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Adjacent Vertex Distinguishing (Avd) Edge Colouring of Permutation Graphs

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Abstract. A graph *G=(V,E)* with vertex set on *N*, the set of natural numbers is called a permutation graph if there exists a permutation $\pi = \{\pi(1),\pi(2),\ldots,\pi(n)\}$ on *N* such that for i,j ϵ *N*, such that either i<j and $\pi^{-1}(i) > \pi^{-1}(j)$ or i >j and $\pi^{-1}(i) < \pi^{-1}(j)$ where $\pi^{-1}(i)$ is the element of *N* which π maps into *i*.

Adjacent vertex distinguishing edge coloring (avd edge colouring in short) applied on different type of permutation graphs. Some theorems of avd edge colouring on permutation graphs and their proofs are established.

Keywords: Permutation graphs, graph colouring, Adjacent Vertex Distinguishing (Avd) Edge Colouring

1. Introduction

The intersection graphs [51] is a very imnportant subclass of graphs due to its wide applications. These graphs also include interval graphs [6,9,37,41-48,56,57,60-63,65], permutation graphs [1,4,8,16,36,39,40,49,50,58,66], circular-arc graphs [31-34,64], trapezoid graphs [2,3,5,10-12,17-25,38,59], etc. Severals problems are solved on these graphs among them graph colouring is one of the most important problem of graph theory. Different kind of graph colouring/labelling problems are also available in literature [7,13,14,22,26-29,35,52-55] and these problems are solved for these graphs. The concept of vertex distinguishing edge colouring was introduced independently by Adjacent vertex distinguishing (avd) edge colouring of graphs are investigated on [30,67,68].

A proper edge colouring of a graph is an assignment of colours to the edges of the graph such that two adjacent edges do not use the same colour.

Adjacent vertex distinguishing (avd) edge colouring

An adjacent vertex distinguishing edge colouring (avd edge colouring) of a graph G is a proper edge colouring $\varphi : E \rightarrow \{c_1, c_2, \ldots, c_k\}$ of a graph G=(V,E) for every pair of adjacent vertices u, v the set of the colours of the edges incident to u differs from the set of colours of the edges incident to v.

Therefore, ϕ is called adjacent vertex distinguishing edge colouring if $\phi(u) \neq \phi(v)$ for all $(u, v) \in E$ where $\phi(v) = \{ \varphi(v, w) | (v, w) \in E \}$.

Avd edge chromatic number

The avd edge chromatic number of G, denoted by $\chi_a'(G)$ is the minimum number of colours needed is an avd edge colouring of G.
Therefore, $\chi_a'(G)$ = min{k| G is k avd edge colourable}.

Example 1. Let us consider the following graph;

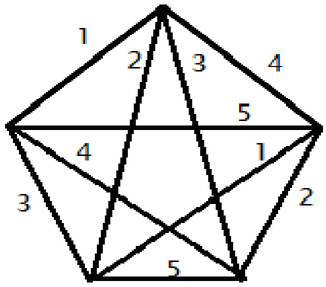

Figure 1: The complete graph K_5

Here $\chi_a'(K_5) = 5$.

Avd total colouring

A proper total colouring of a graph G is a mapping f from V(G)UE(G) to {1,2,...,k} such that

a) For all u, v ∈ V(G) if uv ∈ E(G) then $f(u) \neq f(v)$
b) For all $e_1, e_2 \in E(G)$, $e_1 \neq e_2$,if e_1, e_2 have a common end vertex ,then $f(e_1) \neq f(e_2)$
c) For all u ∈ V(G), e ∈ E(G) if u is the end vertex of e, then $f(u) \neq f(e)$
d) It is called a avd total colouring if $\varphi[u] \neq \varphi[v]$ where $\varphi[u]=\{f(e)|$ e is incident to v $\} \cup \{f(v)\}$.

Avd total chromatic number

The avd total chromatic number of G denoted by $\chi_{at}(G)$, is the minimum no. of colours needed is an avd total colouring of G. Therefore, $\chi_{at}(G)$= min{k |G is avd total k-colourable}.

A cut vertex is a vertex the removal of which would disconnect the remaining graph.

1.1. Permutation graph

An undirected graph G=(V,E) with vertices V={1,2,…,n} is called a permutation graph if there exists a permutation π on N={1,2,3,....} such for all i, j ∈ N

$$(i - j)(\pi^{-1}(i) - \pi^{-1}(j)) < 0$$

if and only if i and j are joined by an edge in G.

Geometrically, the integers 1,2,3,…,n are drawn in order on a real line called an upper line and $\pi(1)$, $\pi(2)$,…, $\pi(n)$ on a line parallel to this upper line called as lower line such that for each i∊ N, i is directly below $\pi(i)$.

Let us consider the permutation σ $=\begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 3 & 4 & 5 & 2 & 1 \end{pmatrix}$.

The permutation representation of σ is

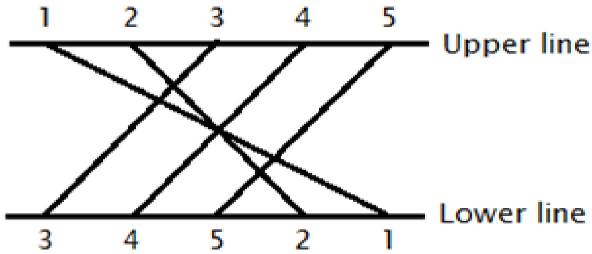

Figure 2: The permutation representation of σ

The corresponding permutation graph is

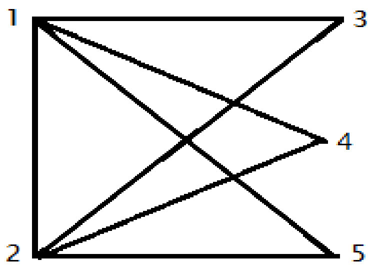

Figure 3: Permutation graph of σ

Complete permutation graph

A permutation graph is called complete permutation graph if it is complete.

For an example, let us consider a permutation σ = $\begin{pmatrix} 1 & 2 & 3 & 4 \\ 4 & 3 & 2 & 1 \end{pmatrix}$.

The permutation representation of σ and the corresponding permutation graph is

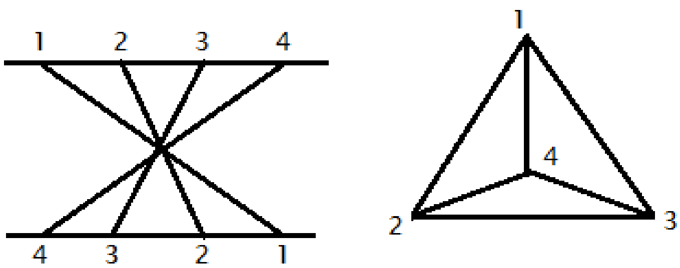

Figure 4: The permutation representation and the complete permutation graph

A permutation graph is called *bipartite permutation graph* if it is bipartite.

Complete bipartite permutation graph
A permutation graph is called a complete bipartite permutation graph if it is complete and bipartite.

One cycle permutation graph (means a permutation graph looking just like a cycle)
Example 2.
Let σ = $\begin{pmatrix} 1 & 2 & 3 \\ 3 & 2 & 1 \end{pmatrix}$. The permutation representation of σ is

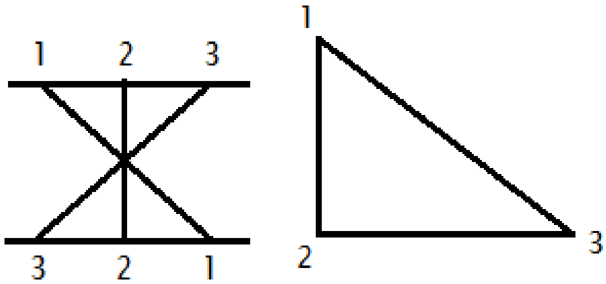

Figure 5: Permutation representation of σ and one cycle permutation graph of length 3

Some important properties of permutation graphs

a) The complement of a permutation graph is also a permutation graph.
b) Permutation graph is transitively orientable.
c) The permutation graph and its complement are comparability graph.
d) Any graph containing k-cycle is not a permutation graph for k ≥5.
e) Permutation graph are perfect.
f) A graph is permutation graph if the graph has a permutation representation.
g) There exist at most four permutation representations for any connected bipartite permutation graph.
h) A permutation graph is an intersection graph of segments between two parallel lines.
i) A bipartite graph is a bipartite permutation graph if and only if it has a strong ordering
j) A permutation graph is the complement of a comparability graph.
k) Every bipartite permutation graph having 2-chromatic number.
l) Complement of a complement permutation graph is the original permutation graph.

2. Avd edge colouring on other graphs
2.1. Avd colouring on cactus graph
Khan et al. [30] have worked on Avd colouring on cactus graphs. The following results are preented from [30].

Definition 2.1. (Cactus graph) A cactus graph is a connected graph in which any two simple cycles have at most one vertex in common.

Conjecture 2.1. If G be a simple connected graph with at least three vertices and $G \neq C_5$ then $\Delta \leq \chi_a'(G) \leq \Delta+2$.

Lemma 2.1. For any star graph $K_{1,\Delta}$, $\chi_a'(K_{1,\Delta}) = \Delta$, where Δ is the degree of the star graph.

Lemma 2.2. For any cycle C_n of length n,

$$\chi_a'(C_n)=\begin{cases}3, & \text{if } n \equiv 0 \pmod 3;\\ 4, & \text{if } n \equiv 1,2 \pmod 3 \text{ and } n \neq 5;\\ 5, & \text{if } n = 5\end{cases}$$

Lemma 2.3. If a graph G contains two cycles of finite lengths and they are joined with a common cut vertex, then

$$\chi_a'(G) = \begin{cases}5, & \text{when two cycles are of length 5;}\\ 4, & \text{otherwise.}\end{cases}$$

Lemma 2.4. Let a graph G contains three cycles of finite lengths and they are joined with a common cut vertex v_0. If $\Delta(=6)$ be the degree of v_0, then $\chi_a'(G) = \Delta$.

Lemma 2.5. If the graph G contains finite cycles of finite lengths, joined with a common cut vertex with degree Δ, then $\chi_a'(G) = \Delta$.

Lemma 2.6. Let G be a graph contains finite no. of cycles of finite lengths and finite no. of edges, joined with a common cut vertex. If Δ be the degree of the cut vertex, then $\chi_a'(G) = \Delta$.

Lemma 2.7. For any sun S_{2n},

$$\chi_a'(S_{2n})=\begin{cases}\Delta+2, & \text{if } n = 5;\\ \Delta+1, & \text{otherwise.}\end{cases} \qquad \text{where } \Delta = 3.$$

Lemma 2.8. Let G be a graph obtained from S_{2n}, by adding an edge to each of the pendent vertex, then $\chi_a'(G)=\begin{cases}5, & if n = 5;\\ 4, & otherwise.\end{cases}$

Lemma 2.9. Let G be a graph contains a cycle of any length and finite no. of edges. If they are joined by a common cut vertex v_0 with degree Δ, then $\chi_a'(G)= \Delta$.

2.2. Avd colouring on Halin graph

Definition 2.2. (Halin graph) A Halin graph is a type of planer graph. It is constructed from a tree that has at least four vertices, none of which have exactly two neighbours (the term neighbour is a node that is attatched to a other node by an edge/path).

Conjecture 2.2. For every connected graph G with order at least 2, we have $\chi_{at}(G) \leq \Delta(G)+3$.

Definition 2.3. (Generalized Halin graph) Suppose G=(V,E) is a plane graph. If after removing all edges of the boundary of a face f_0 (the degree of the vertices of f_0 are all 3), G(V,E) becomes a tree, then G(V,E) is called a generalized Halin graph with f_0 being called the exterior face (others the interior faces) and vertices in $v(f_0)$ being called the exterior vertices (others the interior vertices).

Lemma 2.10. If no two vertices of degree $\Delta(G)$ are adjacent then $\chi_{at}(G) \geq \Delta(G)+1$. Further, if G has two distinct vertices of maximum degree which are adjacent, then $\chi_{at}(G) \geq \Delta(G)+2$.

Theorem 2.1. Let G be a generalized Halin graph. If ΔG) ≥ 6 and any two vertices of degree Δ(G) are not adjacent, then $\chi_{at}(G) = \Delta(G)+1$. Further, if Δ(G) ≥ 5 and there are two vertices of degree Δ(G) which are adjacent, then $\chi_{at}(G) = \Delta(G)+2$.

Conjecture 2.3. If G is a generalized Halin graph with Δ(G) = 3, then $\chi_{at}(G) = 5$.

Conjecture 2.4. Let G be a generalized Halin graph with Δ(G) = 4. If no two vertices of degree 4 are adjacent, then $\chi_{at}(G) = 5$; If there are two vertices of degree 4 which are adjacent then $\chi_{at}(G)=6$.

Conjecture 2.5. Let G be a generalized Halin graph with Δ(G) = 5. If no two vertices of degree 5 are adjacent, then $\chi_{at}(G)= 6$.

2. Main results

Avd labelling of permutation graphs is discussed in this section.

Theorem 3.1. For complete bipartite permutation graph G=(U,V,E)

$$\chi_a'(G) = \begin{cases} \Delta(G)+2, & \text{if } |U| = |V|; \\ \Delta(G), & \text{if } |U| \neq |V|; \end{cases}$$

where Δ(G) is the maximum degree of the graph and |U|=the cardinality of the 1st subset of the vertex set and |V| = the cardinality of the 2nd subset of the vertices.

Proof: Let G=(U,V,E) be a complete bipartite permutation graph.

Case 1: *Let |U| =|V|=n (≥2) [n is any positive integer].*

Let $U=\{u_1,u_2,u_3,\ldots,u_n\}$, $V=\{v_1,v_2,v_3,\ldots,v_n\}$.

Let, $\varphi(u_1)=\{\phi(e_{11}),\phi(e_{12}), \phi(e_{13}),\ldots, \phi(e_{1n})\}$
$=\{ \phi(e_{1j}) | j=1,2,\ldots,n\}$

$\varphi(u_2)=\{ \phi(e_{21}), \phi(e_{22}), \phi(e_{23}),\ldots, \phi(e_{3n})\}$
$=\{ \phi(e_{2j}) | j=1,2,\ldots,n\}$

… … …

$\varphi(u_n)=\{ \phi(e_{n1}), \phi(e_{n2}), \phi(e_{n3}),\ldots, \phi(e_{nn})\}$
$=\{ \phi(e_{nj}) | j=1,2,\ldots,n\}$

And $\varphi(v_1)=\{ \phi(e_{11}), \phi(e_{21}), \phi(e_{31}),\ldots, \phi(e_{n1})\}$
$=\{ \phi(e_{i1}) | i=1,2,\ldots,n\}$

$\varphi(v_2)=\{ \phi(e_{12}), \phi(e_{22}), \phi(e_{32}),\ldots, \phi(e_{n2})\}$
$=\{ \phi(e_{i2}) | i=1,2,\ldots,n\}$

… … …

$\varphi(v_n)=\{ \phi(e_{1n}), \phi(e_{2n}), \phi(e_{3n}),\ldots, \phi(e_{nn})\}$
$=\{ \phi(e_{in}) | i=1,2,\ldots,n\}$

$E=\{ e_{11},e_{12},\ldots, e_{1n}, e_{21}, e_{22},\ldots, e_{2n},\ldots, e_{n1}, e_{n2},\ldots, e_{nn}\}$

$$= \begin{pmatrix} e_{11} & e_{12} & e_{13} & \ldots\ldots\ldots\ldots\ldots\ldots & e_{1n} \\ e_{21} & e_{22} & e_{23} & \ldots\ldots\ldots\ldots\ldots\ldots & e_{2n} \\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\ e_{n1} & e_{n2} & e_{n3} & \ldots\ldots\ldots\ldots\ldots\ldots & e_{nn} \end{pmatrix}$$

where $e_{ij}=(u_i, v_j)$ i ,j=1,2,…,n.

Case 1.1: ***When n is odd***

For avd edge colouring we construct a table such that ith row and j th column of that table must be unequal for i ,j=1,2,…,n but any two of rows or any two of columns may be equal.

The ith row of the table gives ϕ(u_i) for i=1,2,...,n and the j th column of the table gives ϕ(v_i) for j=1,2,…,n and φ(u_i, v_j) represent the colour of the edge (u_i, v_j) i.e e_{ij}. This is shown in Table 1.

φ	v_1	v_2	v_3	…	v_{n-2}	v_{n-1}	v_n	
u_1	1	2	3	…	n-2	n-1	n	φ(u_1)
u_2	3	4	5	…	n	n+1	n+2	φ(u_2)
u_3	5	6	7	…	n+2	1	2	φ(u_3)
…	…	…	…	…	…	…	…	…
$u_{(n+1)/2}$	N	n+1	n+2	…	n-5	n-4	n-3	φ($u_{(n+1)/2}$)
$u_{(n+3)/2}$	2	3	4	…	n-1	n	n+1	φ($u_{(n+3)/2}$)
$u_{(n+5)/2}$	4	5	6	…	n+1	n+2	1	φ($u_{(n+5)/2}$)
…	…	…	…	…	…	…	…	…
u_{n-1}	n-3	n-2	n-1	…	n-8	n-7	n-6	φ(u_{n-1})
u_n	n+2	1	2	…	n-3	n-2	n-1	φ(u_n)
	φ(v_1)	φ(v_2)	φ(v_3)	…	φ(v_{n-2})	φ(v_{n-1})	φ(v_n)	

Table 1: Avd edges colouring of complete bipartite permutation graphs, when *n* is odd

Here ϕ(u_i) and ϕ(v_j) are different for i, j=1,2,...,n.
Hence the required minimum color is n+2 for avd edge coloring.

Case 1.2: W*hen n is even*

We construct the table as per the rule of case 1.1, which is given below :

Φ	v_1	v_2	v_3	…	v_{n-2}	v_{n-1}	v_n	
u_1	1	2	3	…	n-2	n-1	n	ϕ(u_1)
u_2	3	4	5	…	n	n+1	n+2	ϕ(u_2)
u_3	5	6	7	…	n+2	1	2	ϕ(u_3)
…	…	…	…	…	…	…	…	…
$u_{(n+2)/2}$	n+1	n+2	1	…	n-4	n-3	n-2	ϕ($u_{(n+2)/2}$)
$u_{(n+4)/2}$	2	3	4	…	n-1	n	n+1	ϕ($u_{(n+4)/2}$)
$u_{(n+6)/2}$	4	5	6	…	n+1	n+2	1	ϕ($u_{(n+6)/2}$)
…	…	…	…	…	…	…	…	…
u_{n-1}	n-4	n-3	n-2	…	n-9	n-8	n-7	ϕ(u_{n-1})
u_n	n+2	1	2	…	n-3	n-2	n-1	ϕ(u_n)
	ϕ(v_1)	ϕ(v_2)	ϕ(v_3)	…	ϕ(v_{n-2})	ϕ(v_{n-1})	ϕ(v_n)	

Table 2: Avd edges colouring of complete bipartite permutation graphs, when *n* is even

Here also ϕ(u_i)≠ ϕ(v_j) for i, j=1,2,...,n.
Hence the required minimum colour for avd edge colouring is n+2.
Therefore when |U|=|V|= n then $\chi_a'(G) = \Delta(G)+2$.

Case 2: W*hen |U|≠|V|*

Let |U|=m and |V|=n where m$\neq$ n and m and n are positive integer.

Case 2.1: Let m< n .

Let u_i's and v_j 's are the elements of the vertex subset U and V respectively.

The edges are $e_{ij} = (u_i, v_j)$ for i =1,2,…,m and j=1,2,…,n.

The total no. of the edges is mn.

Now we colour the edges as $\varphi(e_{1j}) = j \pmod n$; $\varphi(e_{2j}) = j+1 \pmod n$; $\varphi(e_{3j}) = j+2 \pmod n$; … ; $\varphi(e_{mj})=j+(m-1) \pmod n$ for j=1,2,…,n; and therefore $\varphi(e_{ij})= \{\varphi(e_{(i-1)j})+1\} \pmod n$ or $\varphi(e_{ij})= \{ \varphi(e_{1j})+(i-1)\} \pmod n$ for i=1,2,…,m and j=1,2,…,n.

It can be represent in tabulated form given by below

Φ	v_1	v_2	v_3	…	v_{n-2}	v_{n-1}	v_n	
u_1	1	2	3	…	n-2	n-1	0	$\varphi(u_1)$
u_2	2	3	4	…	n-1	0	1	$\varphi(u_2)$
u_3	3	4	5	…	0	1	2	$\varphi(u_3)$
…	…	…	…	…	…	…	…	…
u_{m-1}	m-1	m	m+1	…	m-4	m-3	m-2	$\varphi(u_{m-1})$
u_m	m	m+1	m+2	…	m-3	m-2	m-1	$\varphi(u_m)$
	$\varphi(v_1)$	$\varphi(v_2)$	$\varphi(v_3)$	…	$\varphi(v_{n-2})$	$\varphi(v_{n-1})$	$\varphi(v_n)$	

Table 3: Avd edges colouring of complete bipartite permutation graphs, when |***U***|<|***V***|

Here ith row represent $\phi(u_i)$ for i=1,2,…,m and j th column represent $\phi(v_j)$ for j=1,2,…,n. So, $\phi(u_i) \neq \phi(v_j)$ and each $\phi(u_i)$ contains n colours and each $\phi(v_j)$ contains {(j+m-1)-j+1} = m colours for i=1,2,…,m and j=1,2,...,n. To fill up the table n minimum colours are required.

Therefore, $\chi_a'(G) = n$ (as n>m).

Case 2.2 *: Let m>n.*

In that case we colour the edges as per the rule of avd edge colouring given in case 2.1.

Then, $\chi_a'(G) = m$(as m>n).

Hence from above cases,

$$\chi_a'(G)=\begin{cases}\Delta(G)+2, & \text{if}|U| = |V|;\\ \Delta(G), & \text{if}|U| \neq |V|;\end{cases}$$

Theorem 3.2. For complete permutation graph

$$\chi_a'(G)=\begin{cases} n, & \text{if n is odd};\\ n+1, & \text{if n is even}.\end{cases}$$

Proof: Let G=(V,E) be a complete permutation graph .Let V={v_i | i=1,2,…,n } is the vertex set, where n is any positive integer and n $\geq$3 .Let e_{1j} be the edges incident to v_1 for j=2,3,…,n; e_{2j} be the edges incident to v_2 for j=1,3,…,n; ...; e_{nj} be the edges incident to v_n for j=1,2,…,n. Or e_{ij} be the edges incident to v_i for j=1,2,…,i-1,i+1,…,n; where $e_{ij}=(u_i, v_j)$ for i$\neq$j and i,j=1,2,…,n.

Case 1: ***Let |V|=n is odd***

For avd edge colouring we construct a table such that any two rows of that table must be unequal and the ith row represent $\phi(v_i)$ i.e the non-vacuous elements of ith row are the elements of $\phi(v_i)$. This is shown in Table 4.

φ	v_1	v_2	v_3	v_4	…	v_{n-1}	v_n	
v_1	-	1	2	3	…	n-2	n-1	$\phi(v_1)$
v_2	1	-	3	4	…	n-1	n	$\phi(v_2)$
v_3	2	3	-	5	…	n	1	$\phi(v_3)$
v_4	3	4	5	-	…	1	2	$\phi(v_4)$
…	…	…	…	…	…	…	…	…
v_{n-1}	n-2	n-1	n	1	…	-	n-3	$\phi(v_{n-1})$
v_n	n-1	n	1	2	…	n-3	-	$\phi(v_n)$

Table 4: Avd edges colouring of complete permutation graphs, when |***V***| is odd

Since there is no edge between the vertices v_i and v_j for i = j and i,j=1,2,…,n; so $\varphi(v_i,v_i)$ is empty and in the table it is denoted by dash(-).
Here $\phi(v_i) \neq \phi(v_j)$ for i≠j & i,j=1,2,…,n.
So to fill up the table n minimum colours are required.

Case 2*: Let |V|=n is even*

For avd edge colouring we construct the table as per the rule of avd edge colouring of case 1 and it is given below :

φ	v_1	v_2	v_3	v_4	…	v_{n-1}	v_n	
v_1	-	1	2	3	…	n-2	n-1	$\phi(v_1)$
v_2	1	-	3	4	…	n-1	n	$\phi(v_2)$
v_3	2	3	-	5	…	n	n+1	$\phi(v_3)$
v_4	3	4	5	-	…	n+1	1	$\phi(v_4)$
…	…	…	…	…	…	…	…	…
v_{n-1}	n-2	n-1	n	n+1	…	-	n-4	$\phi(v_{n-1})$
v_n	n-1	n	n+1	1	…	n-4	-	$\phi(v_n)$

Table 5: Avd edges colouring of complete permutation graphs, when |***V***| is even

Here, also $\phi(v_i) \neq \phi(v_j)$ for i≠j and i,j=1,2,…,n.
So, to fill up the table n+1 minimum colours are required.

Hence, $\chi_a'(G)= \begin{cases} n, & \text{if n is odd;} \\ n+1, & \text{if n is even;} \end{cases}$

Theorem 3.3. For avd edge colouring of one cycle permutation graph(means a permutation graph looking just a cycle):

$$\chi_a'(C_n)= n \ , n=3,4 \text{ where n is the cycle length.}$$

Proof:

Case 1: *when n=3.*

Let $e_0=(v_0,v_1)$, $e_1=(v_1,v_2)$,$e_3=(v_2,v_0)$ be the edges where v_0,v_1,v_2 are the vertices.
Now we colour the edges as $\varphi(e_0)=0$, $\varphi(e_1)=1$, $\varphi(e_2)=2$.

So $\phi(v_i)= \begin{cases} \{0,2\}, & \text{for } i=0; \\ \{0,1\}, & \text{for } i=1; \\ \{1,2\}, & \text{for } i=2; \end{cases}$

Hence $\chi_a'(C_3)=3 = n$.

Case 2: *when n=4* .
Let the edges are $e_0=(v_0,v_1)$, $e_1=(v_1,v_2)$, $e_3=(v_2,v_3)$,$e_4=(v_3,v_0)$ where $\{v_0,v_1,v_2,v_3\}$ are the vertices of C_4.
Now we colour the edges such that $\varphi(e_j) = j$ for j=0,1,2,3.

$$\text{So, } \phi(v_i)=\begin{cases}\{0,3\}, & \text{for } i = 0;\\ \{0,1\}, & \text{for } i = 1;\\ \{1,2\}, & \text{for } i = 2;\\ \{2,3\}, & \text{for } i = 3;\end{cases}$$

Hence $\chi_a'(C_4)= 4 = n$
Therefore from the above cases $\chi_a'(C_n) = n$ for n=3,4.

Theorem 3.4. For permutation graph containing two cycles of length 3, 4 and they are joined with a common cut vertex then $\chi_a'(G) = 4$ (degree of the common cut vertex)
Proof:
Case 1: *Containing two cycles each of length 3*
Then the vertex set is $V=\{v_0,v_1,v_2,v_3,v_4\}$.
Let v_0 is the cut vertex at which two cycles of vertices $\{v_0,v_1,v_2\}$ & $\{v_0,v_3,v_4\}$ respectively are joined.
Denote, $e_0=(v_0,v_1)$, $e_1=(v_1,v_2)$, $e_2=(v_2,v_0)$ & $e_3=(v_0,v_3)$, $e_4=(v_3,v_4)$, $e_5=(v_4,v_0)$.
Set the colours of the edges as $\varphi(e_i) = i \pmod 4$ for i=0,1,2,3,4,5.

$$\text{So } \phi(v_i)=\begin{cases}\{0,1,2,3\}, & if i = 0;\\ \{0,1\}, & if i = 1;\\ \{1,2\}, & if i = 2;\\ \{3,0\}, & if i = 3;\\ \{0,1\}, & if i = 4;\end{cases}$$

Hence $\chi_a'(G)=4$.
Case 2: *contains two cycles each of length 4*
Let v_0 is the cut vertex.
The vertex set of two cycles are $\{v_0,v_1,v_2,v_3\}$ & $\{v_0,v_1',v_2',v_3'\}$.
Let $e_0=(v_0,v_3)$, $e_j=(v_{j-1},v_j)$ for j=1,2,3 & $e_j'=(v_{j-1}',v_j')$ for j=2,3 & $e_0'=(v_0,v_3')$, $e_1'=(v_0,v_1')$.
We colour the edges as
$\varphi(e_j)=j$ for j=0,1,2,3 & $\varphi(e_j')=3-j$ for j=0,1,2,3.

$$\phi(v_j)=\begin{cases}\{1,2\}, & \text{if } j = 1;\\ \{2,3\}, & \text{if } j = 2;\\ \{3,0\}, & \text{if } j = 3;\end{cases}$$

$$\phi(v_j')=\begin{cases}\{2,1\}, & \text{if } j = 1;\\ \{1,0\}, & \text{if } j = 2;\\ \{0,3\}, & \text{if } j = 3;\end{cases}$$

$\phi(v_0)=\{0,1,2,3\}$.

So, χ_a'(G) = 4.
Case 3: Containing two cycles one of whose of length 4 and other is of length 3:
Let v_0 is the cut vertex. The vertex set is V={v_0,v_1,v_2,v_3,v_4,v_5}.
The vertex v_0 joined one cycle of length 4 having vertex set {v_0,v_1,v_2,v_3} and one cycle of length 3 having vertex set {v_0,v_4,v_5}.
Denote, $(v_i,v_{i+1})=e_{i-1}$ for i=1,2 and $(v_3,v_0)=e_2$ and $(v_0,v_1)=e_3$.Also denote $(v_0,v_i)=e_i$ for i=4,5 and $(v_4,v_5)=e_6$.
Now we colour the edges as $\varphi(e_i)$ = i (mod 4) i=0,1,2,3,4,5,6.

$$\text{Hence, } \phi(v_i)=\begin{cases} \{0,1,2,3\}, & for\ i=0; \\ \{0,3\}, & for\ i=1; \\ \{0,1\}, & for\ i=2; \\ \{1,2\}, & for\ i=3; \\ \{0,2\}, & for\ i=4; \\ \{1,2\}, & for\ i=5; \end{cases}$$

So, $\chi_a{}'$(G)= 4.
Therefore, from the above cases, $\chi_a{}'$(G)= 4 (the degree of the common cut vertex).

Conjecture 3.1. For bipartite permutation graph G=(U,V,E), $\Delta(G) \le \chi_a{}'(G) \le \Delta(G)+2$.

4. Conclusion

The tables which are constructed in the theorem1 and theorem2 are not unique. There are many ways to construct the tables but in every way the required minimum no. of colours for avd edge colouring in the table is same as the theorems. Also the tables are constructed in the theorem 2 are symmetric. Any one can work on algorithm of avd edge colouring of any arbitrary permutation graphs.

Question number 1. For any permutation graph G=(V,E), $\Delta(G) \le \chi_a{}'(G) \le \Delta(G)+2$?
Question number 2. If G=(V,E) be a permutation graph containing three cycles of lengths 3 and 4 and they are joined with a common cut vertex v_0. If such graph exist and Δ (=6) be the degree of v_0, then $\chi_a{}'(G)=\Delta$?
Question number 3. If the permutation graph G=(V,E) containing finite cycle of lengths 3 and 4, joined with a common cut vertex with degree Δ exist, then $\chi_a{}'(G)=\Delta$?

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