Progress in Nonlinear Dynamics and Chaos
Vol. 5, No. 1, 2017, 17-23
ISSN: 2321 – 9238 *(online)*
Published on 22 April 2017
www.researchmathsci.org
DOI: http://dx.doi.org/10.22457/pindac.v5n1a3

Progress in Nonlinear Dynamics and Chaos

An Efficient Approach for Finding an Initial Basic Feasible Solution for Transportation Problems

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Received 14 March 2017; accepted 1 April 2017

Abstract: Transportation problem is famous in operation research for its wide application in real life. This is a special kind of the network optimization problems in which goods are transported from a set of sources to a set of destinations subject to the supply and demand of the source and destination, respectively, such that the total cost of transportation is minimized. Finding an initial basic feasible solution is the prime requirement to obtain an optimal solution for the transportation problems. In this article, a new approach named '**Row-Column's Divided Method'** is proposed to find an initial basic feasible solution for the transportation problems. The method is also illustrated with numerical examples and comparison of the results obtained by various methods.

Keywords: Transportation problem, initial basic feasible solution, row-column's divided method, optimal solution.

1. Introduction

This is a type of linear programming problem that may be solved using a simplified version of the simplex technique called transportation method. Because of its major application in solving problems involving several product sources and several destinations of products, this type of problem is frequently called the transportation problem**.**

Transportation problem was first formulated by Hitchcook [1] Charnes et al. [2] Appa [3] Klingman and Russel [4] developed further the basic transportation problem. Basically, the papers of Charnes and Klingman [5] and Szwarc [6] are treated as the sources of transportation paradox for the researchers. In the paper of Charnes and Klingman, they name it "morefor-less" paradox and wrote "The paradox was first observed in the early days of linear programming history (by whom no one knows) and has been a part of the folklore known to some (e.g. Charnes and Cooper), but unknown to the great majority of workers in the field of linear programming". Subsequently, in the paper of Appa, he mentioned that this paradox is known as "Doig Paradox" at the London School of Economics, named after Alison Doig. Gupta et al. [7] established a sufficient condition for a paradox in a linear fractional transportation problem with mixed constraints. Adlakha and Kowalski [8] derived a sufficient condition to identify the cases where the paradoxical situation exists. Ryan [9] developed a goal programming approach to the representation and resolution of the more for less and more for nothing paradoxes in the distribution problem. Deineko et al. [10] developed a necessary and sufficient condition for a cost matrix which is immune against the transportation paradox. Dahiya and Verma [11] considered paradox in a nonlinear capacitated transportation problem. Adlakha et al. [12] developed a simple heuristic algorithm to identify the demand destinations and the supply points to ship more for less in fixed-charge transportation problems. Storoy [13] considered the classical transportation problem and studied the occurrence of the so-called transportation paradox (also called the more-for-less paradox). Joshi and Gupta [14] studied an efficient heuristic algorithm for solving more-for-less paradox and algorithm for finding the initial basic feasible solution for linear plus linear fractional transportation problem.

2. Proposed approach to find an initial basic feasible solution

We, now introduce a new method called '**Row-Column's Divided Method'** for finding an initial basic feasible solution to a transportation problem. The Row-Columns Divided Method' proceeds as follows:

Step 1: Construct a Transportation Table (TT) from the given transportation problem.

Step 2: Ensure whether the TP is balanced or not, if not, make it balanced.

Step 3: Divide each row entries of the transportation table by the respective row minimum and then divide each column entries of the resulting transportation table by respective column minimum.

Step 4: Write the round figure of 1.(write 1 for 1-1.74 and 2 for 1.75-1.99). Now there will be at least one 1 in each row and in each column in the reduced cost matrix.

Step 5: Select the first 1 (row-wise) occurring in the cost matrix. Suppose $(i, j)^{th}$ one(1) is selected, count the total number of one's (excluding the selected one) in the i^{th} row and j^{th} column. Now select the next 1 and count the total number of 1 in the corresponding row and column in the same way. Continue it for all 1's in the cost matrix.

Step 6: Now choose a 1 for which the number of 1 counted in step 5 is minimum and supply maximum possible amount to that cell. If tie occurs for some 1's in step 5 then choose a $(m,n)^{th}$ 1 breaking tie such that the total sum of all the elements in the n^{th} column (excluding the selected cell) in main cost matrix is maximum. Allocate maximum possible amount to that cell (row-wise).

Step 7: After performing step 6, delete the row or column for further calculation where the supply from a given source is depleted or the demand for a given destination is satisfied.

Step 8: Check whether the resultant matrix possesses at least one 1 in each row and in each column. If not, repeat step 3, otherwise go to step 9.

Step 9: Repeat step 4 to step 8 until and unless all the demands are satisfied and all the supplies are exhausted.

3. Numerical example (1) with illustration

Consider the following cost minimizing transportation problem with three origins and four destinations

	Destinations				
Origins	**A**	**B**	**C**	**D**	Supply, a_i
1	4	3	2	5	**6**
2	6	1	4	3	**9**
3	7	2	4	6	**7**
Demand, b_j	**4**	**6**	**6**	**6**	**22**

Solution. Finding initial basic feasible solution according to Row-Columns Divided Method:

Step 1: Divide each row entries of the transportation table by the respective row minimum.

	Destinations				
Origins	**A**	**B**	**C**	**D**	Supply a_i
1	2	1.5	1	2.5	6
2	6	1	4	3	9
3	3.5	1	2	3	7
Demand, b_j	4	6	6	6	**22**

Table 1: Row operation

Step 2: Divide each column entries of the transportation table by the respective column minimum.

	Destinations				
Origins	**A**	**B**	**C**	**D**	Supply a_i
1	1	1.5	1	1	**6**
2	3	1	4	1.2	**9**
3	1.75	1	2	1.2	**7**
Demand, b_j	**4**	**6**	**6**	**6**	**22**

Table 2: Column operation

Step 3: **W**rite 1 for 1-1.74 and 2 for 1.75-1.99

	Destinations				
Origins	**A**	**B**	**C**	**D**	Supply a_i
1	1	1	1	1	**6**
2	3	1	4	1	**9**
3	2	1	2	1	**7**
Demand, b_j	**4**	**6**	**6**	**6**	**22**

Table 3: Rounding 1

Step 4: Now allocated all rim requirements

	Destinations				
Origins	**A**	**B**	**C**	**D**	Supply a_i
1	$\boxed{4}$ 1	1	$\boxed{2}$ 1	1	**6**
2	3	**1** $\boxed{3}$	4	1 $\boxed{6}$	**9**
3	2	$\boxed{3}$ 1	$\boxed{4}$ 2	1	**7**
Demand, b_j	**4**	**6**	**6**	**6**	**22**

Table 4: Final row-column divided allocation

Illustration

- In first row, $(1,A)^{th}$1has the minimum 1 in the corresponding row and columns. So select it and set x_{1A} = min(4,6) = 4 and allocate it in the cell (1,A), therefore demand of destination A is satisfied completely.
- Next Select $(1,C)^{th}$1has the next minimum 1 in the corresponding row and columns. So select it and set x_{1C} = min(2,6) = 2 and allocate it in the cell (1,C); therefore the capacity of 1 row is exhausted.
- In 2^{nd} row there are two 1 in the cell (2,B) and (2,D) respectively, both cell have same number of 1 in the corresponding row and columns but for cell (2,D): total sum of all the elements in the D column (excluding the selected cell) in main cost matrix is maximum. So set X_{2D}= min(9,6)= 6 and allocate it in the cell (2,D);), therefore demand of destination D is satisfied completely.
- Next set X_{2B}= min(3,6)= 3 and allocate it in the cell (2,B),therefore the capacity of 2 row is exhausted.
- Similarly next set X_{3B}= min(0,3)= 3 and allocate it in the cell (3,B),therefore B is satisfied.
- Next set X_{3C}= min(4,4)= 4 and allocate it in the cell (3,C),therefore the capacity of 3 row is exhausted and also all rim requirements are satisfied.

Here the number of basic variable is 6=(3+4-1) which is satisfied the condition (m+n-1)
Total transportation cost is = $(4\times 4 + 2 \times 2 + 1 \times 3 + 3 \times 6+ 2 \times 3 + 4\times 4)$= \$ 63

4. Numerical example (2) without illustration

Consider the following cost minimizing transportation problem with three origins and four destinations.

	Destinations				
Origins	**A**	**B**	**C**	**D**	Supply a_i
O_1	13	18	30	8	**8**
O_2	55	20	25	40	**10**
O_3	30	6	50	10	**11**
Demand, b_j	**4**	**7**	**6**	**12**	**29**

Solution: Initial basic feasible solution according to row-columns divided method:

		Destinations			
Origins	**A**	**B**	**C**	**D**	Supply a_i
O_1	$\boxed{4}$ 1	2.25	3	$\boxed{4}$ 1	**8**
O_2	1	$\boxed{4}$ 1	$\boxed{6}$ 1	2	**10**
O_3	3.07	$\boxed{3}$ 1	6.67	$\boxed{8}$ 1	**11**
Demand, b_j	**4**	**7**	**6**	**12**	**29**

Table 5: Final row-column divided allocation

- Total cost is = (13*4) + (8*4) + (20*4) + (25*6) + (6*3) + (10*8)
  = 52 + 32 + 80 + 150 + 18+ 80
  = $ 412

5. Result analysis and comparison

After obtaining an IBFS by the proposed "**Row Column's Divided Method**", the obtained result is compared with the results obtained by other existing methods is shown in the following table:

Methods	**Results**	
	Example1	Example 2
North-west Corner Method	86	484
Row Minimum Method	73	589
Column Minimum Method	73	476
Least Cost Method	78	516
Vogel's Approximation Method	73	475
New Proposed Method	**63**	**412**

Table 6: Comparison of the results obtained by various methods

As observed from **Table 6,** the proposed allocation table method provides comparatively a better initial basic feasible solution than the results obtained by the traditional algorithms which are either optimal or near to optimal. Again the performance of the solution varies for other methods which may happen also in case of the proposed method.

6. Conclusion

In this article, a new approach named '**Row- Column's Divided Method'** for finding an initial basic feasible solution of transportation problems is proposed. Efficiency of allocation table method has also been tested by solving several number of cost minimizing transportation problems and it is found that the allocation table method yields comparatively a better result. Finally it can be claimed that the allocation table method may provide a remarkable initial basic feasible solution by ensuring minimum transportation cost. This will help to achieve the goal to those who want to maximize their profit by minimizing the transportation cost.

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