to the total scoring table of the players of a bridge club of the city of Patras, who participated in at least five of the six in total events of a simultaneous tournament organized by the HBF, which ended on February 19, 2014 (see results in [5]). Nine men and five women players are included in this table, who obtained the following scores. Men: 57.22%, 54.77%, 54.77%, 54.35%, 54.08%, 50.82%, 50.82%, 49.61%, 47.82%. Women: 59.48%, 54.08%, 53.45%, 53.45%, 47.39%. The above results give a mean percentage of approximately 52.696% for the men and 53.57% for the women players. Therefore the women demonstrated a slightly better mean performance than the men players.

The above results are summarized in Table 1.

Michael Gr. Voskoglou

Table 1: Total scoring of the men and women players

% Scale	Performance	Men	Women
>65%	A	0	0
55-65%	B	1	1
48-55%	C	7	3
40-48%	D	1	1
<40%	F	0	0
Total		9	5

The data of Table 1 provides the following percentages: $y_5=0$, $y_4=\frac{1}{9}$, $y_3=\frac{7}{9}$, $y_2=\frac{1}{9}$, $y_1=0$ for the men players and $y_5=0$, $y_4=\frac{1}{5}$, $y_3=\frac{3}{5}$, $y_2=\frac{1}{5}$, $y_1=0$ for the women players. Therefore, applying formulas (2) we find that $X_c=7(\frac{2}{9} + \frac{21}{9} + \frac{2}{9})-2=19$, $Y_c=\frac{3}{7}(\frac{1}{81} + \frac{49}{81} + \frac{1}{81}) = \frac{51}{189} \approx 0.27$ for the men players and $X_c=7(\frac{2}{5} + \frac{9}{5} + \frac{4}{5})-2=19$, $Y_c=\frac{3}{7}(\frac{1}{25} + \frac{9}{25} + \frac{1}{25}) = \frac{33}{175} \approx 0.19$ for the women players. Hence, according to the second case of our criterion stated in paragraph 6 of the previous section, and in contrast to their mean performance, the men demonstrated better quality performance with respect to the women players.

In concluding, our new assessment method of the bridge players' performance can be used as a complement of the usual scoring methods of the game (match points or IMPs) in cases where one wants to compare (for statistical or other reasons) the overall performance of special groups of players (e.g. men and women, young and old players, players of two or more clubs participating in a big tournament, etc).

3.2. Students' assessment

The students of two different Departments of the School of Management and Economics of the Graduate Technological Educational Institute of Western Greece achieved the following scores (in a climax from 0 to 100) at their common progress exam in the course "Mathematics for Economists I":

Table 2: Students' scores

% Scale	Grade	Department 1	Department 2
89-100	A	3	1
77-88	B	21	10
65-76	C	28	37
53-64	D	22	31
Less than 53	F	16	21
Total		90	100

A New Fuzzy Method for Assessing Human Skills

From Table 2 we obtain the following percentages: $y_5 = \frac{3}{90}$, $y_4 = \frac{21}{90}$, $y_3 = \frac{28}{90}$, $y_2 = \frac{22}{90}$, $y_1 = \frac{16}{90}$ for the first Department and $y_5 = \frac{1}{100}$, $y_4 = \frac{10}{100}$, $y_3 = \frac{37}{100}$, $y_2 = \frac{31}{100}$, $y_1 = \frac{21}{100}$ for the second Department. Therefore, applying formulas (2) we find that $X_c = 7(\frac{16}{90} + \frac{44}{90} + \frac{84}{90} + \frac{84}{90} + \frac{15}{90}) - 2 = \frac{63}{90} = 0.7$ for the first Department and $X_c = 7(\frac{21}{100} + \frac{62}{100} + \frac{111}{100} + \frac{40}{100} + \frac{5}{100}) - 2 = 0.31$ for the second Department. Hence, according to the first case of our criterion stated in paragraph 6 of the previous section the first Department demonstrated a better quality performance than the second one.

4. Conclusions and discussion

In the present paper we developed a general fuzzy method for assessing the overall performance of groups of individuals participating in any kind of human activity. Our method is very simple to its application in practice needing no complicated calculations in its final step, as it happens with other assessment methods (e.g. measurement of the system's uncertainty [13, 14]). For developing this method we represented each of the groups under assessment as a fuzzy subset of a set U of linguistic labels characterizing their members' performance and we used the TRFAM in converting the fuzzy data collected from the corresponding activity to a crisp number. According to the above assessment method the higher is an individual's performance the more its "contribution" to the corresponding group's total performance (weighted performance). Thus, in contrast to the mean of the scores of all the group's members, which is connected to the mean group's performance, our method is connected to the group's *quality performance*. Consequently, when the above two different assessment methods are used in comparing the performance of two or more groups of individuals, the results obtained may differ to each other (e.g. see our bridge application). Two applications were also presented, related to the bridge players' performance and to the students' assessment respectively, illustrating the importance of our assessment method in practice.

Our future plans for further research on the subject aim at applying our new assessment method in more bridge matches (including also games played with IMPs) and problem solving (not only mathematical) applications in order to get statistically safer and more solid conclusions about its applicability and usefulness. In a wider spectre, since our method is actually a general assessment method, it could be interesting to be applied in more sectors of the human activity, including other competitive games (e.g. other card games, chess, backgammon, etc), collective and individual sports, human cognition and learning, Artificial Intelligence, Biomedical Sciences, Management and Economics, etc.

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Michael Gr. Voskoglou

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