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(2,0)-Ideals in Γ-Semirings

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Abstract. In this paper the notions of a (2, 0)-ideal , 0-minimal (2, 0)-ideal, (2, 0)-bi-ideal, 0-minimal (2, 0)-bi-ideal and 0-(2, 0)-bi-simple Γ-semiring are introduced. Several characterizations of a 0-minimal (2, 0)-ideal , 0-minimal (2, 0)-bi-ideal and 0-(2, 0)-bi-simple Γ-semiring are furnished.

Keywords: (2, 0)-ideal, 0-minimal (2, 0)-bi-ideal, (2, 0)-bi-simple Γ-semiring, 0-minimal left ideal.

1. Introduction

The notion of a Γ-semiring was introduced by Rao [10] as a generalization of a ring, Γ-ring and a semiring. Characterizations of ideals in a semigroup were given by Lajos in [6]. The notion of a bi-ideal was first introduced for semigroups by Good and Hughes [2].The concept of a bi-ideal for a ring was given by Lajos and Szasz [7]. Also in [8, 9] Lajos and Szasz discussed some characterizations of bi-ideals in semigroups. Shabir, Ali and Batool in [11] gave some properties of bi-ideals in a semiring. Authors were defined bi-ideals in a Γ-semiring [4] and studied quasi-ideals and minimal quasi-ideals of a Γ-semiring in [3]. (m, n)-ideals in semigroups were introduced and studied by Lajos in [6] and 0-minimal bi-ideals of a semigroup was discussed by Krgovic in [5].

In this paper the concepts of a (2, 0)-ideal , 0-minimal (2, 0)-ideal, (2, 0)-bi-ideal, 0-minimal (2,0)-bi-ideal and 0-(2,0)-bi-simple Γ-semiring are introduced. Characterizations of a 0-minimal (2, 0)-ideal, 0-minimal (2, 0)-bi-ideal and a 0-(2, 0)-bi-simple Γ-semiring are studied.

2. Preliminaries

First we recall some definitions of the basic concepts of Γ-semirings that we need in sequel. For this we follow Dutta and Sardar [1].

Definition 2.1. *Let S and Γ be two additive commutative semigroups. S is called a Γ-semiring if there exists a mapping $S \times \Gamma \times S \longrightarrow S$ whose image is denoted by $a\alpha b$; for all $a, b \in S$ and for all $\alpha \in \Gamma$ satisfying the following conditions:*
(i) $a\alpha(b + c) = (a\alpha b) + (a\alpha c)$
(ii) $(b + c)\alpha a = (b\alpha a) + (c\alpha a)$
(iii) $a(\alpha + \beta)c = (a\alpha c) + (a\beta c)$
(iv) $a\alpha(b\beta c) = (a\alpha b)\beta c$; for all $a, b, c \in S$ and $, \beta \in \Gamma$.

Obviously, every semiring is a Γ-semiring.

Definition 2.2. *An element* $0 \in S$ *is said to be an absorbing zero if* $0\alpha a = 0 = a\alpha 0, a + 0 = 0 + a = a$ *; for all* $a \in S$ *and for all* $\alpha \in \Gamma$.

Definition 2.3. *A non-empty subset T of a* Γ*-semiring S is said to be a sub-*Γ*-semiring of S if (T,+) is a subsemigroup of (S,+) and* $a\alpha b \in T$*; for all* $a, b \in T$ *and for all* $\alpha \in \Gamma$.

Definition 2.4. *A non-empty subset T of a* Γ*-semiring S is called a left (respectively right) ideal of S if T is a subsemigroup of (S,+) and* $x\alpha a \in T$ *(respectively* $a\alpha x \in T$*) for all* $a \in T, x \in S$ *and for all* $\alpha \in \Gamma$.

Definition 2.5. *If a non-empty subset T is both left and right ideal of a* Γ*-semiring S, then T is known as an ideal of S.*

Following results from [3] needed in sequel.

Result 2.6. *For each non-empty subset X of a* Γ*-semiring S the following statements hold.*

(i) $S\Gamma X$ *is a left ideal of S.*

(ii) $X\Gamma S$ *is a right ideal of S.*

(iii) $S\Gamma X\Gamma S$ *is an ideal of S.*

Result 2.7. *For a* Γ*-semiring S and* $a \in S$ *the following statements hold.*

(i) $S\Gamma a$ *is a left ideal of S.*

(ii) $a\Gamma S$ *is a right ideal of S.*

(iii) $S\Gamma a\Gamma S$ *is an ideal of* S.

Definition 2.8 [4]. *A non-empty subset B of a* Γ*-semiring S is a bi-ideal of S if B is a sub-*Γ*-semiring of S and* $B\Gamma S\Gamma B \subseteq B$.

Example 1. Let N be the set of natural numbers and let $\Gamma = 2N$. Then N and Γ both are additive commutative semigroups. An image of a mapping $N \times \Gamma \times N \longrightarrow N$ is denoted by $a\alpha b$ and defined as $a\alpha b =$ product of a, α, b; for all $a, b \in S$ and $\alpha \in \Gamma$.
Then N forms a Γ-semiring. $B = 3N$ is a bi-ideal of N.

Now onwards S denotes a Γ-semiring with absorbing zero and a Γ-semiring S means any Γ-semiring unless otherwise stated.

3. (2,0)-Ideals

Definition 3.1. *A subsemigroup A of a* Γ*-semiring S is said to be a* (2,0)*-ideal of S if* $A\Gamma A\Gamma S \subseteq A$.

Definition 3.2. *A subsemigroup A of a* Γ*-semiring S is said to be a* (2,1)*-ideal of S if* $A\Gamma A\Gamma S\Gamma A \subseteq A$.

Proofs of following theorems are straightforward.

Theorem 3.3. Every right ideal of a Γ-semiring S is a (2, 0)-ideal of S.

Theorem 3.4. Every bi-ideal of a Γ-semiring S is a (1, 2)-ideal of S.

Theorem 3.5. Every bi-ideal of a Γ-semiring S is a (2, 1)-ideal of S.

Theorem 3.6. If A is a (2, 0)-ideal of a Γ-semiring S, then $A\Gamma A\Gamma S$ is a (2,0)-ideal of S.

Theorem 3.7. Arbitrary intersection of (2, 0)-ideals of a Γ-semiring S is either empty or a (2, 0)-ideal of a Γ-semiring S.

Theorem 3.8. If A is a (2, 0)-ideal of a Γ-semiring S and T is a sub-Γ-semiring of S, then $A \cap T$ is a (2, 0)-ideal of T.
Proof: Let A be a (2, 0)-ideal and T be a sub-Γ-semiring of S. Clearly $A \cap T$ is a subsemigroup of $(S, +)$. Now $(A \cap T)\Gamma(A \cap T)\Gamma T \subseteq A\Gamma A\Gamma T \subseteq A\Gamma A\Gamma S \subseteq A$. Also $(A \cap T)\Gamma(A \cap T)\Gamma T \subseteq T\Gamma T\Gamma T \subseteq T$.Therefore $(A \cap T)\Gamma(A \cap T)\Gamma T \subseteq A \cap T$.
Hence $A \cap T$ is a (2,0)-ideal of T. ■

Theorem 3.9. If A is a non-empty subset of a Γ-semiring S, then A is a (2, 0)-ideal of S. If and only if A is a right ideal of some right ideal of S.
Proof: Assume that A is a (2, 0)-ideal of S. Therefore $A\Gamma A\Gamma S \subseteq A$. As $A\Gamma S$ is a right ideal of S, $A\Gamma A\Gamma S \subseteq A$ implies A is a right ideal of some right ideal $A\Gamma S$ of S.
Conversely, suppose that A is a right ideal of some right ideal R of S. Hence $A\Gamma R \subseteq A$. Therefore $A\Gamma A\Gamma S \subseteq A\Gamma R\Gamma S \subseteq A\Gamma R \subseteq A$. This shows that A is a (2, 0)-ideal of S. ■

Theorem 3.10. If A is a non-empty subset of a Γ-semiring S, then following statements are equivalent.
(1) A is a (2, 1)-ideal of S.
(2) A is a right ideal of some bi-ideal of S.
(3) A is a bi-ideal of some right ideal of S.
(4) A is a (2, 0)-ideal of some left ideal of S.
(5) A is a left ideal of some (2, 0)-ideal of S
Proof: (1) ⇒(2) Let A be a (2, 1)-ideal of S. Therefore $A\Gamma A\Gamma S\Gamma A \subseteq A$. We have $A\Gamma S\Gamma A$ is a bi-ideal of S. Hence $A\Gamma(A\Gamma S\Gamma A) \subseteq A$ implies A is a right ideal of some bi-ideal $A\Gamma S\Gamma A$ of S.
(2) ⇒(3) Suppose that A is a right ideal of some bi-ideal B of S. Hence $A\Gamma B \subseteq A$ and $B\Gamma S\Gamma B \subseteq B$. Therefore $A\Gamma(A\Gamma S)\Gamma A \subseteq A\Gamma(B\Gamma S\Gamma B) \subseteq A\Gamma B \subseteq A$. This shows that A is a bi-ideal of some right ideal $A\Gamma S$ of S.
(3) ⇒(4) Assume that A is a bi-ideal of some right ideal R of S. Hence $A\Gamma R\Gamma A \subseteq A$ and $R\Gamma S \subseteq R$. Therefore $A\Gamma A\Gamma(S\Gamma A) \subseteq A\Gamma(R\Gamma S)\Gamma A \subseteq A\Gamma R\Gamma A \subseteq A$. By Result 2.6, $S\Gamma A$ is a left ideal of S. Hence $A\Gamma A\Gamma(S\Gamma A) \subseteq A$ shows that A is a (2, 0)-ideal of some left ideal $S\Gamma A$ of S.
(4) ⇒(5) Suppose that A is a (2, 0)-ideal of some left ideal L of S. Hence $S\Gamma L \subseteq L$ and $A\Gamma A\Gamma L \subseteq A$. Therefore $A\Gamma A\Gamma S\Gamma A \subseteq A\Gamma A\Gamma(S\Gamma L) \subseteq A\Gamma A\Gamma L \subseteq A$. This shows that A is a left ideal of some (2, 0)-ideal $A\Gamma A\Gamma S$ of S.

(5) ⇒(1) Assume that A is a left ideal of some (2, 0)-ideal K of S. Hence $K\Gamma A \subseteq A$ and $K\Gamma K\Gamma S \subseteq K$. Therefore $A\Gamma A\Gamma S\Gamma A \subseteq K\Gamma K\Gamma S\Gamma A \subseteq K\Gamma A \subseteq A$. This shows that A is a (2, 1)-ideal of S. ■

Theorem 3.11. A subsemigroup A of a Γ-semiring S is a (2, 1)-ideal of S if and only if there exist a (2, 0)-ideal I and a left ideal L of S such that $I\Gamma I\Gamma L \subseteq A \subseteq L \cap I$.
Proof: Let A be a (2, 1)-ideal of S. Hence $A\Gamma A\Gamma S\Gamma A \subseteq A$. Let $I = A\Gamma A\Gamma S$ be a (2, 0)-ideal (see Theorem 3.6) and $L = S\Gamma A$ is a left ideal of S (see Result 2.6). Therefore $I\Gamma I\Gamma L = (A\Gamma A\Gamma S)\Gamma(A\Gamma A\Gamma S)\Gamma(S\Gamma A) \subseteq (A\Gamma A\Gamma S)\Gamma S\Gamma A \subseteq A\Gamma A\Gamma S\Gamma A \subseteq A$. Now $A\Gamma A\Gamma L = A\Gamma A\Gamma S\Gamma A \subseteq A$. This shows that A is a (2, 0) ideal of L. Then $I\Gamma A = A\Gamma A\Gamma S\Gamma A \subseteq A$. Hence A is a left ideal of I. Thus we get $A \subseteq L \cap I$. Therefore we have $I\Gamma I\Gamma L \subseteq A \subseteq L \cap I$. Conversely, $A\Gamma A\Gamma S\Gamma A \subseteq (L \cap I)\Gamma(L \cap I)\Gamma S\Gamma(L \cap I) \subseteq I\Gamma I\Gamma(S\Gamma L) \subseteq I\Gamma I\Gamma L \subseteq A$. Therefore A is a (2, 1)-ideal of S. ■

Theorem 3.12. Let R be a 0-minimal right ideal and A be a subsemigroup of S. Then A is a (2, 0)-ideal of S if and only if $A\Gamma A = \{0\}$ or $A = R$.
Proof: Let A be a (2, 0)-ideal of S. If $A\Gamma A = \{0\}$, then theorem holds. Suppose that $A\Gamma A \neq \{0\}$. Let $A \subseteq R$.We have $A\Gamma A\Gamma S$ is a right ideal of S (see Result 2.6). Hence $A\Gamma A\Gamma S \subseteq R\Gamma R\Gamma S \subseteq R$. As R is a 0-minimal right ideal of S and $A\Gamma A \neq \{0\}$, we have $A\Gamma A\Gamma S = R$. Therefore $R = A\Gamma A\Gamma S \subseteq A$. Thus we get $A = R$. ■

Theorem 3.13. If A is a 0-minimal (2, 0)-ideal of S,then $A\Gamma A = \{0\}$ or A is a 0-minimal right ideal S.
Proof: Let A be a 0-minimal (2, 0)-ideal of S. Hence $A\Gamma A\Gamma S \subseteq A$. If $A\Gamma A = \{0\}$, then theorem holds. Assume that $A\Gamma A \neq \{0\}$. By Theorem 3.6, $A\Gamma A\Gamma S$ is a (2, 0)-ideal of S. Therefore $A\Gamma A\Gamma S \subseteq A$ and A is a 0-minimal (2, 0)-ideal imply $A\Gamma A\Gamma S = A$. Let $R \neq \{0\}$ be a right ideal of S such that $R \subseteq A$. Hence R is a (2, 0)-ideal of S (see Theorem 3.3). But A is a 0-minimal (2, 0)-ideal implies $A = R$. Therefore A is a 0-minimal right ideal of S. ■

Theorem 3.14. Let S be a Γ-semiring without zero. Then A is a minimal (2, 0)-ideal of S if and only if A is a minimal right ideal of S.
Proof: Let A be a minimal (2, 0)-ideal of S. Hence $A\Gamma A\Gamma S \subseteq A$. Let R be a right ideal of S such that $R \subseteq A$. Therefore, by Theorem 3.3, R is a (2, 0)-ideal of S. As A is a minimal (2, 0)-ideal , we have $A = R$. Hence A is a minimal right ideal of S. Conversely, suppose that A is a minimal right ideal of S. Therefore A is a (2, 0)-ideal of S (see Theorem 3.3). Let B be a (2, 0)-ideal of S such that $B \subseteq A$. Then we have $B\Gamma B\Gamma S \subseteq B$. By Result 2.6, $B\Gamma B\Gamma S$ is a right ideal of S. Therefore $B\Gamma B\Gamma S \subseteq A$ and A is a minimal right ideal of S imply $B\Gamma B\Gamma S = A$. Hence $A \subseteq B$. Thus we get $A = B$. This shows that A is a minimal (2, 0)-ideal of S. ■

Theorem 3.15. Let S be a Γ-semiring without zero. Then A is a minimal (1, 2)-ideal of S if and only if A is a minimal bi-ideal of S.
Proof: Let A be a minimal (1, 2)-ideal of S. Therefore $A\Gamma S\Gamma A\Gamma A \subseteq A$. We know that $A\Gamma S\Gamma A\Gamma A$ is a (1, 2)-ideal of S.Hence $A\Gamma S\Gamma A\Gamma A \subseteq A$ and A is a minimal (1, 2)-ideal

imply $A\Gamma S\Gamma A\Gamma A = A$. Therefore $A\Gamma S\Gamma A = A\Gamma S\Gamma(A\Gamma S\Gamma A\Gamma A) \subseteq A\Gamma S\Gamma A\Gamma A = A$. This shows that A is a bi-ideal of S. Let B be a bi-ideal of S such that $B \subseteq A$. Then $B\Gamma S\Gamma B\Gamma B$ is a (1, 2)-ideal of S. Now $B\Gamma S\Gamma B\Gamma B \subseteq A\Gamma S\Gamma A\Gamma A = A$. Thus we get $B\Gamma S\Gamma B\Gamma B = A$. Therefore $A = B\Gamma S\Gamma B\Gamma B \subseteq B\Gamma S\Gamma B \subseteq B$. Hence we get $A = B$. Thus A is a minimal bi-ideal of S. Conversely, suppose that A is a minimal bi-ideal of S. By Theorem 3.4, A is a (1, 2)-ideal of S. Let B be a (1, 2)-ideal of S such that $B \subseteq A$. Therefore $B\Gamma S\Gamma B\Gamma B \subseteq B$. Clearly $B\Gamma S\Gamma B\Gamma B$ is a bi-ideal of S. Hence $B\Gamma S\Gamma B\Gamma B \subseteq A\Gamma S\Gamma A\Gamma A = A$ and A is a minimal bi-ideal of S imply $B\Gamma S\Gamma B\Gamma B = A$. Therefore $A \subseteq B$. Thus we get $A = B$. This shows that A is a minimal (1,2)-ideal of S. ■

4. (2, 0)-Bi-ideals

Definition 4.1. *A subsemigroup A of a Γ-semiring S is said to be a (2, 0)-bi-ideal of S if A is a bi-ideal of S and also A is a (2, 0)-ideal of S.*
That is a subsemigroup A of a Γ-semiring S is a (2, 0)-bi-ideal of S if $A\Gamma S\Gamma A \subseteq A$ and $A\Gamma A\Gamma S \subseteq A$.

Definition 4.2. *A (2, 0)-bi-ideal A of S is said to be a 0-minimal (2, 0)-bi-ideal of S if $A \neq \{0\}$ and $\{0\}$ is the only proper (2, 0)-bi-ideal of S contained in A.*

Definition 4.3. *S is said to be 0-(2, 0)-bi-simple Γ-semiring if $S\Gamma S \neq \{0\}$ and $\{0\}$ is the only proper (2, 0)-bi-ideal of S.*

Theorem 4.4. Let A be a non-empty subset of a Γ-semiring S. Then A is a (2, 0)-bi-ideal of S if and only if A is an ideal of some right ideal of S.

Proof: Let A be a (2, 0)-bi-ideal of S. Therefore, $A\Gamma A\Gamma S \subseteq A$ and $A\Gamma S\Gamma A \subseteq A$ by definition. $(A\Gamma S)\Gamma A = A\Gamma S\Gamma A \subseteq A$, shows that A is a left ideal of some right ideal $A\Gamma S$ of S. Now $A\Gamma(A\Gamma S) = A\Gamma A\Gamma S \subseteq A$, which shows that A is a right ideal of some right ideal $A\Gamma S$ of S. Therefore A is an ideal of some right ideal $A\Gamma S$ of S. Conversely, suppose A is an ideal of some right ideal R of S. Hence $A\Gamma R \subseteq A$, $R\Gamma A \subseteq A$ and $R\Gamma S \subseteq R$. Then we consider $A\Gamma A\Gamma S \subseteq A\Gamma(R\Gamma S) \subseteq A\Gamma R \subseteq A$. This shows that A is a (2, 0)-ideal of S.Now $A\Gamma S\Gamma A \subseteq R\Gamma S\Gamma A \subseteq R\Gamma A \subseteq A$, shows that A is a bi-ideal of S. Hence A is a (2, 0)-bi-ideal of S. ■

Theorem 4.5. If A is a 0-minimal (2, 0)-bi-ideal of S, then $A\Gamma A = \{0\}$ or $a\Gamma a\Gamma S = A$, for any $a \in A\backslash\{0\}$.

Proof: Let A be a 0-minimal (2, 0)-bi-ideal of S. Then we have $A\Gamma A\Gamma S \subseteq A$ and $A\Gamma S\Gamma A \subseteq A$. If $A\Gamma A = \{0\}$, then theorem holds. Assume that $A\Gamma A \neq \{0\}$. For any $a \in A\backslash\{0\}$, $a\Gamma a\Gamma S$ is a (2, 0)-bi-ideal of S. Hence $a\Gamma a\Gamma S \subseteq A$ and A is a 0-minimal (2, 0)-bi-ideal imply $a\Gamma a\Gamma S = A$. ■

Theorem 4.6. S is 0-(2, 0)-bi-simple if and only if $a\Gamma a\Gamma S = S$, for any $a \in S\backslash\{0\}$.

Proof: Let S be a 0- (2, 0)-bi-simple Γ-semiring. For any $a \in S\backslash\{0\}$, $a\Gamma a\Gamma S$ is a (2, 0)-bi-ideal of S. $a\Gamma a\Gamma S \subseteq S$ and S is a 0-(2, 0)-bi-simple Γ-semiring imply $a\Gamma a\Gamma S = S$. Conversely, suppose that $a\Gamma a\Gamma S = S$, for any $a \in S\backslash\{0\}$. Let A be a non zero (2, 0)-bi-ideal of S.Then for any $a \in A\backslash\{0\}$, $a\Gamma a\Gamma S$ is a (2, 0)-bi-ideal of S. By assumption

$a\Gamma a\Gamma S = S$. Therefore $S = a\Gamma a\Gamma S \subseteq A\Gamma A\Gamma S \subseteq A$. Hence we get $S = A$. Thus S is a 0-(2, 0)-bi-simple Γ-semiring. ∎

Theorem 4.7. S is 0-(2, 0)-bi-simple if and only if S is right 0-simple.

Proof: Let S be a 0- (2, 0)-bi-simple Γ-semiring. By Theorem3.3, every right ideal of S is a (2, 0)-bi-ideal of S.Therefore S is 0- (2, 0)-bi-simple implies S is right 0-simple. Conversely, suppose that S is a right 0-simple. For any $a \in S\backslash\{0\}$, $a\Gamma S$ is a non zero right ideal of S (see Result 2.7). As S is right 0-simple, we have $a\Gamma S = S$. Now $a\Gamma a\Gamma S = a\Gamma S = S$. Hence by Theorem 4.6, S is a 0-(2,0)-bi-simple Γ-semiring. ∎

Theorem 4.8. If A is a 0-minimal (2, 0)-bi-ideal of S, then $A\Gamma A = \{0\}$ or A is right 0-simple.

Proof: Let A be a 0-minimal (2, 0)-bi-ideal of a Γ-semiring S. Hence $A\Gamma A\Gamma S \subseteq A$ and $A\Gamma S\Gamma A \subseteq A$. If $A\Gamma A = \{0\}$, then theorem holds. Assume that $A\Gamma A \neq \{0\}$.Then by Theorem 4.5, $a\Gamma a\Gamma S = A$, for any $a \in A\backslash\{0\}$. For $a \in A\backslash\{0\}$, $(a\Gamma a\Gamma A)\Gamma(a\Gamma a\Gamma A)\Gamma S \subseteq a\Gamma a\Gamma A\Gamma(A\Gamma S\Gamma A)\Gamma S \subseteq a\Gamma a\Gamma A\Gamma A\Gamma S \subseteq a\Gamma a\Gamma A$ .Hence $a\Gamma a\Gamma A$ is a (2, 0)-ideal of S. Also $(a\Gamma a\Gamma A)\Gamma S\Gamma(a\Gamma a\Gamma A) \subseteq a\Gamma a\Gamma(A\Gamma S\Gamma A)\Gamma a\Gamma A \subseteq a\Gamma a\Gamma A\Gamma a\Gamma A \subseteq a\Gamma a\Gamma A$. Therefore $a\Gamma a\Gamma A$ is a bi-ideal of S. Hence $a\Gamma a\Gamma A$ is a (2, 0)-bi-ideal of S and $a\Gamma a\Gamma A \subseteq A$. Therefore $a\Gamma a\Gamma A = A$. Hence by Theorems 4.6 and 4.7, A is right 0-simple. ∎

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