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Effect of Porous Medium on Unsteady Heat and Mass Transfer Flow of Fluid

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Abstract. The effect of porous medium on unsteady heat and mass transfer flow of fluid in a porous medium and non-porous medium is investigated. The problem is governed by coupled non-linear partial differential equations. The dimensionless equations of the problem have been solved numerically by using explicit finite difference method. The effect of flow parameters on the velocity field, temperature field and concentration field are discussed quantitatively and the obtained solutions are shown graphically.

Keywords: Heat and Mass Transfer, MHD, Porous Medium, Non-porous medium, explicit finite difference method.

1. Introduction

The phenomenon of heat and mass transfer has been the object of extensive research due to its science and technology. Magneto hydrodynamic (MHD) is currently undergoing a period of great enlargement and differentiation of subject matter. The MHD heat and mass transfer processes are of interest in power engineering, metallurgy, astrophysics, and geophysics. Unsteady convective flows in a porous medium have received much attention in recent time due to its wide applications in geothermal and oil reservoir engineering as well as other geophysical and astrophysical studies. The unsteady fluid flow past a moving plate in the presence of free convection and radiation were studied by Cogley et. al. (1968), Das, Deka and Rang Soundarker (1996), Abdus Satter and Hamid Kalim et. al. (1996), Grief et. al. (1971), Ganeasan and Loganathan (2002). All these studies have been confined to unsteady flow in a nonporous medium. From the previous literature survey about unsteady fluid flow, we observe that little papers were done in porous media. The effect of radiation on MHD flow and heat transfer must be considered when high temperatures are reached. Hakiern (2000), studied the unsteady MHD oscillatory flow on free convection-radiation through a porous medium with a vertical infinite surface that absorbs the fluid with a constant velocity. Cookey et. al. (2003), researched the influence of viscous dissipation and radiation on unsteady MHO free-convection flow past on infinite heated vertical plate in a porous medium with time dependent suction. Naby et. al. (2004), studied unsteady MHD convective heat transfer past a semi-infinite vertical porous moving plate. Anwar et. al. (2009), studied and combined the effects of Soret and Dufour diffusion and porous impedance on laminar magneto- hydrodynamic mixed convection heat and mass transfer of an electrically-conducting, Newtonian, Boussinesq fluid from a vertical stretching surface in a Darcian, porous medium under uniform transverse magnetic field. Heat and mass transfer for Soret and Dufour's effect on mixed convection boundary layer flow over a stretching vertical surface in a porous medium filled with a visco-elastic fluid in the presence of magnetic field. Ahmed and Kalita et. al. (2012), is examined and investigate the problem of an oscillatory MHD free convective flow through a porous medium with mass transfer, soret effect and chemical reaction when the temperature as well as concentration at the plate varies periodically with time about a steady mean. Recently, Anuradha and Priyadharshini (2014), studied and combined the effects of unsteady MHD free convective heat and mass transfer on the flow of viscous fluid past an impulsively started semi-infinite vertical plate with Soret effect. In the present analysis, it is proposed to study the effect of heat and mass transfer flow of viscous fluid through a porous media and non-porous media. Ahmmed and Das et. al. (2015), described Analytical Study on Unsteady MHD Free Convection and Mass Transfer Flow Past a Vertical Porous Plate. The closed solutions have been obtained for the velocity, temperature and concentration profile. The effects of various parameters on the velocity, temperature and concentration are obtained graphically.

Hence our aim of this research is to compare the effect the speed of fluid in porous and non-porous medium for non-dimensional velocity *U*, concentration *C*, Temperature profile *T* for different parameters Garsof Number, Modified Garshof Number, Prandtl Number and Schimidt Number.

2. Mathematical formulation

We discussed an unsteady two dimensional natural convection flow of an incompressible viscous fluid past an impulsively started vertical plate through porous medium and non-porous medum. The positive x coordinate is measured along the plate in the direction of fluid motion and the positive y coordinate is measured normal to the plate. A uniform transverse magnetic field B_0 is imposed parallel to y-direction. The magnetic Reynolds number of the flow is taken to be small enough and the magnetic field is negligible in comparison with applied magnetic field and the magnetic lines are fixed relative to the fluid. The level of concentration of foreign mass is assumed to be low, so that the Soret and Dufour effects are negligible.

The level of concentration of foreign mass is assumed to be low, so that the Soret and Dufour effects are negligible..Initially, it is considered that the plate as well as the fluid is at the same temperature T and the concentration level C . .Also it is considered that the fluid and the plate is at rest. Again consider the plate is to be moving with a constant velocity U in it's own plane. Instantaneously at time $t > 0$ the temperature of the plate and species concentration are raised to T_w and C_w respectively, where T_w, C_w are temperature and spices concentration at the wall and T_∞, C_∞ are the temperature and the concentration of the species outside the plate respectively. Then under the above assumptions, the governing boundary equations with Boussinesq's approximation are given below:

Porous medium:

Continuity equation:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{1}$$

Momentum equation:

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=g\beta\left(T-T_{\infty}\right)+g\beta^{*}\left(C-C_{\infty}\right)+\upsilon\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right)-\frac{\sigma' u B_0^2}{\rho}-\frac{\mu}{\rho k}u \tag{2}$$

Energy equation:

$$\frac{\partial T}{\partial t}+u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\frac{k}{\rho c_P}\left(\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}\right) \tag{3}$$

Concentration equation:

$$\frac{\partial C}{\partial t}+u\frac{\partial C}{\partial x}+v\frac{\partial C}{\partial y}=D_m\left(\frac{\partial^2 C}{\partial x^2}+\frac{\partial^2 C}{\partial y^2}\right) \tag{4}$$

In Non- Porous medium:

Continuity equation:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{5}$$

Momentum equation:

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=g\beta\left(T-T_{\infty}\right)+g\beta^{*}\left(C-C_{\infty}\right)+\upsilon\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right)-\frac{\sigma' u B_0^2}{\rho} \tag{6}$$

Energy equation:

$$\frac{\partial T}{\partial t}+u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\frac{k}{\rho c_P}\left(\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}\right) \tag{7}$$

Concentration equation:

$$\frac{\partial C}{\partial t}+u\frac{\partial C}{\partial x}+v\frac{\partial C}{\partial y}=D_m\left(\frac{\partial^2 C}{\partial x^2}+\frac{\partial^2 C}{\partial y^2}\right) \tag{8}$$

The boundary conditions for the problem are:

$$\text{at } \tau=0; u=0; v=0; T=0; C=0$$

$$\text{at } \tau>0\begin{cases} u=0; v=0; T=0; C=0 & \text{at x = 0} \\ u=0; v=0; T=T_w; C=C_w & \text{at y = 0} \\ u=0; v=0; T=0; C=0 & \text{at y} \to \infty \end{cases} \tag{9}$$

where *u*, *v* are the *x*, and *y* components of velocity vector, , where μ is the fluid viscosity, ρ is the density of the fluid, κ is the thermal conductivity, C_p is the specific heat at the constant pressure, D is the coefficient of mass diffusivity. We assume that the temperature difference within the flow are sufficiently small so that the T^4 can be expressed as a linear function after using Taylor series to expand T^4 about the free stream temperature T and neglecting higher-order terms. This result in the following approximation: $T^4 \cong 4T^3 - 3T^4$. To obtain the governing equations and the boundary condition in dimension less form, we have the following non-dimensional parameters are as follows,

$$X = x\frac{U_0}{\nu} \quad Y = y\frac{U_0}{\nu} \quad V = \frac{v}{U_0} \quad \tau = t\frac{{U_0}^2}{\nu} \quad \overline{T} = \frac{T - T_\infty}{T_w - T_\infty} \quad \overline{C} = \frac{C - C_\infty}{C_w - C_\infty} \quad U = \frac{u}{U_0}$$

Substituting the above relations in equation (1) to equation (8) and corresponding boundary conditions (9). Therefore the governing equations in the dimensionless form become equation (10) to equation (17) with the boundary condition (18),

Mathematical Formulation for porous Medium

$$\frac{\partial U}{\partial X} + \frac{\partial V}{\partial Y} = 0 \tag{10}$$

$$\frac{\partial U}{\partial \tau} + U\frac{\partial U}{\partial X} + V\frac{\partial U}{\partial Y} = G_r\overline{T} + G_m\overline{C} + \frac{\partial^2 U}{\partial Y^2} - MU - KU \tag{11}$$

$$\frac{\partial \overline{T}}{\partial \tau} + U\frac{\partial \overline{T}}{\partial X} + V\frac{\partial \overline{T}}{\partial Y} = \frac{1}{P_r}\frac{\partial^2 \overline{T}}{\partial Y^2} \tag{12}$$

$$\frac{\partial \overline{C}}{\partial \tau} + U\frac{\partial \overline{C}}{\partial X} + V\frac{\partial \overline{C}}{\partial Y} = \frac{1}{S_c}\frac{\partial^2 \overline{C}}{\partial Y^2} \tag{13}$$

Mathematical formulation for non-porous Medium

$$\frac{\partial U}{\partial X} + \frac{\partial V}{\partial Y} = 0 \tag{14}$$

$$\frac{\partial U}{\partial \tau} + U\frac{\partial U}{\partial X} + V\frac{\partial U}{\partial Y} = G_r\overline{T} + G_m\overline{C} + \frac{\partial^2 U}{\partial Y^2} - MU \tag{15}$$

$$\frac{\partial \overline{T}}{\partial \tau} + U\frac{\partial \overline{T}}{\partial X} + V\frac{\partial \overline{T}}{\partial Y} = \frac{1}{P_r}\frac{\partial^2 \overline{T}}{\partial Y^2} \tag{16}$$

$$\frac{\partial \overline{C}}{\partial \tau} + U\frac{\partial \overline{C}}{\partial X} + V\frac{\partial \overline{C}}{\partial Y} = \frac{1}{S_c}\frac{\partial^2 \overline{C}}{\partial Y^2} \tag{17}$$

Also the associated initial and boundary conditions become

$$\begin{aligned} &\text{at } \tau = 0; \quad U = 0; V = 0; T = 0; C = 0 \\ &\text{at } \tau > 0; \quad \left\{ \begin{array}{ll} U = 0; V = 0; T = 0; C = 0 & \text{at X} = 0 \\ U = 0; V = 0; T = 1; C = 1 & \text{at Y} = 0 \\ U = 0; V = 0; T = 0; C = 0 & \text{at Y} \to \infty \end{array} \right\} \end{aligned} \tag{18}$$

3. Numerical solurions

In order to solve the non-dimensional system by the implicit finite difference technique, it is required a set of finite difference equations. In this case, the region within the boundary layer is divided by some perpendicular lines of Y -axis, where Y -axis is normal to the medium as shown in Fig. 2. It is assumed that the maximum length of boundary layer is $Y_{\max} = 25$ as corresponds to $Y \to \infty$ i.e. Y varies from 0 to 25 and the number of grid spacing in Y directions is $P = 400$, hence the constant mesh size along Y axis becomes $\Delta Y = 0.0625 (0 < y < 25)$ with a smaller time-step $\Delta t = 0.001$. Let U', T', C' denote the values of U, T, C at the end of a time-step respectively. Using the implicit finite difference approximation, the following appropriate set of finite difference equations are obtained as,

For porous medium:

$$\frac{U_{i,j} - U_{i-1,j}}{\Delta X} + \frac{V_{i,j} - V_{i,j-1}}{\Delta Y} = 0 \tag{19}$$

$$\frac{U'_{i,j} - U_{i,j}}{\Delta \tau} + U_{i,j}\frac{U_{i,j} - U_{i-1,j}}{\Delta X} + V_{i,j}\frac{U_{i,j+1} - U_{i,j}}{\Delta Y} = \frac{U_{i,j+1} - 2U_{i,j} + U_{i,j-1}}{(\Delta Y)^2} + G_r\overline{T} + G_m\overline{C} - MU_{i.j} - KU_{i.j} \tag{20}$$

$$\frac{T'_{i,j} - \overline{T}_{i,j}}{\Delta \tau} + U_{i,j}\frac{\overline{T}_{i,j} - \overline{T}_{i-1,j}}{\Delta X} + V_{i,j}\frac{\overline{T}_{i,j+1} - \overline{T}_{i,j}}{\Delta Y} = \frac{1}{P_r}\frac{\overline{T}_{i,j+1} - 2\overline{T}_{i,j} + T_{i,j-1}}{(\Delta Y)^2} \tag{21}$$

$$\frac{\overline{C}'_{i,j} - \overline{C}_{i,j}}{\Delta \tau} + U_{i,j}\frac{\overline{C}_{i,j} - \overline{C}_{i-1,j}}{\Delta X} + V_{i,j}\frac{\overline{C}_{i,j+1} - \overline{C}_{i,j}}{\Delta Y} = \frac{1}{S_c}\frac{\overline{C}_{i,j+1} - 2\overline{C}_{i,j} + \overline{C}_{i,j-1}}{(\Delta Y)^2} \tag{22}$$

For non-porous medium:

$$\frac{U_{i,j} - U_{i-1,j}}{\Delta X} + \frac{V_{i,j} - V_{i,j-1}}{\Delta Y} = 0 \tag{23}$$

$$\frac{U'_{i,j} - U_{i,j}}{\Delta \tau} + U_{i,j}\frac{U_{i,j} - U_{i-1,j}}{\Delta X} + V_{i,j}\frac{U_{i,j+1} - U_{i,j}}{\Delta Y} = \frac{U_{i,j+1} - 2U_{i,j} + U_{i,j-1}}{(\Delta Y)^2} + G_r\overline{T} + G_m\overline{C} - MU_{i.j} \tag{24}$$

$$\frac{T'_{i,j} - \overline{T}_{i,j}}{\Delta \tau} + U_{i,j}\frac{\overline{T}_{i,j} - \overline{T}_{i-1,j}}{\Delta X} + V_{i,j}\frac{\overline{T}_{i,j+1} - \overline{T}_{i,j}}{\Delta Y} = \frac{1}{P_r}\frac{\overline{T}_{i,j+1} - 2\overline{T}_{i,j} + T_{i,j-1}}{(\Delta Y)^2} \tag{25}$$

$$\frac{\overline{C}'_{i,j} - \overline{C}_{i,j}}{\Delta \tau} + U_{i,j}\frac{\overline{C}_{i,j} - \overline{C}_{i-1,j}}{\Delta X} + V_{i,j}\frac{\overline{C}_{i,j+1} - \overline{C}_{i,j}}{\Delta Y} = \frac{1}{S_c}\frac{\overline{C}_{i,j+1} - 2\overline{C}_{i,j} + \overline{C}_{i,j-1}}{(\Delta Y)^2} \tag{26}$$

The initial and boundary conditions with the finite difference scheme are

at $\tau = 0;\quad u = 0; v = 0; T = 0; C = 0$

$$\text{at } \tau > 0; \left\{ \begin{array}{l} u = 0; v = 0; T = 0; C = 0 \quad \text{at x = 0} \\ u = 0; v = 0; T = T_w; C = C_w \quad \text{at y = 0} \\ u = 0; v = 0; T = 0; C = 0 \quad \text{at y} \to \infty \end{array} \right\}$$

4. Results and discussion

To observe the physical situation of the problem, the steady-state solutions have been illustrated. These results show that the velocity, distribution increase with the increase of Garsof Number G_r, Modified Garshof Number G_m, Prandtl Number P_r and decrease for Magnetic number *M* and Schimidt Number S_c both in porous and non-porous medium. The effect of Magnetic number *M* and Schimidt Number S_c on the velocity, temperature and concentration distributions are respectively. These results show that the velocity distributions decrease both in porous and non-porous medium with the increase of time $\tau = 10,\ 30,\ 80$ but in porous medium it becomes steady with the increase of time and in non-porous, unsteady. The temperature and concentration distributions have a bit impact with the increase of Garsof Number G_r, Magnetic number *M* and for Schimidt Number *Sc* both in porous medium and in non-porous medium the thickness expands but in non-porous medium it expands more than porous. It is noted that the concentration decreases *with* the increase of *Sc* in porous, where thickness increase in non-porous. It is noted that the temperature and concentration decreases with increase of Prandlt number *Pr and* Garsof Number *Gr*, where concentration distribution has no impact for Prandlt number *Pr* both in porous and non-porous medium respectively.

A qualitative comparison for the parameters G_m, G_r, M, P_r *and* S_c are presented in Table 1. For porous and non-porous medium.

Increased Parameter τ=10-80	Increase	In Porous medium				In Non-Porous medium			
		Steady	***U***	***T***	***C***	Steady	***U***	***T***	***C***
M	Inc	Unsteady	Dec	No imp	No imp	Steady	Dec	expand	expand
G_m	Inc	Unsteady	Inc	No imp	No imp	Steady	Inc	expand	expand
G_r	Inc	Unsteady	Inc	Dec	Dec	Steady	Inc	Dec	Dec
P_r	Inc	Unsteady	Inc	Dec	No imp	Steady	Inc	Dec	No imp
Sc	Inc	Unsteady	Dec	No imp	Dec	Steady	Dec	expand	expand

Table 1: A qualitative comparison for the parameters G_m, G_r, M, P_r *and* S_c

5. Conclusion

In this paper the effects of porous medium on unsteady heat and mass transfer flow of fluid have been studied numerically. Explicit finite difference method is employed to solve the equations governing the flow. From the present numerical investigation, following conclusions have been discussed:

a) The result of the computations shows that the fluid speed goes higher for all parameters like G_m, G_r, M, P_r *and* S_c in non-porous medium and with increase of time the speed of flow in non-porous medium goes twice and sometimes goes more than twice from porous medium.

b) For temperature profile it is observed that the flow of fluid expends in non-porous medium with increase of time but in porous medium the flow has no impact for parameter G_m and M. But for P_r and G_r the flow expands in both medium but it expands more in non-porous medium than porous medium.

c) Other important effects of flow are shown for concentration profile for different values of parameter G_r G_m, P_r and S_c at time $\tau = 10,\ 30,\ 80$ the flow of fluid expands in non-porous medium and it increases with the increase of time but in porous medium it has no effect. The same case arises for G_m. On the other hand the fluid flow spreads in non-porous medium for G_r than from porous medium and with the increase of time it expands more in non-porous medium than porous medium and It shows no effect in porous medium for the parameter P_r but shows a bit expansion in non-porous medium. Lastly for S_c the flow expands more in porous medium than from non-porous medium.

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